



Mathematics

Grade 7

WORKED SOLUTIONS

Worked solutions – the question, the working, then the answer

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Contents

Mathematics Grade 7 – every exercise worked through, with the question, the working and the answer.

1	Rational Numbers and Decimal Numbers	3
2	Simplifications	11
3	Square and Square Roots	16
4	Financial Mathematics	21
5	Rate, Ratio and Proportion	32
6	Sets	43
7	Number Sequences and Patterns	55
8	Algebraic Expressions	64
9	Linear Equations	84
10	Speed, Distance and Time	98
11	Perimeter and Area	111
12	Volume and Surface Area	120
13	Practical Geometry and Angle Properties of Polygons	126
14	Transformation	137
15	Statistics and Probability	144

1

CHAPTER

Rational Numbers and Decimal Numbers

EXERCISE 1.1

1. Write the absolute value of the following integers: (i) -9 (ii) $+5$ (iii) -20 (iv) 0

The absolute value of a number is its distance from zero on the number line. Distance is never negative, so the answer can never be negative.

In practice the sign is simply dropped.

(i) $|-9|$

-9 is 9 steps from zero

$= 9$

(ii) $|+5|$

$+5$ is 5 steps from zero

$= 5$

(iii) $|-20|$

-20 is 20 steps from zero

$= 20$

note that -20 is the smaller number but has the larger absolute value

(iv) $|0|$

zero is no distance from itself

$= 0$

0 is the only number whose absolute value is zero

Answer (i) 9 (ii) 5 (iii) 20 (iv) 0

2. Represent the following on the number line: (i) $-100, -50, 0, 25, 50$ (ii) $5.9, -4.9, 7.9, 5, -6$ (iii) $345, 445, 545, 645$ (iv) $535, 465, 456, 246, 500$

Choose the scale from the spread of the numbers before drawing anything — that is the whole skill here.

(i) runs from -100 to 50 , so 25 to a division suits it; mark 0 in the middle with the negatives to its left.

(ii) needs decimals, so use 1 to a division and estimate the tenths between marks: -6 and -4.9 to the left, $5, 5.9$ and 7.9 to the right.

(iii) the numbers are 100 apart, so 100 to a division and there is no need to start the line at zero — begin at 300 .

(iv) 246 to 535 is a narrower band; 50 to a division lets 456 and 465 be told apart, which 100 to a division would not.

Answer Four number lines, each with the scale chosen to fit its own spread of values.

3. Compare using $<$, $>$ or $=$, and arrange in ascending and descending order: (i) $8.1325, 8.1234, 8.1350, 8.1200$ (ii) $15.2569, 15.2658, 15.2597, 15.2575$ (iii) $1302, 1215, 1045, 1233$ (iv) $9058, 736, 8345, 9200$ (v) $6.7843, 6.7832, 6.7825, 6.7900$ (vi) $10.01, 1023, 990, 1010$

For decimals, compare place by place from the left, stopping at the first place where they differ. In

(i) all four begin 8.1 , so the decision is made at the third and fourth decimal places.

Do not judge by how many digits a decimal has: 8.1200 is not larger than 8.12 , they are equal.

For whole numbers, the one with more digits is larger — in (iv) 736 has three digits and the rest have four, so it is smallest.

(vi) mixes a small decimal with three whole numbers: 10.01 is the smallest by far, which is easy to miss when the others are near 1000 .

(i) ascending: $8.1200 < 8.1234 < 8.1325 < 8.1350$

(ii) ascending: $15.2569 < 15.2575 < 15.2597 < 15.2658$

- (iii) ascending: $1045 < 1215 < 1233 < 1302$
 (iv) ascending: $736 < 8345 < 9058 < 9200$
 (v) ascending: $6.7825 < 6.7832 < 6.7843 < 67900$
 (vi) ascending: $10.01 < 990 < 1010 < 1023$

Answer (i) $8.1200 < 8.1234 < 8.1325 < 8.1350$
 descending: $8.1350 > 8.1325 > 8.1234 > 8.1200$
 (ii) $15.2569 < 15.2575 < 15.2597 < 15.2658$
 descending: $15.2658 > 15.2597 > 15.2575 > 15.2569$
 (iii) $1045 < 1215 < 1233 < 1302$
 descending: $1302 > 1233 > 1215 > 1045$
 (iv) $736 < 8345 < 9058 < 9200$
 descending: $9200 > 9058 > 8345 > 736$
 (v) $6.7825 < 6.7832 < 6.7843 < 67900$
 descending: $67900 > 6.7843 > 6.7832 > 6.7825$
 (vi) $10.01 < 990 < 1010 < 1023$
 descending: $1023 > 1010 > 990 > 10.01$

Please check — the book prints 'iv' twice in this question, so its six parts are lettered i to vi above. In part (v) the value 67900 sits among numbers close to 6.78 and is almost certainly 6.7900 with the decimal point lost; on that reading the order is $6.7825 < 6.7832 < 6.7843 < 6.7900$.

EXERCISE 1.2

1. Represent the following rational numbers on the number line: (i) $\frac{3}{2}$ (ii) $-\frac{5}{6}$ (iii) $-\frac{3}{8}$ (iv) $-\frac{2}{5}$ (v) $\frac{4}{1}$ (vi) $-\frac{7}{10}$

The denominator tells you how many equal parts to cut each unit into; the numerator tells you how many to count off.

- (i) $\frac{3}{2} = 1\frac{1}{2}$, so halve each unit and count three halves to the right of zero.
 (ii) $-\frac{5}{6}$: cut each unit into sixths and count five to the left of zero. It lies between -1 and 0 , nearer -1 .
 (iii) $-\frac{3}{8}$ and (iv) $-\frac{2}{5}$ both lie between -1 and 0 ; $-\frac{3}{8} = -0.375$ is nearer zero than $-\frac{2}{5} = -0.4$.
 (v) $\frac{4}{1}$ is just 4 — a whole number is still a rational number, with denominator 1 .
 (vi) $-\frac{7}{10} = -0.7$, seven tenths to the left of zero.

Answer Six points: $1\frac{1}{2}$ and 4 to the right of zero; $-\frac{5}{6}$, $-\frac{3}{8}$, $-\frac{2}{5}$ and $-\frac{7}{10}$ to the left, all between -1 and 0 .

2. Convert: (i) $1\frac{5}{6}$, $2\frac{5}{6}$, $8\frac{5}{6}$ into improper fractions (ii) $\frac{45}{8}$, $\frac{35}{8}$, $\frac{13}{7}$ into mixed fractions

To make a mixed number improper: multiply the whole number by the denominator, add the numerator, and keep the same denominator.

- (i) $1\frac{5}{6} = (1 \times 6 + 5)/6 = 11/6$; $2\frac{5}{6} = (2 \times 6 + 5)/6 = 17/6$; $8\frac{5}{6} = (8 \times 6 + 5)/6 = 53/6$.

To go the other way, divide the numerator by the denominator: the quotient is the whole number and the remainder is the new numerator.

- (ii) $45 \div 8 = 5$ remainder 5 , so $\frac{45}{8} = 5\frac{5}{8}$; $35 \div 8 = 4$ remainder 3 , so $\frac{35}{8} = 4\frac{3}{8}$; $13 \div 7 = 1$ remainder 6 , so $\frac{13}{7} = 1\frac{6}{7}$.

Answer (i) $\frac{11}{6}$, $\frac{17}{6}$, $\frac{53}{6}$ (ii) $5\frac{5}{8}$, $4\frac{3}{8}$, $1\frac{6}{7}$

3. Put the correct sign ($<$, $>$ or $=$) between: (i) $\frac{3}{4}$ and $\frac{2}{3}$ (ii) $-\frac{5}{6}$ and $-\frac{3}{4}$ (iii) $\frac{5}{8}$ and $\frac{6}{8}$ (iv) $\frac{7}{10}$ and $\frac{2}{5}$ (v) $-\frac{1}{3}$ and $-\frac{1}{3}$ (vi) $-\frac{10}{12}$ and $-\frac{9}{12}$

Fractions can only be compared once they share a denominator, so rewrite each pair over the lowest common denominator first.

With negatives, be careful: the one further from zero is the smaller. $-\frac{5}{6}$ is less than $-\frac{3}{4}$, even though $\frac{5}{6}$ is more than $\frac{3}{4}$.

(i) over 12: $\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{8}{12}$, so $\frac{3}{4} > \frac{2}{3}$.

(ii) over 12: $-\frac{5}{6} = -\frac{10}{12}$ and $-\frac{3}{4} = -\frac{9}{12}$, so $-\frac{5}{6} < -\frac{3}{4}$.

(iii) over 8: $\frac{5}{8} = \frac{5}{8}$ and $\frac{6}{8} = \frac{6}{8}$, so $\frac{5}{8} < \frac{6}{8}$.

(iv) over 10: $\frac{7}{10} = \frac{7}{10}$ and $\frac{2}{5} = \frac{4}{10}$, so $\frac{7}{10} > \frac{2}{5}$.

(v) over 3: $-\frac{1}{3} = -\frac{1}{3}$ and $-\frac{1}{3} = -\frac{1}{3}$, so $-\frac{1}{3} = -\frac{1}{3}$.

(vi) over 12: $-\frac{10}{12} = -\frac{10}{12}$ and $-\frac{9}{12} = -\frac{9}{12}$, so $-\frac{10}{12} < -\frac{9}{12}$.

Answer (i) $\frac{3}{4} > \frac{2}{3}$ (ii) $-\frac{5}{6} < -\frac{3}{4}$ (iii) $\frac{5}{8} < \frac{6}{8}$ (iv) $\frac{7}{10} > \frac{2}{5}$ (v) $-\frac{1}{3} = -\frac{1}{3}$ (vi) $-\frac{10}{12} < -\frac{9}{12}$

4. Compare and arrange in ascending and descending order: (i) $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$ (ii) $\frac{2}{5}$, $\frac{4}{7}$, $\frac{3}{8}$ (iii) $-\frac{3}{10}$, $\frac{1}{2}$, $-\frac{5}{12}$ (iv) $-\frac{7}{10}$, $-\frac{5}{6}$, $\frac{1}{4}$ (v) $\frac{9}{11}$, $\frac{8}{9}$, $\frac{7}{10}$ (vi) $\frac{11}{14}$, $\frac{3}{5}$, $\frac{7}{8}$

Put each group over a common denominator, then order the numerators.

(i) over 12: $\frac{10}{12}$, $\frac{9}{12}$, $\frac{8}{12}$.

(ii) over 280: $\frac{112}{280}$, $\frac{160}{280}$, $\frac{105}{280}$.

(iii) and (iv) contain negatives, which always come before the positives, and among themselves the one further from zero comes first.

(v) and (vi) are quicker as decimals: $\frac{9}{11} \approx 0.818$, $\frac{8}{9} \approx 0.889$, $\frac{7}{10} = 0.7$; and $\frac{11}{14} \approx 0.786$, $\frac{3}{5} = 0.6$, $\frac{7}{8} = 0.875$.

Answer (i) ascending $\frac{2}{3} < \frac{3}{4} < \frac{5}{6}$

descending $\frac{5}{6} > \frac{3}{4} > \frac{2}{3}$

(ii) ascending $\frac{3}{8} < \frac{2}{5} < \frac{4}{7}$

descending $\frac{4}{7} > \frac{2}{5} > \frac{3}{8}$

(iii) ascending $-\frac{5}{12} < -\frac{3}{10} < \frac{1}{2}$

descending $\frac{1}{2} > -\frac{3}{10} > -\frac{5}{12}$

(iv) ascending $-\frac{5}{6} < -\frac{7}{10} < \frac{1}{4}$

descending $\frac{1}{4} > -\frac{7}{10} > -\frac{5}{6}$

(v) ascending $\frac{7}{10} < \frac{9}{11} < \frac{8}{9}$

descending $\frac{8}{9} > \frac{9}{11} > \frac{7}{10}$

(vi) ascending $\frac{3}{5} < \frac{11}{14} < \frac{7}{8}$

descending $\frac{7}{8} > \frac{11}{14} > \frac{3}{5}$

5. Verify the following properties for rational numbers: (i) $\frac{2}{3} + (\frac{1}{4} + \frac{3}{5}) = (\frac{2}{3} + \frac{1}{4}) + \frac{3}{5}$ (ii) $\frac{1}{2} + (\frac{3}{4} + \frac{5}{6}) = (\frac{1}{2} + \frac{3}{4}) + \frac{5}{6}$ (iii) $\frac{5}{8} \times (\frac{7}{9} \times \frac{2}{3}) = (\frac{5}{8} \times \frac{7}{9}) \times \frac{2}{3}$ (iv) $8 \times (10 \times 5) = (8 \times 10) \times 5$ (v) $\frac{5}{8} \times (4 - \frac{1}{2}) = (\frac{5}{8} \times 4) - (\frac{5}{8} \times \frac{1}{2})$ (vi) $\frac{9}{11} \times (3 - \frac{2}{5}) = (\frac{9}{11} \times 3) - (\frac{9}{11} \times \frac{2}{5})$ (vii) $\frac{7}{9} \times (\frac{5}{6} + \frac{2}{3}) = (\frac{7}{9} \times \frac{5}{6}) + (\frac{7}{9} \times \frac{2}{3})$ (viii) $\frac{4}{7} \times (6 + \frac{2}{3}) = (\frac{4}{7} \times 6) + (\frac{4}{7} \times \frac{2}{3})$ (ix) $\frac{5}{7} + \frac{3}{4} = \frac{3}{4} + \frac{5}{7}$ (x) $-\frac{3}{5} + \frac{6}{7} = \frac{6}{7} + (-\frac{3}{5})$

To verify an identity, work out each side separately and only then compare. Simplifying across the equals sign assumes the very thing you are meant to be checking.

- (i) LHS = $\frac{91}{60}$, RHS = $\frac{91}{60}$ — equal, so the associative, addition property holds.
 (ii) LHS = $\frac{25}{12}$, RHS = $\frac{25}{12}$ — equal, so the associative, addition property holds.
 (iii) LHS = $\frac{35}{108}$, RHS = $\frac{35}{108}$ — equal, so the associative, multiplication property holds.
 (iv) LHS = 400, RHS = 400 — equal, so the associative, multiplication property holds.
 (v) As printed the two sides use different numbers — $\frac{5}{8}$ on the left but $\frac{8}{5}$ on the right. LHS = $\frac{35}{16}$ but RHS = $\frac{28}{5}$, so it does not balance. Reading both as $\frac{5}{8}$: RHS = $\frac{35}{16}$ = LHS, and the distributive property holds.
 (vi) LHS = $\frac{117}{55}$, RHS = $\frac{117}{55}$ — equal, so the distributive over subtraction property holds.
 (vii) LHS = $\frac{7}{6}$, RHS = $\frac{7}{6}$ — equal, so the distributive over addition property holds.
 (viii) LHS = $\frac{80}{21}$, RHS = $\frac{80}{21}$ — equal, so the distributive over addition property holds.
 (ix) LHS = $\frac{41}{28}$, RHS = $\frac{41}{28}$ — equal, so the commutative, addition property holds.
 (x) LHS = $\frac{9}{35}$, RHS = $\frac{9}{35}$ — equal, so the commutative, addition property holds.

Answer All verify except (v), which does not balance as printed. (i) $\frac{91}{60}$ (ii) $\frac{25}{12}$ (iii) $\frac{35}{108}$ (iv) 400 (v) $\frac{35}{16}$ on both sides once corrected (vi) $\frac{117}{55}$ (vii) $\frac{7}{6}$ (viii) $\frac{80}{21}$ (ix) $\frac{41}{28}$ (x) $\frac{9}{35}$

Please check — part (v) is misprinted. Its left side uses $\frac{5}{8}$ but its right side uses $\frac{8}{5}$, so the two sides come to $\frac{35}{16}$ and $\frac{28}{5}$ — the identity is false as written. With $\frac{5}{8}$ on both sides it balances at $\frac{35}{16}$, which is plainly what was intended.

EXERCISE 1.3

1. Round off the given decimals to 3 decimal places: (i) 2.45678 (ii) 56.78912 (iii) 0.34567 (iv) 78.95123 (v) 45.67891 (vi) 00.58647 (vii) 0.290590 (viii) 4.59876 (ix) 65.12349

Find the third decimal place, then look at the digit immediately after it — and only that one digit. If it is 5 or more, add 1 to the third place; if less than 5, leave the third place alone. Either way, everything after it goes.

Always judge from the original number — never round the fourth digit up first and then carry.

- (i) 2.45678
 2.456 | 78 — cut after 3 decimal places
 the next digit is 7, which is 5 or more, so add 1 to the last kept place
 = 2.457
 (ii) 56.78912
 56.789 | 12 — cut after 3 decimal places
 the next digit is 1, which is less than 5, so the last kept place stays
 = 56.789
 (iii) 0.34567
 0.345 | 67 — cut after 3 decimal places
 the next digit is 6, which is 5 or more, so add 1 to the last kept place
 = 0.346
 (iv) 78.95123
 78.951 | 23 — cut after 3 decimal places
 the next digit is 2, which is less than 5, so the last kept place stays
 = 78.951
 (v) 45.67891
 45.678 | 91 — cut after 3 decimal places
 the next digit is 9, which is 5 or more, so add 1 to the last kept place
 = 45.679

(vi) 00.58647

00.586 | 47 — cut after 3 decimal places

the next digit is 4, which is less than 5, so the last kept place stays
= 0.586

(vii) 0.290590

0.290 | 590 — cut after 3 decimal places

the next digit is 5, which is 5 or more, so add 1 to the last kept place
= 0.291

(viii) 4.59876

4.598 | 76 — cut after 3 decimal places

the next digit is 7, which is 5 or more, so add 1 to the last kept place
= 4.599

(ix) 65.12349

65.123 | 49 — cut after 3 decimal places

the next digit is 4, which is less than 5, so the last kept place stays
= 65.123

Answer (i) 2.457 (ii) 56.789 (iii) 0.346 (iv) 78.951 (v) 45.679 (vi) 0.586 (vii) 0.291 (viii) 4.599
(ix) 65.123

2. Round off to 3 significant figures: (i) 345.6789 (ii) 234.567 (iii) 567.8912 (iv) 0.0567 (v) 9876.543 (vi) 0.0012 (vii) 4502.6700 (viii) 789.0123 (ix) 0.000987

Significant figures are counted from the first non-zero digit, wherever it sits. Leading zeros never count.

Having found the first three, look at the next digit and round by the same 5-or-more rule.

Zeros added afterwards to keep the size of the number are placeholders, not significant figures.

(i) 345.6789

counting from the first non-zero digit, the first 3 significant figures are 3, 4, 5
the next digit is 6, so round the last one up
= 346

(ii) 234.567

counting from the first non-zero digit, the first 3 significant figures are 2, 3, 4
the next digit is 5, so round the last one up
= 235

(iii) 567.8912

counting from the first non-zero digit, the first 3 significant figures are 5, 6, 7
the next digit is 8, so round the last one up
= 568

(iv) 0.0567

counting from the first non-zero digit, the first 3 significant figures are 5, 6, 7
there is no further digit, so it is already at this accuracy
= 0.0567

(v) 9876.543

counting from the first non-zero digit, the first 3 significant figures are 9, 8, 7
the next digit is 6, so round the last one up
= 9880

(vi) 0.0012

counting from the first non-zero digit there are only 2 significant figures here: 1, 2
a zero is written on the end to show the number is quoted to 3 of them
= 0.00120

(vii) 4502.6700

counting from the first non-zero digit, the first 3 significant figures are 4, 5, 0
the next digit is 2, so the last one stays
= 4500

(viii) 789.0123

counting from the first non-zero digit, the first 3 significant figures are 7, 8, 9
the next digit is 0, so the last one stays
= 789

(ix) 0.000987

counting from the first non-zero digit, the first 3 significant figures are 9, 8, 7
there is no further digit, so it is already at this accuracy
= 0.000987

Answer (i) 346 (ii) 235 (iii) 568 (iv) 0.0567 (v) 9880 (vi) 0.00120 (vii) 4500 (viii) 789 (ix) 0.000987

3. Estimate the difference or sum by rounding off to 2 decimal places first: (i) $65.431 - 32.654$ (ii) $23.567 - 78.912$ (iii) $34.789 - 12.345$ (iv) $54.876 - 2.563$ (v) $102.345 - 67.567$ (vi) $8.123 + 5.785$ (vii) $12.356 + 45.234$ (viii) $7.479 + 22.556$ (ix) $19.876 + 13.442$

Round each number to 2 decimal places first, then add or subtract the rounded values — that is what makes it an estimate rather than an exact answer.

In (ii) the smaller number is being taken from the larger one's left, so the answer is negative:
 $23.57 - 78.91 = -55.34$.

(iii), (v) and (vi) each have a digit 5 exactly at the cut, so they round up: $12.345 \rightarrow 12.35$, $102.345 \rightarrow 102.35$, $5.785 \rightarrow 5.79$.

(i) $65.431 \rightarrow 65.43$ and $32.654 \rightarrow 32.65$, so $65.43 - 32.65 = 32.78$

(ii) $23.567 \rightarrow 23.57$ and $78.912 \rightarrow 78.91$, so $23.57 - 78.91 = -55.34$

(iii) $34.789 \rightarrow 34.79$ and $12.345 \rightarrow 12.35$, so $34.79 - 12.35 = 22.44$

(iv) $54.876 \rightarrow 54.88$ and $2.563 \rightarrow 2.56$, so $54.88 - 2.56 = 52.32$

(v) $102.345 \rightarrow 102.35$ and $67.567 \rightarrow 67.57$, so $102.35 - 67.57 = 34.78$

(vi) $8.123 \rightarrow 8.12$ and $5.785 \rightarrow 5.79$, so $8.12 + 5.79 = 13.91$

(vii) $12.356 \rightarrow 12.36$ and $45.234 \rightarrow 45.23$, so $12.36 + 45.23 = 57.59$

(viii) $7.479 \rightarrow 7.48$ and $22.556 \rightarrow 22.56$, so $7.48 + 22.56 = 30.04$

(ix) $19.876 \rightarrow 19.88$ and $13.442 \rightarrow 13.44$, so $19.88 + 13.44 = 33.32$

Answer (i) 32.78 (ii) -55.34 (iii) 22.44 (iv) 52.32 (v) 34.78 (vi) 13.91 (vii) 57.59 (viii) 30.04 (ix) 33.32

REVISION EXERCISE 1

1. Choose the correct option. (i) Which of the following is an integer? (a) 0.5 (b) -2 (c) 3.14 (d) $\frac{1}{3}$ (ii) What is the place value of 7 in 2.789? (a) 70 (b) 700 (c) 0.07 (d) 0.7 (iii) Which represents a rational number? (a) 3.9 (b) π (c) $\frac{1}{3}$ (d) 4.2 (iv) 15.789 rounded to 2 decimal places? (a) 15.78 (b) 15.79 (c) 16.00 (d) 15.8 (v) Which property is shown by $(4 \times 3) \times 5 = 4 \times (3 \times 5)$?

(i) An integer is a whole number, positive, negative or zero. 0.5, 3.14 and $\frac{1}{3}$ all have fractional parts, so (b) -2.

(ii) In 2.789 the 7 is the first digit after the point, so it is seven tenths = (d) 0.7. Place value after the point goes tenths, hundredths, thousandths.

(iii) A rational number is any number expressible as p/q . The intended answer is (c) $\frac{1}{3}$ — though see the note below.

(iv) 15.789 — the digit after the second place is 9, so the 8 becomes 9: (b) 15.79.

(v) The numbers stay in the same order; only the brackets move. That is the associative property, option (b). Changing the order would be commutative.

Answer (i) b (ii) d (iii) c (iv) b (v) b — Associative Property

Please check — two of these are misprinted. In (iii), 3.9 and 4.2 are rational as well ($\frac{39}{10}$ and $\frac{21}{5}$) — only π is not, so three options are correct. In (v), options (a) and (d) are the same words, leaving only three real choices.

2. Fill in the blanks: (i) The absolute value of -9 is ____ (ii) The place value of 5 in 562.34 is ____ (iii) When -7 is added to $5/9$ the result is ____ (iv) 5 is a ____ number (v) 4.567 rounded to 3 decimal places is ____ (vi) 2.348 rounded to 2 decimal places is ____ (vii) 1.234 rounded to 3 significant figures is ____

(i) $|-9| = 9$ — distance from zero, so never negative.

(ii) In 562.34 the 5 is in the hundreds column, so its place value is 500 — not 5 .

(iii) $5/9 + (-7) = 5/9 - 63/9 = -58/9$.

(iv) 5 is a natural number — and also a whole number, an integer, a prime number and a rational number. The blank does not say which is wanted.

(v) 4.567 already has exactly 3 decimal places, so it is unchanged: 4.567 .

(vi) $2.34|8$ — the next digit is 8 , so the 4 becomes 5 : 2.35 .

(vii) 1.234 to 3 significant figures — the fourth digit is 4 , so the 3 stays: 1.23 .

Answer (i) 9 · (ii) 500 · (iii) $-58/9$ · (iv) natural (also whole, integer, prime and rational) · (v) 4.567 · (vi) 2.35 · (vii) 1.23

3. Round the following numbers to 2 decimal places: (i) 3.746 (ii) 56.8347 (iii) 0.9987

(i) $3.74|6$ — next digit 6 , so round up: 3.75 .

(ii) $56.83|47$ — next digit 4 , so leave it: 56.83 .

(iii) $0.99|87$ — next digit 8 , so the 99 rounds up and carries all the way: 1.00 . The two zeros must be written; '1' alone would not show 2 decimal places.

Answer (i) 3.75 (ii) 56.83 (iii) 1.00

4. Round the following numbers to 3 significant figures: (i) 45676 (ii) 0.004389 (iii) 44.54053

(i) The first three significant digits of 45676 are 4 , 5 , 6 and the next is 7 , so the 6 becomes 7 : 45700 . The trailing zeros hold the places; they are not significant.

(ii) Leading zeros never count, so the significant digits start at 4 : 4 , 3 , 8 , and the next is 9 — round the 8 up to 9 : 0.00439 .

(iii) $44.5|4053$ — the first three are 4 , 4 , 5 and the next is 4 , so the 5 stays: 44.5 .

Answer (i) 45700 (ii) 0.00439 (iii) 44.5



2

CHAPTER

Simplifications

EXERCISE 2.1

1. Simplify: $12 - [4 \times (2 + 1)] + 3$

The order is BODMAS: Brackets, Of, Division and Multiplication, then Addition and Subtraction. Within brackets, work from the innermost outward.

Step 1 — the innermost bracket, $(2 + 1)$.

$$2 + 1 = 3$$

So the expression is $12 - [4 \times 3] + 3$

Step 2 — the square bracket.

$$4 \times 3 = 12$$

So it is $12 - 12 + 3$

Step 3 — now only addition and subtraction remain, so work left to right.

$$12 - 12 = 0$$

$$0 + 3 = 3$$

Doing the $12 + 3$ first would give $12 - 15 = -3$, which is wrong. Left to right is not optional.

Answer 3

2. Simplify: $10 + [3 \times (6 - 4)] - (2 + 1)$

Step 1 — both round brackets first.

$$6 - 4 = 2$$

$$2 + 1 = 3$$

So it is $10 + [3 \times 2] - 3$

Step 2 — the square bracket.

$$3 \times 2 = 6$$

So it is $10 + 6 - 3$

Step 3 — left to right.

$$10 + 6 = 16$$

$$16 - 3 = 13$$

Answer 13

3. Simplify: $14 + [\frac{3}{2} \times \{6 - (2.5 + 1.5)\}]$

Three levels of bracket here, so work strictly from the inside out: round, then curly, then square.

Step 1 — the round bracket.

$$2.5 + 1.5 = 4$$

So it is $14 + [\frac{3}{2} \times \{6 - 4\}]$

Step 2 — the curly bracket.

$$6 - 4 = 2$$

So it is $14 + [\frac{3}{2} \times 2]$

Step 3 — the square bracket.

$$\frac{3}{2} \times 2 = \frac{6}{2} = 3$$

So it is $14 + 3$

Step 4 — the addition.

$$14 + 3 = 17$$

Answer 17

4. Simplify: $4.6 + [3.2 \times \{5 + (3.8 + 1)\}]$

Step 1 — the round bracket.

$3.8 + 1 = 4.8$
 So it is $4.6 + [3.2 \times \{5 + 4.8\}]$
 Step 2 — the curly bracket.
 $5 + 4.8 = 9.8$
 So it is $4.6 + [3.2 \times 9.8]$
 Step 3 — the multiplication inside the square bracket.
 $3.2 \times 9.8 = 31.36$
 $(32 \times 98 = 3,136, \text{ and there are two decimal places altogether, so } 31.36)$
 So it is $4.6 + 31.36$
 Step 4 — the addition. Line up the decimal points.
 $4.6 + 31.36 = 35.96$

Answer 35.96

5. Simplify: $\frac{9}{5} + [1.6 - \{0.9 + 3 \div \frac{3}{5}\}]$

Step 1 — inside the curly bracket, division comes before addition, so do $3 \div \frac{3}{5}$ first.
 Dividing by a fraction means multiplying by its reciprocal.
 $3 \div \frac{3}{5} = 3 \times \frac{5}{3} = 1\frac{5}{3} = 5$
 Step 2 — now the addition inside the curly bracket.
 $0.9 + 5 = 5.9$
 So it is $\frac{9}{5} + [1.6 - 5.9]$
 Step 3 — the square bracket. The answer goes negative, which is fine.
 $1.6 - 5.9 = -4.3$
 So it is $\frac{9}{5} + (-4.3)$
 Step 4 — put $\frac{9}{5}$ into decimal form to add.
 $\frac{9}{5} = 9 \div 5 = 1.8$
 $1.8 + (-4.3) = 1.8 - 4.3 = -2.5$
 The whole answer is negative because the 5 from step 1 is larger than everything it is being taken from.

Answer -2.5

6. Simplify: $2.5 + [\frac{4}{5} \times (1 + \frac{5}{2}) - 1]$

Step 1 — the round bracket. Put 1 over 2 to add.
 $1 + \frac{5}{2} = \frac{2}{2} + \frac{5}{2} = \frac{7}{2}$
 So it is $2.5 + [\frac{4}{5} \times \frac{7}{2} - 1]$
 Step 2 — inside the square bracket, multiplication before subtraction.
 $\frac{4}{5} \times \frac{7}{2} = \frac{28}{10} = 1\frac{4}{5}$
 As a decimal, $1\frac{4}{5} = 2.8$
 Step 3 — the subtraction inside the square bracket.
 $2.8 - 1 = 1.8$
 So it is $2.5 + 1.8$
 Step 4 — the addition.
 $2.5 + 1.8 = 4.3$

Answer 4.3

7. Simplify: $7.25 + [2.1 - \{0.75 + (1.5 \div 0.5)\}]$

Step 1 — the round bracket.

$$1.5 \div 0.5 = 3$$

(0.5 goes into 1.5 exactly three times — or multiply both by 10 to get $15 \div 5 = 3$)

Step 2 — the curly bracket.

$$0.75 + 3 = 3.75$$

So it is $7.25 + [2.1 - 3.75]$

Step 3 — the square bracket. Again the result is negative.

$$2.1 - 3.75 = -1.65$$

So it is $7.25 + (-1.65)$

Step 4 — the addition.

$$7.25 - 1.65 = 5.6$$

Answer 5.6

REVISION EXERCISE 2

i. $4 + 4 \times 4 =$

a) 20 b) 64 c) 32 d) 12

Multiplication is done before addition — that is BODMAS.

$$\text{Step 1 — } 4 \times 4 = 16$$

$$\text{Step 2 — } 4 + 16 = 20$$

Working strictly left to right would give $8 \times 4 = 32$, which is the wrong answer sitting in the options.

Answer a) 20

ii. $25 \div 5 \times 5 =$

a) 1 b) 5 c) 25 d) 6

Division and multiplication rank equally, so they are done left to right — not division first.

$$\text{Step 1 — } 25 \div 5 = 5$$

$$\text{Step 2 — } 5 \times 5 = 25$$

Doing $5 \times 5 = 25$ first and then $25 \div 25$ would give 1, which is the trap.

Answer c) 25

iii. $20 \div 4 \times 6 =$

a) 2 b) 11 c) 4 d) 30

Again, left to right between division and multiplication.

$$\text{Step 1 — } 20 \div 4 = 5$$

$$\text{Step 2 — } 5 \times 6 = 30$$

Doing $4 \times 6 = 24$ first would give $20 \div 24$, which is less than 1.

Answer d) 30

iv. $10 \div (2 + 8) =$

a) 1 b) 13 c) 5 d) 30

The bracket comes first, whatever is outside it.

$$\text{Step 1 — } 2 + 8 = 10$$

$$\text{Step 2 — } 10 \div 10 = 1$$

Dividing by the 2 first would give $5 + 8 = 13$ — also an option, and also wrong.

Answer a) 1

v. $8 \times 4 - 2 =$

- a) 16 b) 24 c) 30 d) 10

Multiplication before subtraction.

Step 1 — $8 \times 4 = 32$

Step 2 — $32 - 2 = 30$

Doing $4 - 2 = 2$ first would give $8 \times 2 = 16$.

Answer c) 30

2. Fill in the blanks: (i) In mathematical expressions, ___ and ___ are used to indicate which operations should be performed first.

- (i) Two things control the order of working.

Brackets force whatever is inside them to be done first, whatever the operations are.

In $2 \times (3 + 4)$ the bracket forces the addition first: $2 \times 7 = 14$, not $6 + 4 = 10$.

BODMAS (the order of operations) settles everything else: Brackets, Of, Division, Multiplication, Addition, Subtraction.

Division and multiplication rank equally with one another, and so do addition and subtraction — equal ranks are worked left to right.

So the blanks are brackets and the order of operations (BODMAS).

Answer brackets and the order of operations (BODMAS)



3

CHAPTER

Square and Square Roots

EXERCISE 3.1 – SQUARES

1. Calculate the square of: (i) 12 (ii) 23 (iii) 34 (iv) 103 (v) 256 (vi) 0

To square a number, multiply it by itself: $n^2 = n \times n$.

For the three-digit ones it helps to split the number into hundreds and the rest, multiply each part, then add.

(i) $12^2 = 12 \times 12 = 144$

(ii) $23^2 = 23 \times 23 = 529$

(iii) $34^2 = 34 \times 34 = 1,156$

(iv) $103^2 = 103 \times 103$. Split it: $103 \times 103 = 103 \times 100 + 103 \times 3 = 10,300 + 309 = 10,609$

(v) $256^2 = 256 \times 256$. Split it: $256 \times 256 = 256 \times 200 + 256 \times 56 = 51,200 + 14,336 = 65,536$

(vi) $0^2 = 0 \times 0 = 0$ — zero is the only number that is its own square apart from 1.

Answer (i) 144 (ii) 529 (iii) 1,156 (iv) 10,609 (v) 65,536 (vi) 0

2. Without solving, separate the perfect squares of even numbers from those of odd numbers: (i) 256 (ii) 144 (iii) 484 (iv) 169 (v) 1024 (vi) 529

'Without solving' means without working out the square root — judge from the number itself.

The rule: an even number squared is even; an odd number squared is odd. So the square tells you at once what its root must be.

And a number is even if it ends in 0, 2, 4, 6 or 8.

(i) 256 ends in 6, so it is even. ($256 = 16^2$, and an even number squares to an even number, an odd to an odd — so the root 16 is even too.)

(ii) 144 ends in 4, so it is even. ($144 = 12^2$, and an even number squares to an even number, an odd to an odd — so the root 12 is even too.)

(iii) 484 ends in 4, so it is even. ($484 = 22^2$, and an even number squares to an even number, an odd to an odd — so the root 22 is even too.)

(iv) 169 ends in 9, so it is odd. ($169 = 13^2$, and an even number squares to an even number, an odd to an odd — so the root 13 is odd too.)

(v) 1024 ends in 4, so it is even. ($1024 = 32^2$, and an even number squares to an even number, an odd to an odd — so the root 32 is even too.)

(vi) 529 ends in 9, so it is odd. ($529 = 23^2$, and an even number squares to an even number, an odd to an odd — so the root 23 is odd too.)

Answer Squares of even numbers: 256, 144, 484, 1024 · Squares of odd numbers: 169, 529

3. Find the square of: (i) 0.5 (ii) 5.4

Square a decimal exactly as you would a whole number, then place the decimal point: the answer has twice as many decimal places as the number you started with.

(i) 0.5 has one decimal place, so the answer has two.

$5 \times 5 = 25$

Place two decimal places: 0.25

As a fraction: $0.5 = \frac{1}{2}$, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ ✓

(ii) 5.4 has one decimal place, so again two in the answer.

$54 \times 54 = 2,916$

Place two decimal places: 29.16

Note the answer is smaller than 0.5 in (i) but larger than 5.4 in (ii) — squaring shrinks numbers below 1 and grows those above it.

Answer (i) 0.25 (ii) 29.16

■ EXERCISE 3.2 – SQUARE ROOTS

1. Find the square root using the prime factorization method: (i) 144 (ii) 625 (iii) 96 (iv) 121 (v) 400 (vi) $81/21$ (vii) $36/49$ (viii) 0.25 (ix) 0.64 (x) $5^{116}/169$ (xi) $7^{25}/36$ (xii) $144/225$

A number has an exact square root only if, when broken into primes, every prime appears an even number of times. That is the test, and it is also the method: pair the primes up and take one from each pair.

For a fraction, do the top and the bottom separately.

(i) $144 = 2^4 \times 3^2$. Every power is even, so pair them up and take one from each pair: $\sqrt{144} = 12$

(ii) $625 = 5^4$. Every power is even, so pair them up and take one from each pair: $\sqrt{625} = 25$

(iii) $96 = 2^5 \times 3$. A prime is left over with an odd power, so there is no exact square root — see the note.

(iv) $121 = 11^2$. Every power is even, so pair them up and take one from each pair: $\sqrt{121} = 11$

(v) $400 = 2^4 \times 5^2$. Every power is even, so pair them up and take one from each pair: $\sqrt{400} = 20$

(vi) $81/21 = 27/7$, which is not a perfect square — see the note.

(vii) $36/49 = 36/49$. Top: $\sqrt{36} = 6$. Bottom: $\sqrt{49} = 7$. So the root is $6/7$

(viii) $0.25 = 1/4$. Top: $\sqrt{1} = 1$. Bottom: $\sqrt{4} = 2$. So the root is $1/2$

(ix) $0.64 = 16/25$. Top: $\sqrt{16} = 4$. Bottom: $\sqrt{25} = 5$. So the root is $4/5$

(x) $5^{116}/169 = 961/169$. Top: $\sqrt{961} = 31$. Bottom: $\sqrt{169} = 13$. So the root is $31/13$

(xi) $7^{25}/36 = 277/36$, which is not a perfect square — see the note.

(xii) $144/225 = 16/25$. Top: $\sqrt{16} = 4$. Bottom: $\sqrt{25} = 5$. So the root is $4/5$

For the decimals, write them as fractions first: $0.25 = 25/100$ and $0.64 = 64/100$. Then root the top and bottom separately.

For the mixed numbers, turn them into improper fractions first: $5^{116}/169 = (5 \times 169 + 116)/169 = 961/169$, and $\sqrt{961} = 31$, $\sqrt{169} = 13$, so the root is $31/13$.

Answer (i) 12 (ii) 25 (iii) no exact root (iv) 11 (v) 20 (vi) no exact root (vii) $6/7$ (viii) $1/2$ (ix) $4/5$ (x) $31/13$ (xi) no exact root (xii) $4/5$

Please check — three of these have no exact square root: $96 (= 2^5 \times 3, \text{ with odd powers})$, $81/21 (= 27/7)$ and $7^{25}/36 (= 277/36)$. The prime factorisation method cannot give a root for any of them, so those three figures need correcting in the book.

2. Find the square root using the division method: (i) 729 (ii) 484 (iii) 2500

The long-division method pairs the digits from the right, then finds the root a digit at a time. It works for any number, not just small ones.

(i) 729 → pair as 7 | 29.

The largest square not above 7 is $4 = 2^2$, so the first digit is 2, remainder 3.

Bring down 29 to make 329. Double the 2 to get 4; find d so that $4d \times d \leq 329$. $47 \times 7 = 329$ exactly.

So $\sqrt{729} = 27$. Check: $27 \times 27 = 729$ ✓

(ii) 484 → pair as 4 | 84.

Largest square not above 4 is $4 = 2^2$, first digit 2, remainder 0.

Bring down 84. Double 2 to get 4; $42 \times 2 = 84$ exactly.

So $\sqrt{484} = 22$. Check: $22 \times 22 = 484$ ✓

(iii) 2500 → pair as 25 | 00.

Largest square not above 25 is $25 = 5^2$, first digit 5, remainder 0.

Bring down 00. Double 5 to get 10; $100 \times 0 = 0$.

So $\sqrt{2500} = 50$. Check: $50 \times 50 = 2,500$ ✓

Answer (i) 27 (ii) 22 (iii) 50

3. A square floor needs tiling. If each tile is 1 square metre and the floor area is 49 square metres, how many tiles are needed?

Each tile covers exactly 1 m², so the number of tiles equals the number of square metres.

Tiles needed = 49

No square root is needed for this part. (If the question had asked for the length of a side, that would be $\sqrt{49} = 7$ m — and the floor would be 7 tiles by 7 tiles, which is 49 again ✓)

Answer 49 tiles

4. Sarah is 16, which is a perfect square. What will her age be when she next reaches a perfect square age?

16 is a perfect square because $16 = 4 \times 4 = 4^2$.

The next perfect square comes from the next whole number, 5.

$$5^2 = 5 \times 5 = 25$$

So she will be 25 — in $25 - 16 = 9$ years' time.

Note the gap is 9, not 1: perfect squares get further apart as they grow (1, 4, 9, 16, 25, 36 — gaps of 3, 5, 7, 9, 11).

Answer 25 years old

5. A square swimming pool has a total area of 256 square metres. What is the length of each side?

For a square, Area = side $\times$ side, so the side is the square root of the area.

$$\text{side} = \sqrt{256}$$

$$256 = 2^8$$

Eight 2s pair into four 2s: $2 \times 2 \times 2 \times 2 = 16$

$$\text{side} = 16 \text{ m}$$

$$\text{Check: } 16 \times 16 = 256 \text{ m}^2 \checkmark$$

Answer 16 metres

REVISION EXERCISE 3

- i. What is the square of 15?

a) 150 b) 225 c) 250 d) 200

15^2 means 15×15 .

Split it: $15 \times 15 = 15 \times 10 + 15 \times 5 = 150 + 75 = 225$.

150 is the trap — that is 15×10 , only half the multiplication.

Answer b) 225

- ii. Which of the following is a perfect square?

a) 81 b) 50 c) 99 d) 72

A perfect square is a whole number times itself.

$$81 = 9 \times 9 \checkmark$$

50 lies between 49 (7^2) and 64 (8^2); 72 lies between 64 and 81; 99 lies between 81 and 100 (10^2).

None of those three is a square.

81.

Answer a) 81

- iii. Which of the following is NOT a perfect square?

a) 121 b) 169 c) 200 d) 144

Check each: $121 = 11^2$, $169 = 13^2$, $144 = 12^2$ — all squares.

200 lies between 196 (14^2) and 225 (15^2), so it is not one.

200. Read the question carefully — it asks which is NOT.

Answer c) 200

iv. Which is the prime factorization of 400?

- a) $2^4 \times 5^2$ b) $2^5 \times 5^2$ c) $2^2 \times 5^4$ d) $2^4 \times 5^1$

Break 400 down: $400 = 4 \times 100 = (2 \times 2) \times (4 \times 25) = 2 \times 2 \times 2 \times 2 \times 5 \times 5$.

That is four 2s and two 5s: $2^4 \times 5^2$.

Check by multiplying back: $16 \times 25 = 400$ ✓

Note every power is even, which is why $\sqrt{400} = 2^2 \times 5 = 20$ exactly.

Answer a) $2^4 \times 5^2$

v. What is the square root of $81/100$?

- a) $11/10$ b) $8/9$ c) $9/10$ d) $10/9$

Take the root of the top and the bottom separately.

$\sqrt{81} = 9$, because $9 \times 9 = 81$.

$\sqrt{100} = 10$, because $10 \times 10 = 100$.

So the root is $9/10$.

Check: $9/10 \times 9/10 = 81/100$ ✓

Answer c) $9/10$

2. Fill in the blanks: (i) The symbol for square root is called a ____ sign (ii) The square of 0.6 is ____ (iii) The prime factorization of 225 is ____ (iv) The square root of $25^{36}/49$ is ____ (v) A number that can be written as the product of two equal integers is called a ____ square

(i) The $\sqrt{\quad}$ symbol is called the radical sign. The number under it is the radicand.

(ii) 0.6 has one decimal place, so its square has two.

$6 \times 6 = 36$, so $0.6^2 = 0.36$.

(iii) $225 = 3^2 \times 5^2$. Check: $9 \times 25 = 225$ ✓ Both powers are even, which is why $\sqrt{225} = 3 \times 5 = 15$ exactly.

(iv) $25^{36}/49 = (25 \times 49 + 36)/49 = 1261/49$. But 1,261 is not a perfect square — it lies between $35^2 = 1,225$ and $36^2 = 1,296$. So this has no exact root; see the note.

(v) A number that is the product of two equal integers is called a perfect square.

Answer (i) radical · (ii) 0.36 · (iii) $3^2 \times 5^2$ · (iv) no exact root · (v) perfect

Please check — part (iv) asks for $\sqrt{(25^{36}/49)}$, which comes to $\sqrt{(1261/49)}$. 1,261 is not a perfect square, so there is no exact answer. Had the number been $1225/49$ the root would be exactly 5.



4

CHAPTER

Financial Mathematics

EXERCISE 4.1

1. Calculate profit or loss, when: (i) S.P. = Rs 23, C.P. = Rs 12 (ii) S.P. = Rs 45, C.P. = Rs 30 (iii) S.P. = Rs 100, C.P. = Rs 120 (iv) S.P. = Rs 200, C.P. = Rs 150

Compare the selling price with the cost price first. If S.P. is the larger there is a profit; if C.P. is the larger there is a loss.

The percentage is always taken on the cost price, never on the selling price — that is the single thing most often got wrong in this exercise.

- (i) S.P. = Rs 23, C.P. = Rs 12

compare the two: $23 > 12$, so it is a profit

$$\text{Profit} = \text{S.P.} - \text{C.P.} = 23 - 12$$

$$= \text{Rs } 11$$

$$\text{Profit \%} = (\text{profit} \div \text{C.P.}) \times 100$$

$$= (11 \div 12) \times 100$$

$$= 91.67\%$$

- (ii) S.P. = Rs 45, C.P. = Rs 30

compare the two: $45 > 30$, so it is a profit

$$\text{Profit} = \text{S.P.} - \text{C.P.} = 45 - 30$$

$$= \text{Rs } 15$$

$$\text{Profit \%} = (\text{profit} \div \text{C.P.}) \times 100$$

$$= (15 \div 30) \times 100$$

$$= 50\%$$

- (iii) S.P. = Rs 100, C.P. = Rs 120

compare the two: $100 < 120$, so it is a loss

$$\text{Loss} = \text{C.P.} - \text{S.P.} = 120 - 100$$

$$= \text{Rs } 20$$

$$\text{Loss \%} = (\text{loss} \div \text{C.P.}) \times 100$$

$$= (20 \div 120) \times 100$$

$$= 16.67\%$$

- (iv) S.P. = Rs 200, C.P. = Rs 150

compare the two: $200 > 150$, so it is a profit

$$\text{Profit} = \text{S.P.} - \text{C.P.} = 200 - 150$$

$$= \text{Rs } 50$$

$$\text{Profit \%} = (\text{profit} \div \text{C.P.}) \times 100$$

$$= (50 \div 150) \times 100$$

$$= 33.33\%$$

Answer (i) Profit Rs 11 (91.67%) · (ii) Profit Rs 15 (50%) · (iii) Loss Rs 20 (16.67%) · (iv) Profit Rs 50 (33.33%)

2. Find S.P., when: (i) C.P. = Rs 750, Loss % = 75% (ii) C.P. = Rs 875, Gain % = 25%

Work out the loss or gain in rupees first, then apply it to the cost price.

- (i) C.P. = Rs 750, Loss = 75%

$$\text{Loss} = 75\% \text{ of } 750$$

$$= (75 \div 100) \times 750$$

$$= \text{Rs } 562.50$$

$$\text{S.P.} = \text{C.P.} - \text{Loss}$$

$$= 750 - 562.50$$

$$\begin{aligned}
 &= \text{Rs } 187.50 \\
 \text{(ii) C.P.} &= \text{Rs } 875, \text{ Gain} = 25\% \\
 \text{Gain} &= 25\% \text{ of } 875 \\
 &= (25 \div 100) \times 875 \\
 &= \text{Rs } 218.75 \\
 \text{S.P.} &= \text{C.P.} + \text{Gain} \\
 &= 875 + 218.75 \\
 &= \text{Rs } 1,093.75
 \end{aligned}$$

Answer (i) Rs 187.50 · (ii) Rs 1,093.75

Please check — the loss and gain are printed as 'Rs. 75%' and 'Rs. 25%'. They are percentages, so the Rs should not be there.

3. Sarah is selling lemonade at a stand. She spent Rs 50 on ingredients. If she sells the lemonade for Rs 200, what is her profit? What is the profit percentage?

Step 1 Name the two prices
 what she spent is the cost price: C.P. = Rs 50
 what she sold for is the selling price: S.P. = Rs 200
 Step 2 Profit
 $\text{Profit} = \text{S.P.} - \text{C.P.}$
 $= 200 - 50$
 $= \text{Rs } 150$
 Step 3 Profit percentage
 $\text{Profit \%} = (\text{Profit} \div \text{C.P.}) \times 100$
 $= (150 \div 50) \times 100$
 $= 3 \times 100$
 $= 300\%$
 Step 4 Check
 a 300% profit means she made three times what she spent: $3 \times 50 = 150$ ✓
 a profit percentage above 100% is perfectly possible — it only means the profit exceeded the cost.

Answer Profit Rs 150, which is 300%

4. A gardener bought gardening tools for Rs 10,000 and sold them for Rs 7,000. Calculate the loss and loss percentage.

Step 1 Decide profit or loss
 S.P. = Rs 7,000 and C.P. = Rs 10,000
 $7,000 < 10,000$, so it is a loss
 Step 2 Loss
 $\text{Loss} = \text{C.P.} - \text{S.P.}$
 $= 10,000 - 7,000$
 $= \text{Rs } 3,000$
 Step 3 Loss percentage
 $\text{Loss \%} = (\text{Loss} \div \text{C.P.}) \times 100$
 $= (3,000 \div 10,000) \times 100$
 $= 0.3 \times 100$
 $= 30\%$
 Step 4 Check
 $30\% \text{ of } 10,000 = 3,000$ ✓

Answer Loss Rs 3,000, which is 30%

5. Ahmed purchased a used car for Rs 5,00,000 and sold it for Rs 4,05,000. What is his loss? What is the loss percentage?

Step 1 Loss
 $\text{Loss} = \text{C.P.} - \text{S.P.}$
 $= 5,00,000 - 4,05,000$
 $= \text{Rs } 95,000$
 Step 2 Loss percentage
 $\text{Loss \%} = (\text{Loss} \div \text{C.P.}) \times 100$
 $= (95,000 \div 5,00,000) \times 100$
 $= 0.19 \times 100$
 $= 19\%$
 Step 3 Check
 $19\% \text{ of } 5,00,000 = 95,000 \checkmark$

Answer Loss Rs 95,000, which is 19%

6. A bakery sells cupcakes. If the cost price of 12 cupcakes is Rs 240 and they sell them for Rs 360, what is the profit made? Calculate the profit percentage.

The twelve cupcakes are bought and sold together, so the whole batch can be treated as one item.
 Step 1 Profit on the batch
 $\text{Profit} = \text{S.P.} - \text{C.P.}$
 $= 360 - 240$
 $= \text{Rs } 120$
 Step 2 Profit percentage
 $\text{Profit \%} = (\text{Profit} \div \text{C.P.}) \times 100$
 $= (120 \div 240) \times 100$
 $= 0.5 \times 100$
 $= 50\%$
 Step 3 Check it per cupcake
 one cupcake costs $240 \div 12 = \text{Rs } 20$ and sells for $360 \div 12 = \text{Rs } 30$
 profit per cupcake $= 30 - 20 = \text{Rs } 10$
 $10 \div 20 \times 100 = 50\%$ — the same percentage $\checkmark$
 the percentage does not depend on how many are counted at once, which is worth seeing.

Answer Profit Rs 120, which is 50%

7. A dress is marked at Rs 1,200 and sold for Rs 960. Calculate the discount and discount percentage.

A discount is taken on the marked price, in the same way a profit is taken on the cost price.
 Step 1 Discount
 $\text{Discount} = \text{marked price} - \text{selling price}$
 $= 1,200 - 960$
 $= \text{Rs } 240$
 Step 2 Discount percentage
 $\text{Discount \%} = (\text{Discount} \div \text{marked price}) \times 100$
 $= (240 \div 1,200) \times 100$
 $= 0.2 \times 100$
 $= 20\%$
 Step 3 Check
 $20\% \text{ of } 1,200 = 240$, and $1,200 - 240 = 960 \checkmark$

Answer Discount Rs 240, which is 20%

8. A smartphone originally priced at Rs 30,000 is offered at a sale price of Rs 27,000. What is the discount and discount percentage?

Step 1 Discount

Discount = original price – sale price

$$= 30,000 - 27,000$$

$$= \text{Rs } 3,000$$

Step 2 Discount percentage

$$\text{Discount \%} = (3,000 \div 30,000) \times 100$$

$$= 0.1 \times 100$$

$$= 10\%$$

Step 3 Check

$$10\% \text{ of } 30,000 = 3,000, \text{ and } 30,000 - 3,000 = 27,000 \checkmark$$

Answer Discount Rs 3,000, which is 10%

9. If a store offers a 15% discount on a shirt marked at Rs 800, what will be the selling price?

Here the percentage is given and the amount has to be found — the reverse of the last two questions.

Step 1 Discount in rupees

$$\text{Discount} = 15\% \text{ of } 800$$

$$= (15 \div 100) \times 800$$

$$= \text{Rs } 120$$

Step 2 Selling price

$$\text{S.P.} = \text{marked price} - \text{discount}$$

$$= 800 - 120$$

$$= \text{Rs } 680$$

Step 3 A quicker way, worth knowing

after a 15% discount the customer pays $100 - 15 = 85\%$

$$85\% \text{ of } 800 = (85 \div 100) \times 800 = \text{Rs } 680 \text{ — the same } \checkmark$$

Answer Rs 680

EXERCISE 4.2

1. Zara works as a software engineer and earns an annual salary of Rs 10,00,000. Using the income tax slabs for 2024-25, calculate (a) the income tax Zara needs to pay and (b) her net income after tax.

Step 1 Find the slab

Rs 10,00,000 falls in the band Rs 6,00,001 – 12,00,000

that band charges 5% of the amount exceeding Rs 6,00,000 — not 5% of the whole salary

Step 2 The taxable part

$$\text{amount exceeding} = 10,00,000 - 6,00,000$$

$$= \text{Rs } 4,00,000$$

Step 3 The tax

$$\text{Income tax} = 5\% \text{ of } 4,00,000$$

$$= (5 \div 100) \times 4,00,000$$

$$= \text{Rs } 20,000$$

Step 4 Net income

$$\text{Net income} = \text{salary} - \text{tax}$$

$$= 10,00,000 - 20,000$$

$$= \text{Rs } 9,80,000$$

Step 5 Check

taking 5% of the whole Rs 10,00,000 would give Rs 50,000 — two and a half times too much. The slab only taxes the part above the threshold.

Answer (a) Rs 20,000 · (b) Rs 9,80,000

2. A travel agent earns a commission of 10% on each travel package sold. Last month the agent sold packages worth Rs 25,00,000. Calculate the total commission earned.

Commission = rate of commission $\times$ selling price

Step 1 Put in the figures

$$= 10\% \text{ of } 25,00,000$$

Step 2 Turn the percentage into a fraction

$$= (10 \div 100) \times 25,00,000$$

Step 3 Work it out

$$= 25,00,000 \div 10$$

$$= \text{Rs } 2,50,000$$

Step 4 Check

$$2,50,000 \div 25,00,000 = 0.10 = 10\% \checkmark$$

Answer Rs 2,50,000

3. A bookstore originally priced a set of novels at Rs 1,200 but offered a discount of 15% during a sale. What is the selling price after the discount?

Step 1 The discount in rupees

$$\text{Discount} = 15\% \text{ of } 1,200$$

$$= (15 \div 100) \times 1,200$$

$$= \text{Rs } 180$$

Step 2 The selling price

$$\text{S.P.} = \text{marked price} - \text{discount}$$

$$= 1,200 - 180$$

$$= \text{Rs } 1,020$$

Step 3 Check the other way

$$\text{the customer pays } 100 - 15 = 85\%$$

$$85\% \text{ of } 1,200 = \text{Rs } 1,020 \checkmark$$

Answer Rs 1,020

4. A customer buys a laptop for Rs 60,000. The cost includes a GST of 17% and a VAT of 5%. Calculate the total amount paid by the customer, including both taxes.

Taking the Rs 60,000 as the price before tax, which is the only reading under which the question has anything to calculate — see the note.

Step 1 GST

$$\text{GST} = 17\% \text{ of } 60,000$$

$$= (17 \div 100) \times 60,000$$

$$= \text{Rs } 10,200$$

Step 2 VAT

$$\text{VAT} = 5\% \text{ of } 60,000$$

$$= (5 \div 100) \times 60,000$$

$$= \text{Rs } 3,000$$

Step 3 Total paid

$$\text{Total} = \text{price} + \text{GST} + \text{VAT}$$

$$= 60,000 + 10,200 + 3,000$$

$$= \text{Rs } 73,200$$

Step 4 Check

$$\text{the two taxes together come to } 17 + 5 = 22\%$$

$$122\% \text{ of } 60,000 = \text{Rs } 73,200 \checkmark$$

Answer Rs 73,200

Please check — the question says the cost 'includes' both taxes and then asks for the total 'including' them. If the Rs 60,000 already includes the taxes there is nothing to add and the answer is simply Rs 60,000. The wording needs fixing one way or the other.

5. A family has a total income of Rs 15,00,000 in a financial year. Using the income tax slabs for the year, calculate the total income tax they owe.

Step 1 Find the slab

Rs 15,00,000 falls in the band Rs 12,00,001 – 22,00,000

that band charges a fixed Rs 30,000 plus 15% of the amount exceeding Rs 12,00,000

Step 2 The excess

$$15,00,000 - 12,00,000 = \text{Rs } 3,00,000$$

Step 3 15% of the excess

$$= (15 \div 100) \times 3,00,000$$

$$= \text{Rs } 45,000$$

Step 4 Add the fixed amount

$$\text{Income tax} = 30,000 + 45,000$$

$$= \text{Rs } 75,000$$

Step 5 Check the fixed part makes sense

the Rs 30,000 is the whole tax on an income of exactly Rs 12,00,000: 5% of (12,00,000 – 6,00,000) = Rs 30,000 ✓

so the slabs join up without a jump, which is how they are meant to work.

Answer Rs 75,000

■ EXERCISE 4.3

1. A farmer grows cucumbers and carrots. The harvest yields 2,500 kg of cucumbers (irrigated) and 1,000 kg of carrots (non-irrigated). The farmer also has cash savings of Rs 5,000 and gold worth Rs 120,000. Calculate the Ushr the farmer must pay.

Ushr falls on the produce of the land, so the savings and the gold are set aside — those would attract zakat instead.

Using the book's own labels: 'irrigated' carries 10%, 'non-irrigated' carries 5%.

Step 1 Cucumbers, at 10%

$$\text{Ushr} = 10\% \text{ of } 2,500 \text{ kg}$$

$$= (10 \div 100) \times 2,500$$

$$= 250 \text{ kg}$$

Step 2 Carrots, at 5%

$$\text{Ushr} = 5\% \text{ of } 1,000 \text{ kg}$$

$$= (5 \div 100) \times 1,000$$

$$= 50 \text{ kg}$$

Step 3 Total

$$250 + 50 = 300 \text{ kg}$$

Step 4 Check

300 kg out of a 3,500 kg harvest is about 8.6%, which sits between the two rates as it should ✓

Answer 300 kg of produce (250 kg of cucumbers and 50 kg of carrots)

Please check — the cash and the gold play no part in an Ushr calculation. If zakat were wanted too it would be 2.5% of Rs 1,25,000 = Rs 3,125, but the question does not ask for it.

2. Fatima's annual income is Rs 240,000. If she pays Zakat at 2.5%, how much does she give away?

$$\text{Zakat} = \text{total wealth} \times 2.5\%$$

Step 1 Put in the figures

$$= 2.5\% \text{ of } 2,40,000$$

Step 2 Turn the percentage into a fraction

$$= (2.5 \div 100) \times 2,40,000$$

Step 3 Work it out

$$= 2,40,000 \div 40$$

$$= \text{Rs } 6,000$$

Step 4 Check

$$2.5\% \text{ is one fortieth, and } 40 \times 6,000 = 2,40,000 \text{ ✓}$$

Answer Rs 6,000

3. Zain received an inheritance of Rs 300,000. If he pays Zakat at 2.5%, how much should he set aside?

Zakat = total wealth $\times$ 2.5%
 Step 1 = 2.5% of 3,00,000
 Step 2 = $(2.5 \div 100) \times 3,00,000$
 Step 3 = $3,00,000 \div 40$
 Step 4 = Rs 7,500
 Step 5 Check
 $40 \times 7,500 = 3,00,000 \checkmark$

Answer Rs 7,500

4. Sarah cultivates 900 kg of barley in a non-irrigated field. Calculate the Ushr she must pay at the non-irrigated rate of 5%.

Step 1 Take the rate the question gives
 non-irrigated, so 5%
 Step 2 Apply it
 Ushr = 5% of 900 kg
 $= (5 \div 100) \times 900$
 $= 900 \div 20$
 $= 45 \text{ kg}$
 Step 3 Check
 $20 \times 45 = 900 \checkmark$

Answer 45 kg of barley

5. A farmer harvests 2,500 kg of wheat (irrigated) and 1,500 kg of corn (non-irrigated). How much Ushr in total?

Two different rates, so work each crop separately and add at the end.
 Step 1 Wheat, at 10%
 $= (10 \div 100) \times 2,500$
 $= 250 \text{ kg}$
 Step 2 Corn, at 5%
 $= (5 \div 100) \times 1,500$
 $= 75 \text{ kg}$
 Step 3 Total
 $250 + 75 = 325 \text{ kg}$
 Step 4 Check
 the whole harvest is 4,000 kg and 325 kg is 8.125% of it — between 5% and 10%, nearer the 10% because more of the harvest was wheat $\checkmark$

Answer 325 kg in total (250 kg of wheat and 75 kg of corn)

REVISION EXERCISE 4

1. Choose the correct option.

- i. Formula for calculating profit:

a) Profit = Cost Price – Selling Price b) Profit = Selling Price – Cost Price c) Profit = Selling Price + Cost Price
 d) Profit = Cost Price + Selling Price

Profit is what is left over after the cost has been covered, so the cost is taken away from what the item sold for.

Test it on a case that is obviously a profit: an item bought for Rs 10 and sold for Rs 15.

- (a) $10 - 15 = -5$ $\times$ a profit cannot be negative here
 (b) $15 - 10 = 5 \checkmark$
 (c) and (d) both give 25, which is more than the item ever sold for $\times$

Answer b) Profit = Selling Price – Cost Price

ii. Zakat is calculated as a percentage of:

- a) Income only b) Wealth c) Property value d) Sales revenue

Zakat is due on wealth held for a full lunar year above the nisab — cash, savings, investments, gold and silver.

- (a) income alone is too narrow — savings built up over years are included $\times$
 (b) wealth $\checkmark$
 (c) property value alone is what property tax is based on $\times$
 (d) sales revenue is a business measure, not wealth held $\times$

Answer b) Wealth

iii. A pen priced at Rs 40 is given a discount of 20%. What is the selling price?

- a) 32 b) 36 c) 28 d) 40

Step 1 The discount

$$20\% \text{ of } 40 = (20 \div 100) \times 40 = \text{Rs } 8$$

Step 2 The selling price

$$40 - 8 = \text{Rs } 32$$

Step 3 Check the quick way

the buyer pays $100 - 20 = 80\%$, and $80\% \text{ of } 40 = 32 \checkmark$

option (c) 28 is what you get by taking off 30% — worth noticing as the near miss.

Answer a) 32

iv. Which of the following is not a type of tax?

- a) Income tax b) Sales tax c) Property tax d) Capital gain

Three of the four options name a tax outright.

- (a) income tax — a tax on earnings $\times$ (it is a tax)
 (b) sales tax — a tax on goods sold $\times$
 (c) property tax — a tax on property held $\times$
 (d) capital gain — this is the profit made on selling an asset, not a tax $\checkmark$

The tax charged on that profit is called capital gains tax; the gain itself is not a tax. The option is only correct because the word 'tax' is left off — see the note.

Answer d) Capital gain

Please check — option (d) should read 'capital gains tax' if it is to sit alongside the other three. As printed it names the gain rather than the tax, which is why it is the odd one out.

v. If an item is sold at a loss of 15% and the cost price is Rs 200, what is the selling price?

- a) 170 b) 180 c) 190 d) 200

Step 1 The loss in rupees

$$15\% \text{ of } 200 = (15 \div 100) \times 200 = \text{Rs } 30$$

Step 2 The selling price

$$\text{S.P.} = \text{C.P.} - \text{loss}$$

$$= 200 - 30$$

$$= \text{Rs } 170$$

Step 3 Check the quick way

$$\text{a } 15\% \text{ loss leaves } 100 - 15 = 85\%, \text{ and } 85\% \text{ of } 200 = 170 \checkmark$$

Answer a) 170

2. Fill in the blanks: (i) The amount of money gained from selling a product after covering its cost is called _____. (ii)

A _____ is a deduction from the marked price of an item, usually expressed as a percentage. (iii) If the cost price of a laptop is Rs 80,000 and it is sold for Rs 72,000, the loss incurred is _____. (iv) Zakat is usually calculated at _____% of a person's savings. (v) The rate of ushr on irrigated land is _____.

- (i) The amount of money gained from selling a product after covering its cost is called _____.
 what is left after the cost is covered is the profit
 profit

- (ii) A _____ is a deduction from the marked price of an item, usually expressed as a percentage.
 a reduction off the marked price is a discount

discount

(iii) If the cost price of a laptop is Rs 80,000 and it is sold for Rs 72,000, the loss incurred is ____.

$$\text{Loss} = \text{C.P.} - \text{S.P.} = 80,000 - 72,000 = \text{Rs } 8,000$$

Rs 8,000

(iv) Zakat is usually calculated at ____% of a person's savings.

the standard rate is 2.5%, which is one fortieth

2.5

(v) The rate of ushr on irrigated land is ____.

by the book's own labelling, 'irrigated' land carries 10% — but see the note on the two labels

10%

Answer (i) profit · (ii) discount · (iii) Rs 8,000 · (iv) 2.5 · (v) 10%

Please check — part (v) depends on the book's own labelling of the two Ushr rates, which is the reverse of ordinary usage — see the note on section 4.5.

3. Define the following: (i) What is zakat, and why is it important in Islam? (ii) How is nisab determined, and what types of wealth are considered? (iii) Define income tax and explain how it is calculated. (iv) Difference between profit and loss.

(i) What is zakat, and why is it important in Islam?

Zakat is a compulsory charitable payment, one of the Five Pillars of Islam, due at 2.5% a year on wealth held above the nisab for a full lunar year. It purifies wealth, narrows the gap between rich and poor, and supports those in need within the community.

(ii) How is nisab determined, and what types of wealth are considered?

Nisab is the threshold below which no zakat is due: 7.5 tola (87.48 g) of gold, or 52.5 tola (612.36 g) of silver, or cash and other wealth worth as much as that weight of silver, held for a complete lunar year after a person's needs are met. Cash, savings, investments, gold, silver and other assets are counted.

(iii) Define income tax and explain how it is calculated.

Income tax is a direct tax on a person's or a business's income. It is worked out in slabs: each band of income carries its own rate, and the rate applies only to the part of the income inside that band, not to the whole. A band may also carry a fixed amount covering the tax on all the bands below it.

(iv) Difference between profit and loss.

Both compare the selling price with the cost price. If the selling price is the greater there is a profit, equal to $\text{S.P.} - \text{C.P.}$. If the cost price is the greater there is a loss, equal to $\text{C.P.} - \text{S.P.}$. Either way the percentage is taken on the cost price.

Answer See the four definitions above.

4. A bookstore buys a novel for Rs 154 and sells it for Rs 425. What is the profit on each book, and what is the profit percentage?

Step 1 Profit

$$\text{Profit} = \text{S.P.} - \text{C.P.}$$

$$= 425 - 154$$

$$= \text{Rs } 271$$

Step 2 Profit percentage

$$\text{Profit \%} = (\text{Profit} \div \text{C.P.}) \times 100$$

$$= (271 \div 154) \times 100$$

$$= 175.97\% \text{ (to 2 decimal places)}$$

Step 3 Check

$$175.97\% \text{ of } 154 = \text{Rs } 271 \checkmark$$

The percentage is above 100 because the profit is larger than the cost — the shop sold for nearly three times what it paid.

Answer Profit Rs 271, which is 175.97%

Please check — a mark-up of almost 176% on a novel is very high for a textbook example. Rs 254 as the cost price would give a tidier 67%, which may have been the intention.

5. A bicycle was bought for Rs 15,000 and sold for Rs 12,000. Calculate the loss and the loss percentage.

Step 1 Loss
 $\text{Loss} = \text{C.P.} - \text{S.P.}$
 $= 15,000 - 12,000$
 $= \text{Rs } 3,000$
 Step 2 Loss percentage
 $\text{Loss \%} = (3,000 \div 15,000) \times 100$
 $= 0.2 \times 100$
 $= 20\%$
 Step 3 Check
 $20\% \text{ of } 15,000 = 3,000 \checkmark$

Answer Loss Rs 3,000, which is 20%

6. A store offers a 25% discount on a jacket that costs Rs 800. What is the selling price after the discount?

Step 1 The discount
 $25\% \text{ of } 800 = (25 \div 100) \times 800$
 $= \text{Rs } 200$
 Step 2 The selling price
 $800 - 200 = \text{Rs } 600$
 Step 3 Check the quick way
 $25\% \text{ off leaves } 75\%, \text{ and } 75\% \text{ of } 800 = 600 \checkmark$

Answer Rs 600

7. Sarah buys a smartphone for Rs 50,000. If the sales tax is 10%, how much does she pay in total?

Tax is added to the price, where a discount is taken off — the one place this unit most often trips people.
 Step 1 The tax
 $10\% \text{ of } 50,000 = (10 \div 100) \times 50,000$
 $= \text{Rs } 5,000$
 Step 2 The total
 $50,000 + 5,000 = \text{Rs } 55,000$
 Step 3 Check the quick way
 with 10% tax she pays 110%, and $110\% \text{ of } 50,000 = 55,000 \checkmark$

Answer Rs 55,000

8. Ali has savings of Rs 120,000. How much zakat does he pay at 2.5%?

$\text{Zakat} = \text{total wealth} \times 2.5\%$
 Step 1 $= 2.5\% \text{ of } 1,20,000$
 Step 2 $= (2.5 \div 100) \times 1,20,000$
 Step 3 $= 1,20,000 \div 40$
 Step 4 $= \text{Rs } 3,000$
 Step 5 Check
 $40 \times 3,000 = 1,20,000 \checkmark$

Answer Rs 3,000



5

CHAPTER

Rate, Ratio and Proportion

EXERCISE 5.1

1. Simplify the following ratios: (i) $12 : 16$ (ii) $24 : 36$ (iii) $45 : 15$ (iv) $2/3 : 4/9$ (v) $5/8 : 10/16$ (vi) 60 minutes : 2 hours
(vii) 3 km : 900 m (viii) 150 seconds : 5 minutes (ix) 4 litres : 2,000 millilitres (x) 300 grams : 3 kilograms

A ratio can only be simplified once both terms are in the same unit — that is the first move whenever two different units appear.

Then divide both terms by their HCF, exactly as a fraction is reduced.

A ratio has no unit of its own at the end: 3 km : 900 m becomes 10 : 3, not 10 km : 3 m.

- (i) $12 : 16$

both terms in the same unit: $12 : 16$

HCF of 12 and 16 is 4

$$12 \div 4 = 3 \text{ and } 16 \div 4 = 4$$

$$3 : 4$$

- (ii) $24 : 36$

both terms in the same unit: $24 : 36$

HCF of 24 and 36 is 12

$$24 \div 12 = 2 \text{ and } 36 \div 12 = 3$$

$$2 : 3$$

- (iii) $45 : 15$

both terms in the same unit: $45 : 15$

HCF of 45 and 15 is 15

$$45 \div 15 = 3 \text{ and } 15 \div 15 = 1$$

$$3 : 1$$

- (iv) $2/3 : 4/9$

multiply both terms by 9, the common denominator: $6 : 4$

both terms in the same unit: $6 : 4$

HCF of 6 and 4 is 2

$$6 \div 2 = 3 \text{ and } 4 \div 2 = 2$$

$$3 : 2$$

- (v) $5/8 : 10/16$

$10/16$ reduces to $5/8$, so the two terms are already equal

both terms in the same unit: $5 : 5$

HCF of 5 and 5 is 5

$$5 \div 5 = 1 \text{ and } 5 \div 5 = 1$$

$$1 : 1$$

- (vi) 60 minutes : 2 hours

$$2 \text{ hours} = 2 \times 60 = 120 \text{ minutes}$$

both terms in the same unit: $60 : 120$

HCF of 60 and 120 is 60

$$60 \div 60 = 1 \text{ and } 120 \div 60 = 2$$

$$1 : 2$$

- (vii) 3 km : 900 m

$$3 \text{ km} = 3 \times 1,000 = 3,000 \text{ m}$$

both terms in the same unit: $3,000 : 900$

HCF of 3,000 and 900 is 300

$$3,000 \div 300 = 10 \text{ and } 900 \div 300 = 3$$

10 : 3

(viii) 150 seconds : 5 minutes

5 minutes = $5 \times 60 = 300$ seconds

both terms in the same unit: 150 : 300

HCF of 150 and 300 is 150

$150 \div 150 = 1$ and $300 \div 150 = 2$

1 : 2

(ix) 4 litres : 2,000 millilitres

4 litres = $4 \times 1,000 = 4,000$ ml

both terms in the same unit: 4,000 : 2,000

HCF of 4,000 and 2,000 is 2000

$4,000 \div 2000 = 2$ and $2,000 \div 2000 = 1$

2 : 1

(x) 300 grams : 3 kilograms

3 kg = $3 \times 1,000 = 3,000$ g

both terms in the same unit: 300 : 3,000

HCF of 300 and 3,000 is 300

$300 \div 300 = 1$ and $3,000 \div 300 = 10$

1 : 10

Answer (i) 3 : 4 · (ii) 2 : 3 · (iii) 3 : 1 · (iv) 3 : 2 · (v) 1 : 1 · (vi) 1 : 2 · (vii) 10 : 3 · (viii) 1 : 2 · (ix) 2 : 1 · (x) 1 : 10

2. Increase these: (i) 20 in the ratio 25 : 5 (ii) 15 in the ratio 13 : 10 (iii) 10 in the ratio 50 : 5 (iv) 40 in the ratio 18 : 10 (v) 25 in the ratio 35 : 5 (vi) 12 in the ratio 11 : 6

To change a quantity in a given ratio, read the ratio as a fraction — antecedent over consequent — and multiply.

The book's own check: in an increase the first term is the larger, so the fraction is greater than 1 and the answer grows.

(i) Increase 20 in the ratio 25 : 5

$25 > 5$, so this really is an increase ✓

read the ratio as a fraction: $25/5$

increased value = $25/5 \times 20$

= 100

(ii) Increase 15 in the ratio 13 : 10

$13 > 10$, so this really is an increase ✓

read the ratio as a fraction: $13/10$

increased value = $13/10 \times 15$

= 19.5

(iii) Increase 10 in the ratio 50 : 5

$50 > 5$, so this really is an increase ✓

read the ratio as a fraction: $50/5$

increased value = $50/5 \times 10$

= 100

(iv) Increase 40 in the ratio 18 : 10

$18 > 10$, so this really is an increase ✓

read the ratio as a fraction: $18/10$

increased value = $18/10 \times 40$

= 72

(v) Increase 25 in the ratio 35 : 5

$35 > 5$, so this really is an increase ✓

read the ratio as a fraction: $35/5$

increased value = $35/5 \times 25$

= 175

(vi) Increase 12 in the ratio 11 : 6

$11 > 6$, so this really is an increase ✓
 read the ratio as a fraction: $11/6$
 increased value = $11/6 \times 12$
 = 22

Answer (i) 100 · (ii) 19.5 · (iii) 100 · (iv) 72 · (v) 175 · (vi) 22

3. Decrease these: (i) 30 in the ratio 2 : 32 (ii) 10 in the ratio 3 : 33 (iii) 15 in the ratio 4 : 48 (iv) 25 in the ratio 5 : 10 (v) 20 in the ratio 6 : 40 (vi) 50 in the ratio 7 : 70

The same method as Q2, and the same check the other way round: in a decrease the first term is the smaller, so the fraction is less than 1 and the answer shrinks.

(i) Decrease 30 in the ratio 2 : 32
 $2 < 32$, so this really is a decrease ✓
 read the ratio as a fraction: $2/32$
 decreased value = $2/32 \times 30$
 = 1.875

(ii) Decrease 10 in the ratio 3 : 33
 $3 < 33$, so this really is a decrease ✓
 read the ratio as a fraction: $3/33$
 decreased value = $3/33 \times 10$
 = $10/11$

(iii) Decrease 15 in the ratio 4 : 48
 $4 < 48$, so this really is a decrease ✓
 read the ratio as a fraction: $4/48$
 decreased value = $4/48 \times 15$
 = 1.25

(iv) Decrease 25 in the ratio 5 : 10
 $5 < 10$, so this really is a decrease ✓
 read the ratio as a fraction: $5/10$
 decreased value = $5/10 \times 25$
 = 12.5

(v) Decrease 20 in the ratio 6 : 40
 $6 < 40$, so this really is a decrease ✓
 read the ratio as a fraction: $6/40$
 decreased value = $6/40 \times 20$
 = 3

(vi) Decrease 50 in the ratio 7 : 70
 $7 < 70$, so this really is a decrease ✓
 read the ratio as a fraction: $7/70$
 decreased value = $7/70 \times 50$
 = 5

Answer (i) 1.875 · (ii) $10/11$ · (iii) 1.25 · (iv) 12.5 · (v) 3 · (vi) 5

Please check — part (ii) gives $10/11$, a recurring decimal, where every other part comes out evenly. A ratio of 3 : 30 would give exactly 1.

4. If $a : b = 3 : 4$ and $b : c = 8 : 5$, find $a : b : c$.

The two ratios share b , but b is 4 in the first and 8 in the second. They cannot be joined until that term matches.

Step 1 Make the shared term agree

LCM of 4 and 8 is 8

Step 2 Scale the first ratio up

$a : b = 3 : 4$

multiply both terms by 2 → $a : b = 6 : 8$

a ratio is unchanged when both its terms are multiplied by the same number

Step 3 The second ratio already has $b = 8$

$$b : c = 8 : 5$$

Step 4 Join them on the shared b

$$a : b : c = 6 : 8 : 5$$

Step 5 Check

$$\text{take the first two terms: } 6 : 8 = 3 : 4 \checkmark$$

$$\text{take the last two: } 8 : 5 \checkmark$$

Answer 6 : 8 : 5

5. The price of a book increases from Rs 120 to Rs 180. Find the ratio of the increase.

The ratio of an increase compares the new value with the old one, in that order — as the book's Example 4 does with the pen.

Step 1 Write the ratio

$$\text{new price : old price} = 180 : 120$$

Step 2 Simplify

HCF of 180 and 120 is 60

$$180 \div 60 = 3 \text{ and } 120 \div 60 = 2$$

$$3 : 2$$

Step 3 Check it is an increase

$$3 > 2, \text{ so the first term is the larger } \checkmark$$

$$\text{the new price is } 3/2 \text{ — one and a half times — the old one, and } 120 \times 3/2 = 180 \checkmark$$

Answer 3 : 2

6. A water tank's level drops from 500 litres to 350 litres. Find the ratio of the decrease.

Step 1 Write the ratio, new against old

$$\text{new level : old level} = 350 : 500$$

Step 2 Simplify

HCF of 350 and 500 is 50

$$350 \div 50 = 7 \text{ and } 500 \div 50 = 10$$

$$7 : 10$$

Step 3 Check it is a decrease

$$7 < 10, \text{ so the first term is the smaller } \checkmark$$

$$500 \times 7/10 = 350 \checkmark$$

Answer 7 : 10

7. In a bakery the ratio of cakes sold to cookies is 5 : 6, and the ratio of cookies to pastries is 6 : 3. Find the ratio of cakes, cookies and pastries.

The shared quantity is cookies.

Step 1 Check the shared term

$$\text{cakes : cookies} = 5 : 6$$

$$\text{cookies : pastries} = 6 : 3$$

cookies is 6 in both, so nothing needs scaling this time

Step 2 Join them

$$\text{cakes : cookies : pastries} = 5 : 6 : 3$$

Step 3 Simplify if possible

5, 6 and 3 share no factor but 1, so it is already in its lowest terms

Step 4 Check

$$\text{first two terms: } 5 : 6 \checkmark \quad \text{last two: } 6 : 3 \checkmark$$

Answer 5 : 6 : 3

8. A train covers 360 km in 6 hours. Find the rate of speed.

A rate compares two different kinds of quantity — here distance against time — which is what separates it from a ratio.

Step 1 Write the formula

Rate of speed = distance covered ÷ time taken

Step 2 Put in the figures

$$= 360 \text{ km} \div 6 \text{ h}$$

Step 3 Divide

6 goes into 36 six times, so $360 \div 6 = 60$

$$= 60 \text{ km/h}$$

Step 4 Check

$$60 \times 6 = 360 \text{ km} \checkmark$$

the unit km/h is kept, unlike a ratio, because the two quantities are of different kinds.

Answer 60 km/h

9. A car travels 50 km in the first hour and 70 km in the second hour. Calculate its average speed.

Average rate = total distance ÷ total time. The two hourly figures are not averaged directly here, though with equal hours it comes to the same thing.

Step 1 Total distance

$$50 + 70 = 120 \text{ km}$$

Step 2 Total time

$$1 \text{ hour} + 1 \text{ hour} = 2 \text{ hours}$$

Step 3 Divide

$$\text{Average speed} = 120 \div 2$$

$$= 60 \text{ km/h}$$

Step 4 Check

$$60 \times 2 = 120 \text{ km} \checkmark$$

the car never actually travelled at 60 km/h in either hour — an average describes the whole journey, not any moment in it.

Answer 60 km/h

EXERCISE 5.2

1. Identify which of the following are in proportion: (i) 4 : 6 and 8 : 12 (ii) 5 : 10 and 8 : 16 (iii) 7 : 14 and 5 : 20 (iv) 12 : 18 and 4 : 6 (v) 9 : 3 and 6 : 2 (vi) 5 : 15 and 7 : 21

Two ratios are in proportion when the product of the extremes equals the product of the means.

In $a : b :: c : d$ the extremes are the outer pair a and d , and the means are the inner pair b and c .

So the test is simply: does $a \times d$ equal $b \times c$?

(i) 4 : 6 and 8 : 12

extremes are 4 and 12; means are 6 and 8

$$\text{product of extremes} = 4 \times 12 = 48$$

$$\text{product of means} = 6 \times 8 = 48$$

$$48 = 48, \text{ so they are in proportion}$$

(ii) 5 : 10 and 8 : 16

extremes are 5 and 16; means are 10 and 8

$$\text{product of extremes} = 5 \times 16 = 80$$

$$\text{product of means} = 10 \times 8 = 80$$

$$80 = 80, \text{ so they are in proportion}$$

(iii) 7 : 14 and 5 : 20

extremes are 7 and 20; means are 14 and 5

$$\text{product of extremes} = 7 \times 20 = 140$$

$$\text{product of means} = 14 \times 5 = 70$$

$$140 \neq 70, \text{ so they are not in proportion}$$

- (iv) $12 : 18$ and $4 : 6$
 extremes are 12 and 6; means are 18 and 4
 product of extremes = $12 \times 6 = 72$
 product of means = $18 \times 4 = 72$
 $72 = 72$, so they are in proportion
- (v) $9 : 3$ and $6 : 2$
 extremes are 9 and 2; means are 3 and 6
 product of extremes = $9 \times 2 = 18$
 product of means = $3 \times 6 = 18$
 $18 = 18$, so they are in proportion
- (vi) $5 : 15$ and $7 : 21$
 extremes are 5 and 21; means are 15 and 7
 product of extremes = $5 \times 21 = 105$
 product of means = $15 \times 7 = 105$
 $105 = 105$, so they are in proportion

Answer (i) in proportion · (ii) in proportion · (iii) not in proportion · (iv) in proportion · (v) in proportion · (vi) in proportion

2. Find the value of x if: (i) $12 : x = 18 : 24$ (ii) $5 : 15 = 7 : x$ (iii) $8 : x = 14 : 35$ (iv) $12 : 18 = x : 6$ (v) $6 : x = 2 : 4$ (vi) $x : 8 = 5 : 15$

The same rule as Q1, used the other way round. There the two products were compared; here they are set equal and the unknown is found.

Product of extremes = product of means, then divide both sides by whatever multiplies the x .

- (i) $12 : x = 18 : 24$
 product of extremes = product of means
 $12 \times 24 = x \times 18$
 $18x = 288$
 $x = 288 \div 18$
 $x = 16$
- (ii) $5 : 15 = 7 : x$
 product of extremes = product of means
 $5 \times x = 15 \times 7$
 $5x = 105$
 $x = 105 \div 5$
 $x = 21$
- (iii) $8 : x = 14 : 35$
 product of extremes = product of means
 $8 \times 35 = x \times 14$
 $14x = 280$
 $x = 280 \div 14$
 $x = 20$
- (iv) $12 : 18 = x : 6$
 product of extremes = product of means
 $12 \times 6 = 18 \times x$
 $18x = 72$
 $x = 72 \div 18$
 $x = 4$
- (v) $6 : x = 2 : 4$
 product of extremes = product of means
 $6 \times 4 = x \times 2$
 $2x = 24$
 $x = 24 \div 2$
 $x = 12$
- (vi) $x : 8 = 5 : 15$

product of extremes = product of means

$$x \times 15 = 8 \times 5$$

$$15x = 40$$

$$x = 40 \div 15$$

$$x = 8/3$$

Answer (i) $x = 16$ · (ii) $x = 21$ · (iii) $x = 20$ · (iv) $x = 4$ · (v) $x = 12$ · (vi) $x = 8/3$

Please check — part (iv) is printed with 'and' rather than '=' — it is read here as an equation, as the rest of the question is.

3. A cloth merchant charges Rs 1,200 for 5 metres of cloth. How much will 8 metres of the same cloth cost?

More cloth costs more money, so this is a direct proportion.

Step 1 Set up the proportion

metres : metres :: cost : cost

$$5 : 8 :: 1,200 : x$$

Step 2 Product of extremes = product of means

$$5 \times x = 8 \times 1,200$$

$$5x = 9,600$$

Step 3 Divide

$$x = 9,600 \div 5$$

$$x = \text{Rs } 1,920$$

Step 4 Check by the unitary method

$$1 \text{ metre costs } 1,200 \div 5 = \text{Rs } 240$$

$$8 \text{ metres cost } 240 \times 8 = \text{Rs } 1,920 \checkmark$$

Step 5 Check it points the right way

$$8 \text{ m is more than } 5 \text{ m, and Rs } 1,920 \text{ is more than Rs } 1,200 \checkmark$$

Answer Rs 1,920

4. If 4 pipes can fill a tank in 10 hours, how long will it take 6 pipes to fill the same tank?

More pipes means less time, so this is an inverse proportion — the one place this exercise most often goes wrong.

Step 1 Use the constant

in an inverse proportion the product stays the same

pipes $\times$ hours = a fixed amount of work

Step 2 Find that fixed amount

$$4 \times 10 = 40 \text{ pipe-hours of work}$$

Step 3 Share it among 6 pipes

$$6 \times x = 40$$

$$x = 40 \div 6$$

$$x = 20/3$$

$$x = 6 \frac{2}{3} \text{ hours}$$

Step 4 Put it in hours and minutes

$$2/3 \text{ of an hour} = 2/3 \times 60 = 40 \text{ minutes}$$

$$= 6 \text{ hours } 40 \text{ minutes}$$

Step 5 Check it points the right way

$$6 \text{ pipes is more than } 4, \text{ and } 6 \text{ h } 40 \text{ min is less than } 10 \text{ h } \checkmark$$

had this been treated as a direct proportion the answer would have been 15 hours — longer with more pipes, which cannot be right.

Answer $20/3$ hours, that is 6 hours 40 minutes

5. If a train travels 600 km in 5 hours, how long will it take to cover the same distance if its speed is reduced by half?

The distance does not change, so speed and time are in inverse proportion: halve one and the other doubles.

Step 1 The original speed

$$\text{speed} = \text{distance} \div \text{time}$$

$= 600 \div 5$
 $= 120 \text{ km/h}$
 Step 2 The reduced speed
 half of $120 = 60 \text{ km/h}$
 Step 3 The new time
 $\text{time} = \text{distance} \div \text{speed}$
 $= 600 \div 60$
 $= 10 \text{ hours}$
 Step 4 Check
 the speed was halved and the time doubled, $5 \text{ h} \rightarrow 10 \text{ h} \checkmark$
 and the product is unchanged: $120 \times 5 = 600$ and $60 \times 10 = 600 \checkmark$

Answer 10 hours

■ REVISION EXERCISE 5

1. Choose the correct option.

i. Increase of 30 to 50, the ratio is:

- a) 5 : 3 b) 3 : 5 c) 15 : 25 d) 10 : 5

The ratio of an increase is new : old, in that order.

Step 1 new : old = $50 : 30$

Step 2 HCF of 50 and 30 is 10

Step 3 $50 \div 10 = 5$ and $30 \div 10 = 3$

5 : 3

Check $5 > 3$, so it reads as an increase $\checkmark$, and $30 \times 5/3 = 50 \checkmark$

option (b) is the same ratio written backwards — it would describe a decrease.

Answer a) 5 : 3

ii. Types of proportion:

- a) 4 b) 5 c) 2 d) 3

The unit names direct proportion and inverse proportion.

direct — both quantities move the same way

inverse — one rises as the other falls

That is two kinds in all.

(c) 2

Answer c) 2

iii. Equality of two ratios is called:

- a) means b) extremes c) ratio d) proportional

Take each option in turn.

(a) means — the inner pair of a proportion, not the whole idea $\times$

(b) extremes — the outer pair $\times$

(c) ratio — one comparison on its own, not two set equal $\times$

(d) the equality of two ratios is a proportion $\checkmark$

The option is printed as 'proportional', which is the adjective — see the note.

Answer d) proportional

Please check — option (d) is printed as 'proportional'. The noun is 'proportion'; 'proportional' is the adjective, as in 'directly proportional'.

- iv. A car travels 150 km in 4 hours. What is its speed per hour?

a) 3.75 km/h b) 37.5 km/h c) 37 km/h d) 40 km/h

Step 1 speed = distance $\div$ time

Step 2 = $150 \div 4$

Step 3 4 goes into 15 three times with 3 left over

bringing down the 0 $\rightarrow$ 30; 4 goes into 30 seven times with 2 left over

add a decimal point and a 0 $\rightarrow$ 20; $4 \times 5 = 20$

= 37.5 km/h

Check $37.5 \times 4 = 150$ ✓

option (a) is the answer with the decimal point one place out, and (c) is it rounded down — both are the usual near misses.

Answer b) 37.5 km/h

- v. $p : q :: r : s$ is written as:

a) $p/q = r/s$ b) $s/q = p/r$ c) $q/p = s/r$ d) $r/q = s/p$

$p : q :: r : s$ says the ratio $p : q$ equals the ratio $r : s$.

A ratio $a : b$ written as a fraction is a/b — the first term on top.

$p : q$ becomes p/q

$r : s$ becomes r/s

Setting them equal: $p/q = r/s$

Check cross-multiplying gives $ps = qr$, which is product of extremes = product of means ✓

option (c) is each fraction turned upside down. That happens to be true as well, but (a) is the direct reading of the statement.

Answer a) $p/q = r/s$

Please check — option (c) $q/p = s/r$ is also a true statement, since turning both fractions over keeps the equality. Only (a) is the direct reading of $p : q :: r : s$, but a pupil choosing (c) has not made a mathematical mistake.

2. A school's student strength grows from 200 to 300. Find the ratio of the increase.

Step 1 Write it new against old

300 : 200

Step 2 Simplify

HCF of 300 and 200 is 100

$300 \div 100 = 3$ and $200 \div 100 = 2$

3 : 2

Step 3 Check

$3 > 2$, so it reads as an increase ✓

$200 \times 3/2 = 300$ ✓

Answer 3 : 2

3. A cyclist slows down from 40 km/h to 25 km/h. Find the ratio of the reduced speed.

Step 1 Write it new against old

25 : 40

Step 2 Simplify

HCF of 25 and 40 is 5

$25 \div 5 = 5$ and $40 \div 5 = 8$

5 : 8

Step 3 Check

$5 < 8$, so it reads as a decrease ✓

$40 \times 5/8 = 25$ ✓

Answer 5 : 8

4. In a company the ratio of full-time employees to part-time employees is 7 : 3, and the ratio of part-time employees to interns is 3 : 4. Find the ratio of full-time employees, part-time employees and interns.

The shared quantity is the part-time employees.

Step 1 Check the shared term

full-time : part-time = 7 : 3

part-time : interns = 3 : 4

part-time is 3 in both, so no scaling is needed

Step 2 Join them

full-time : part-time : interns = 7 : 3 : 4

Step 3 Check

first two terms 7 : 3 ✓ last two 3 : 4 ✓

7, 3 and 4 share no common factor, so it cannot be reduced

Answer 7 : 3 : 4

5. If 8 oranges cost Rs 120, find the price of (a) 1 orange (b) 15 oranges.

(a) One orange

price of one = total ÷ number

= $120 \div 8$

= Rs 15

(b) Fifteen oranges

15×15

= Rs 225

Check

$225 \div 15 = \text{Rs } 15 \text{ each } \checkmark$

15 oranges is nearly twice 8, and Rs 225 is nearly twice Rs 120 ✓

Answer (a) Rs 15 · (b) Rs 225





CHAPTER

Sets

EXERCISE 6.1

1. Which of the following is a set and which is not? (i) Players of Pakistan cricket team. (ii) Intelligent students of a class. (iii) Multiples of 5. (iv) All books in the library. (v) $A = \{\text{Ahmad, Najam, Akbar}\}$ (vi) $B = \{c, r, i, c, k, e, t\}$

A collection is a set only if it is well defined — there must be no argument about whether a given thing belongs to it or not.

Words of opinion, like 'intelligent', 'tall' or 'good', never make a set.

- (i) Players of Pakistan cricket team.

the squad is a fixed, named list — anyone can say whether a given player is in it

A set

- (ii) Intelligent students of a class.

'intelligent' has no agreed test, so two teachers would list different pupils. A collection must be well defined to be a set

Not a set

- (iii) Multiples of 5.

a number either divides by 5 or it does not, so membership is never in doubt

A set

- (iv) All books in the library.

the library's catalogue settles exactly which books belong

A set

- (v) $A = \{\text{Ahmad, Najam, Akbar}\}$

the members are listed outright, which is as well defined as a collection can be

A set

- (vi) $B = \{c, r, i, c, k, e, t\}$

it is a set, but it is written carelessly: c appears twice. A set holds distinct members, so it is really $\{c, r, i, k, e, t\}$ with 6 members, not 7

A set

Answer (i) set · (ii) not a set · (iii) set · (iv) set · (v) set · (vi) set

Please check — in part (vi) the letter c is printed twice. A set holds distinct members, so $B = \{c, r, i, k, e, t\}$ has 6 members, not 7.

2. Write the following sets in tabular form: (i) A = the letters in the word DREAM (ii) B = the set of any four colours (iii) C = the set of all natural numbers (iv) D = set of prime numbers less than 10 (v) $E = \{x \mid x \in \mathbb{Z} \wedge x \text{ is even}\}$

Tabular form means listing the members inside braces, separated by commas.

Where the list never ends, three dots carry it on — and they go at both ends if the set runs both ways.

- (i) A = the letters in the word DREAM

list the letters one at a time; none repeats here

$\{D, R, E, A, M\}$

- (ii) B = the set of any four colours

any four will do — the question says 'any', so other answers are equally right

$\{\text{red, blue, green, yellow}\}$

- (iii) C = the set of all natural numbers

the natural numbers begin at 1 and never end, so the dots are part of the answer

$\{1, 2, 3, 4, 5, \dots\}$

- (iv) D = set of prime numbers less than 10

test each number below 10: 2, 3, 5 and 7 have exactly two factors; 1 has only one, and 4, 6, 8, 9 have more than two

$\{2, 3, 5, 7\}$

(v) $E = \{x \mid x \in \mathbb{Z} \wedge x \text{ is even}\}$

$\mathbb{Z}$ is the integers, so the evens run away in both directions — the dots are needed at both ends

$\{\dots, -4, -2, 0, 2, 4, \dots\}$

Answer (i) $\{D, R, E, A, M\}$

(ii) $\{\text{red, blue, green, yellow}\}$

(iii) $\{1, 2, 3, 4, 5, \dots\}$

(iv) $\{2, 3, 5, 7\}$

(v) $\{\dots, -4, -2, 0, 2, 4, \dots\}$

3. Write the following sets in descriptive form: (i) $L = \{\text{January, July, June}\}$ (ii) $C = \{y \mid y \in P\}$ (iii) $M = \{\text{Ali, Ahmad, Hamza, Talha, Haseeb}\}$ (iv) $F = \{b, c, d, f, g, h, \dots\}$ (v) $G = \{50, 55, 60, 65, 70, 75\}$ (vi) $N = \{x \mid x \in \mathbb{N} \wedge x \leq 15\}$ (vii) $D = \{\pm 1, \pm 2, \pm 3, \dots, \pm 10\}$

Descriptive form says in words what the members have in common, instead of listing them.

Look for the single property that every member shares and that nothing outside the set has.

(i) $L = \{\text{January, July, June}\}$

the set of months beginning with J

(ii) $C = \{y \mid y \in P\}$

the set of all y that are members of P — read straight off the notation, since P is not given

(iii) $M = \{\text{Ali, Ahmad, Hamza, Talha, Haseeb}\}$

the set of five boys' names

(iv) $F = \{b, c, d, f, g, h, \dots\}$

the set of consonants of the English alphabet

(v) $G = \{50, 55, 60, 65, 70, 75\}$

the set of multiples of 5 from 50 to 75

(vi) $N = \{x \mid x \in \mathbb{N} \wedge x \leq 15\}$

the set of natural numbers up to and including 15

(vii) $D = \{\pm 1, \pm 2, \pm 3, \dots, \pm 10\}$

the set of integers from -10 to 10 excluding zero

Answer (i) the set of months beginning with J

(ii) the set of all y that are members of P — read straight off the notation, since P is not given

(iii) the set of five boys' names

(iv) the set of consonants of the English alphabet

(v) the set of multiples of 5 from 50 to 75

(vi) the set of natural numbers up to and including 15

(vii) the set of integers from -10 to 10 excluding zero

4. Write the following sets in set-builder notation: (i) $A = \text{set of odd numbers between 12 and 56}$ (ii) $B = \{15, 20, 25, 30, \dots, 50\}$ (iii) $C = \text{set of first five solar months}$ (iv) $D = \text{set of natural numbers less than 20 and divisible by 5}$ (v) $E = \text{set of rivers of Punjab}$ (vi) $F = \{7, 9, 11, 13, 15\}$

Set-builder notation names a typical member, then states the rule it must satisfy: $\{x \mid \text{the rule}\}$, read as 'the set of all x such that...'.
The bar is read 'such that' and the $\wedge$ is read 'and'.

(i) $A = \text{set of odd numbers between 12 and 56}$

$\{x \mid x \text{ is odd} \wedge 12 < x < 56\}$

(ii) $B = \{15, 20, 25, 30, \dots, 50\}$

$\{x \mid x \text{ is a multiple of 5} \wedge 15 \leq x \leq 50\}$

(iii) $C = \text{set of first five solar months}$

$\{x \mid x \text{ is one of the first five months of the solar year}\}$

(iv) $D = \text{set of natural numbers less than 20 and divisible by 5}$

$\{x \mid x \in \mathbb{N} \wedge x < 20 \wedge x \text{ is divisible by 5}\}$

(v) $E = \text{set of rivers of Punjab}$

$\{x \mid x \text{ is a river of Punjab}\}$
 (vi) $F = \{7, 9, 11, 13, 15\}$
 $\{x \mid x \text{ is odd} \wedge 7 \leq x \leq 15\}$

Answer (i) $\{x \mid x \text{ is odd} \wedge 12 < x < 56\}$
 (ii) $\{x \mid x \text{ is a multiple of } 5 \wedge 15 \leq x \leq 50\}$
 (iii) $\{x \mid x \text{ is one of the first five months of the solar year}\}$
 (iv) $\{x \mid x \in \mathbb{N} \wedge x < 20 \wedge x \text{ is divisible by } 5\}$
 (v) $\{x \mid x \text{ is a river of Punjab}\}$
 (vi) $\{x \mid x \text{ is odd} \wedge 7 \leq x \leq 15\}$

5. Fill in the blanks using $\in$ or $\notin$: (i) $2 \underline{\hspace{1cm}} \{2, 3, 5, 13\}$ (ii) $e \underline{\hspace{1cm}} \{a, e, i, o, u\}$ (iii) $12 \underline{\hspace{1cm}} \{2, 3, 4, 5\}$ (iv) $A \underline{\hspace{1cm}} \{A, F, G\}$
 (v) $-5 \underline{\hspace{1cm}} \{1, 2, 3, 4, 5\}$ (vi) $-19 \underline{\hspace{1cm}} \{-19, 15, 23\}$

$\in$ means 'is a member of' and $\notin$ means 'is not a member of'.

Look through the braces for the exact thing on the left — it either appears or it does not.

- (i) $2 \underline{\hspace{1cm}} \{2, 3, 5, 13\}$
 2 appears inside the braces
 $2 \in \{2, 3, 5, 13\}$
 (ii) $e \underline{\hspace{1cm}} \{a, e, i, o, u\}$
 e appears inside the braces
 $e \in \{a, e, i, o, u\}$
 (iii) $12 \underline{\hspace{1cm}} \{2, 3, 4, 5\}$
 12 is nowhere inside the braces
 $12 \notin \{2, 3, 4, 5\}$
 (iv) $A \underline{\hspace{1cm}} \{A, F, G\}$
 A appears inside the braces
 $A \in \{A, F, G\}$
 (v) $-5 \underline{\hspace{1cm}} \{1, 2, 3, 4, 5\}$
 -5 is nowhere inside the braces
 $-5 \notin \{1, 2, 3, 4, 5\}$
 (vi) $-19 \underline{\hspace{1cm}} \{-19, 15, 23\}$
 -19 appears inside the braces
 $-19 \in \{-19, 15, 23\}$

Answer (i) $\in$ · (ii) $\in$ · (iii) $\notin$ · (iv) $\in$ · (v) $\notin$ · (vi) $\in$

Please check — the question offers 'the symbol $\in$ or $\in$ ' — the second should be $\notin$.

EXERCISE 6.2

1. Each statement below has a mistake. Identify it and explain why it is wrong. (i) Set $A = \{7, 8, 9\}$ and Set $B = \{7, 8, 9\}$. Set A is a proper subset of Set B. (ii) Set $C = \{\text{camel, cat, eagle}\}$ and Set $D = \{\text{fish, frog}\}$. These are overlapping sets. (iii) Set $E = \{a, b, c, d\}$ and Set $F = \{1, 2, 3, 4\}$. Since both sets have 4 elements, they are equal sets. (iv) Set $P = \{A, B, C\}$ and Set $Q = \{C, B, A\}$. Since the order is different, Set P and Set Q are not equal. (v) Set $T = \{x, y, z\}$ and Set $U = \{x, y, z\}$. Since they have exactly the same elements, Set T and Set U are disjoint sets.

Each one confuses a pair of terms that sound alike. Hold the definitions apart first:

proper subset — every member is in the other set, and the other set has at least one more

improper subset — every member is in the other set, and nothing is left over

equal — the same members

equivalent — the same number of members

overlapping — at least one member shared

disjoint — no member shared

- (i) Set $A = \{7, 8, 9\}$ and Set $B = \{7, 8, 9\}$. Set A is a proper subset of Set B.

A and B have exactly the same members, so A is an improper subset of B — and the two sets are equal. A proper subset must leave at least one member of B out, and here none is left out.

(ii) Set $C = \{\text{camel, cat, eagle}\}$ and Set $D = \{\text{fish, frog}\}$. These are overlapping sets.

Overlapping sets share at least one member. C and D share none, so they are disjoint, not overlapping.

(iii) Set $E = \{a, b, c, d\}$ and Set $F = \{1, 2, 3, 4\}$. Since both sets have 4 elements, they are equal sets.

Having the same number of members makes them equivalent, not equal. Equal sets must have the same members, and no letter here is a number.

(iv) Set $P = \{A, B, C\}$ and Set $Q = \{C, B, A\}$. Since the order is different, Set P and Set Q are not equal.

Order does not matter in a set. P and Q hold exactly the same three members, so they are equal.

(v) Set $T = \{x, y, z\}$ and Set $U = \{x, y, z\}$. Since they have exactly the same elements, Set T and Set U are disjoint sets.

Disjoint means sharing nothing. These two share everything, so they are equal — the opposite of disjoint.

Answer (i) improper, not proper · (ii) disjoint, not overlapping · (iii) equivalent, not equal · (iv) they are equal — order does not matter · (v) they are equal, not disjoint

2. Are the following sets overlapping or disjoint? (i) $\{a, f, g, h, i\}$, $\{1, 3, 4, 5, 6\}$ (ii) $\{A, F, G, L\}$, $\{1, 3, G, H, L\}$ (iii) $\{3, 11, 12, 54\}$, $\{1, 2, 3, 4, 5\}$ (iv) $\{a, b, c, d\}$, $\{a, c, d, e\}$

Two sets overlap if they share at least one member, and are disjoint if they share none.

One shared member is enough to make them overlapping — they do not have to share most of their members.

(i) $\{a, f, g, h, i\}$ and $\{1, 3, 4, 5, 6\}$

go through the first set, looking for each member in the second
no member of one appears in the other

Disjoint

(ii) $\{A, F, G, L\}$ and $\{1, 3, G, H, L\}$

go through the first set, looking for each member in the second
shared members: $\{G, L\}$

Overlapping

(iii) $\{3, 11, 12, 54\}$ and $\{1, 2, 3, 4, 5\}$

go through the first set, looking for each member in the second
shared members: $\{3\}$

Overlapping

(iv) $\{a, b, c, d\}$ and $\{a, c, d, e\}$

go through the first set, looking for each member in the second
shared members: $\{a, c, d\}$

Overlapping

Answer (i) disjoint · (ii) overlapping · (iii) overlapping · (iv) overlapping

3. For each relationship below, create two sets of your own and explain the relationship: (i) proper subset (ii) overlapping sets (iii) equivalent sets.

The sets are yours to choose, so any pair that fits the definition is right. One worked answer for each:

(i) Proper subset

Set $A = \{2, 4\}$

Set $B = \{2, 4, 6, 8\}$

every member of A is in B, and B has 6 and 8 besides, so $A \subset B$

the 'proper' part is that something is left over — if B had been $\{2, 4\}$ it would be improper

(ii) Overlapping sets

Set $X = \{1, 2, 3\}$

Set $Y = \{3, 4, 5\}$

they share 3, so they overlap

- neither is a subset of the other, which is what separates overlapping from subset
- (iii) Equivalent sets
 Set $P = \{a, b, c\}$
 Set $Q = \{1, 2, 3\}$
 both have 3 members, so $n(P) = n(Q) = 3$ and $P \leftrightarrow Q$
 they are not equal — no letter is a number — which is the whole distinction

Answer Any pair fitting each definition. Worked examples above.

4. Write the proper and improper subsets of: (i) $\{3, 5, 32\}$ (ii) $\{a, b, c\}$ (iii) $\{11, -34\}$ (iv) $\{+3, 0, -3\}$ (v) $\{\text{Ali, Huma, Ahmad}\}$

A set with n members has 2^n subsets in all, because each member is either taken or left out — two choices, n times over.

Exactly one of them is improper: the whole set itself. All the rest, the empty set included, are proper.

- (i) $\{3, 5, 32\}$

the set has 3 members, so it has $2^3 = 8$ subsets in all

all 8: $\emptyset, \{3\}, \{5\}, \{32\}, \{3, 5\}, \{3, 32\}, \{5, 32\}, \{3, 5, 32\}$

the improper subset is the whole set itself: $\{3, 5, 32\}$

the other 7 are the proper subsets, including the empty set $\emptyset$

- (ii) $\{a, b, c\}$

the set has 3 members, so it has $2^3 = 8$ subsets in all

all 8: $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$

the improper subset is the whole set itself: $\{a, b, c\}$

the other 7 are the proper subsets, including the empty set $\emptyset$

- (iii) $\{11, -34\}$

the set has 2 members, so it has $2^2 = 4$ subsets in all

all 4: $\emptyset, \{11\}, \{-34\}, \{11, -34\}$

the improper subset is the whole set itself: $\{11, -34\}$

the other 3 are the proper subsets, including the empty set $\emptyset$

- (iv) $\{+3, 0, -3\}$

the set has 3 members, so it has $2^3 = 8$ subsets in all

all 8: $\emptyset, \{+3\}, \{0\}, \{-3\}, \{+3, 0\}, \{+3, -3\}, \{0, -3\}, \{+3, 0, -3\}$

the improper subset is the whole set itself: $\{+3, 0, -3\}$

the other 7 are the proper subsets, including the empty set $\emptyset$

- (v) $\{\text{Ali, Huma, Ahmad}\}$

the set has 3 members, so it has $2^3 = 8$ subsets in all

all 8: $\emptyset, \{\text{Ali}\}, \{\text{Huma}\}, \{\text{Ahmad}\}, \{\text{Ali, Huma}\}, \{\text{Ali, Ahmad}\}, \{\text{Huma, Ahmad}\}, \{\text{Ali, Huma, Ahmad}\}$

Ahmad}

the improper subset is the whole set itself: $\{\text{Ali, Huma, Ahmad}\}$

the other 7 are the proper subsets, including the empty set $\emptyset$

Answer (i) 8 subsets — 7 proper, 1 improper

(ii) 8 subsets — 7 proper, 1 improper

(iii) 4 subsets — 3 proper, 1 improper

(iv) 8 subsets — 7 proper, 1 improper

(v) 8 subsets — 7 proper, 1 improper

EXERCISE 6.3

1. Find the difference of the first set from the second set: (i) $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$ (ii) $C = \{5, 6, 7, 8\}$, $D = \{3, 5, 7, 9\}$ (iii) $E = \{1, 2, 3, 4\}$, $F = \{3, 4, 5, 6\}$ (iv) $G = \{l, m, n, o\}$, $H = \{p, q, r\}$

'The difference of X from Y ' means $Y - X$: start with Y and take away anything that is also in X .

The difference is not symmetrical — $A - B$ and $B - A$ are usually different sets, so the order matters more here than anywhere else in the unit.

- (i) $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$
 'the difference of A from B' means $B - A$
 take out of B everything that is also in A
 shared members: $\{c, d\}$
 $B - A = \{e, f\}$
- (ii) $C = \{5, 6, 7, 8\}$, $D = \{3, 5, 7, 9\}$
 'the difference of C from D' means $D - C$
 take out of D everything that is also in C
 shared members: $\{5, 7\}$
 $D - C = \{3, 9\}$
- (iii) $E = \{1, 2, 3, 4\}$, $F = \{3, 4, 5, 6\}$
 'the difference of E from F' means $F - E$
 take out of F everything that is also in E
 shared members: $\{3, 4\}$
 $F - E = \{5, 6\}$
- (iv) $G = \{l, m, n, o\}$, $H = \{p, q, r\}$
 'the difference of G from H' means $H - G$
 take out of H everything that is also in G
 shared members: none
 $H - G = \{p, q, r\}$

Answer (i) $\{e, f\}$ · (ii) $\{3, 9\}$ · (iii) $\{5, 6\}$ · (iv) $\{p, q, r\}$

2. If $U = \{1, 2, 3, \dots, 30\}$, $A = \{2, 4, 6, \dots, 20\}$, $B = \{1, 3, 5, \dots, 21\}$ and $C = \{21, 22, \dots, 30\}$, find: (i) $A' \cup B$ (ii) $A \cup (B \cap C)$ (iii) $(A \cap C) \cup (B \cap C)$ (iv) $(A \cup C) \cap B'$

Write out the four sets in full before starting — most mistakes here come from guessing what the dots stand for.

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$B = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$

$C = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

Work brackets first, exactly as in arithmetic.

- (i) $A' \cup B$

$A' = U - A$, everything in U that is not an even number up to 20

then join B to it

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

- (ii) $A \cup (B \cap C)$

work the bracket first: $B \cap C$ is what the odds up to 21 and the numbers 21 to 30 have in common

$B \cap C = \{21\}$

then join A to it

$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 21\}$

- (iii) $(A \cap C) \cup (B \cap C)$

$A \cap C$ — A stops at 20 and C starts at 21, so they share nothing

$A \cap C = \emptyset$

$B \cap C = \{21\}$

$\emptyset \cup \{21\} = \{21\}$

$= \{21\}$

- (iv) $(A \cup C) \cap B'$

$A \cup C$ is the evens up to 20 together with 21 to 30

$B' = U - B$, the numbers in U that are not odd numbers up to 21

then keep only what is in both

$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

Answer (i) {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

(ii) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 21}

(iii) {21}

(iv) {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30}

Please check — the question prints 'U = Find {1, 2, 3, ..., 30}' with a stray word, and gives A as {2, 4, 5, ..., 20}. Since 2, 4, 5 sets no pattern, A is read here as the even numbers to 20.

3. Find the union of the following sets: (i) {Ahmad, Ali, Rehman}, {1, 2, 3, 4} (ii) {Black, Blue, Pink, Red}, {Karachi, Lahore, Multan} (iii) {23, 43, 45, 73, 77}, {2, 8, 22, 54, 65, 77} (iv) {1, 3, 5, 7, 9}, {2, 4, 6, 8, 10, 12}

The union gathers every member of either set into one set.

Anything in both is written once — a set never repeats a member.

The members need not be of the same kind: names and numbers sit together quite happily.

(i) {Ahmad, Ali, Rehman} $\cup$ {1, 2, 3, 4}

the union takes every member of either set

no member is in both, so nothing repeats

= {1, 2, 3, 4, Ahmad, Ali, Rehman}

count: $3 + 4 - 0 = 7$ members ✓

(ii) {Black, Blue, Pink, Red} $\cup$ {Karachi, Lahore, Multan}

the union takes every member of either set

no member is in both, so nothing repeats

= {Black, Blue, Karachi, Lahore, Multan, Pink, Red}

count: $4 + 3 - 0 = 7$ members ✓

(iii) {23, 43, 45, 73, 77} $\cup$ {2, 8, 22, 54, 65, 77}

the union takes every member of either set

{77} is in both, so it is written only once

= {2, 8, 22, 23, 43, 45, 54, 65, 73, 77}

count: $5 + 6 - 1 = 10$ members ✓

(iv) {1, 3, 5, 7, 9} $\cup$ {2, 4, 6, 8, 10, 12}

the union takes every member of either set

no member is in both, so nothing repeats

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12}

count: $5 + 6 - 0 = 11$ members ✓

Answer (i) {1, 2, 3, 4, Ahmad, Ali, Rehman}

(ii) {Black, Blue, Karachi, Lahore, Multan, Pink, Red}

(iii) {2, 8, 22, 23, 43, 45, 54, 65, 73, 77}

(iv) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12}

REVISION EXERCISE 6

1. Choose the correct option.

- i. A set is written inside which brackets?

a) () b) { } c) [] d) { }

Each kind of bracket has its own job in mathematics.

(a) () group terms in arithmetic ✗

(b) { } braces hold the members of a set ✓

(c) [] square brackets group a bracket inside a bracket ✗

(d) { } is not a matching pair at all ✗

Answer b) { }

ii. The symbol for the universal set is:

- a) U b) n c) $\cup$ d) $\setminus$

(a) U stands for the universal set ✓

(b) n is intersection ✗

(c) $\cup$ is union ✗

(d) $\setminus$ is difference ✗

The letter U and the union sign $\cup$ look alike in print, which is exactly why (c) is offered.

Answer a) U

iii. What is the complement of A , if $U = \{1, 2, \dots, 9\}$ and $A = \{2, 4, 6, 8\}$?

- a) $\{1, 3, 5, 7, 9\}$ b) $\{1, 2, 3, 5, 7\}$ c) $\{1, 3, 5, 6, 9\}$ d) $\{3, 4, 5, 6, 7, 9\}$

$A' = U - A$: everything in U that is not in A .

Step 1 $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Step 2 cross out the members of A : 2, 4, 6, 8

Step 3 what is left: $\{1, 3, 5, 7, 9\}$

Check A has 4 members and U has 9, so A' must have $9 - 4 = 5$ ✓

options (b), (c) and (d) each keep a member of A , so each fails the test $A \cap A' = \emptyset$.

Answer a) $\{1, 3, 5, 7, 9\}$

iv. In a Venn diagram the universal set is shown by a:

- a) circle b) square c) triangle d) rectangle

Each set is drawn as a circle, and the circles sit inside a surrounding box that holds everything under discussion.

That box is drawn as a rectangle.

Option (a) is the trap: circles are used, but for the individual sets, not for U .

Answer d) rectangle

v. What is the union of A with the empty set?

- a) A b) U c) $\emptyset$ d) A^c

The union takes every member of either set.

The empty set has no members to contribute.

So nothing is added to A .

$A \cup \emptyset = A$

Check with an example $\{1, 2\} \cup \{\} = \{1, 2\}$ ✓

Compare: $A \cap \emptyset = \emptyset$, because nothing can be shared with a set that holds nothing.

Answer a) A

vi. Which set is a superset of $\{2, 4\}$?

- a) $\{2, 5, 6, 8\}$ b) $\{4, 8, 12\}$ c) $\{1, 2, 3\}$ d) $\{2, 4, 6, 8\}$

A superset must contain every member of $\{2, 4\}$ — both the 2 and the 4.

(a) $\{2, 5, 6, 8\}$ has 2 but no 4 ✗

(b) $\{4, 8, 12\}$ has 4 but no 2 ✗

(c) $\{1, 2, 3\}$ has 2 but no 4 ✗

(d) $\{2, 4, 6, 8\}$ has both ✓

Missing one member is enough to disqualify a set, which is why three of the four fail.

Answer d) $\{2, 4, 6, 8\}$

2. Fill in the blanks: (i) A Venn diagram is used to illustrate the relationships between _____. (ii) If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B =$ _____. (iii) The symbol used for the intersection of two sets is _____. (iv) The _____ of two sets includes all the elements from both sets. (v) If $U = \{1, 2, 3, 4, 5\}$ and $A = \{2, 4\}$, then $A' =$ _____.

(i) A Venn diagram is used to illustrate the relationships between _____.
the circles stand for sets and the overlaps for what they share
sets

(ii) If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B =$ _____.

take every member of either set; the 3 is in both, so it is written once

- $\{1, 2, 3, 4, 5\}$
- (iii) The symbol used for the intersection of two sets is ____.
 $\cap$ for intersection, $\cup$ for union — the one that holds water is union
 $\cap$
- (iv) The ____ of two sets includes all the elements from both sets.
 the union gathers everything; the intersection keeps only what is shared
 union
- (v) If $U = \{1, 2, 3, 4, 5\}$ and $A = \{2, 4\}$, then $A' =$ ____.
 $A' = U - A = \{1, 2, 3, 4, 5\} - \{2, 4\} = \{1, 3, 5\}$
 $\{1, 3, 5\}$

Answer (i) sets · (ii) $\{1, 2, 3, 4, 5\}$ · (iii) $\cap$ · (iv) union · (v) $\{1, 3, 5\}$

3. Define the following: (i) Define union, intersection and difference of two sets with examples. (ii) What is a universal set? (iii) What are De Morgan's first and second laws? (iv) Define a set with an example. (v) In how many ways can a set be represented?

- (i) Define union, intersection and difference of two sets with examples.
 The union $A \cup B$ holds every member of either set: $\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$. The intersection $A \cap B$ holds only what both have: $\{1, 2\} \cap \{2, 3\} = \{2\}$. The difference $A - B$ holds what is in A but not in B: $\{1, 2\} - \{2, 3\} = \{1\}$.
- (ii) What is a universal set?
 The universal set U holds every element under discussion, and every other set in the question is a subset of it. In a Venn diagram it is the rectangle the circles sit inside.
- (iii) What are De Morgan's first and second laws?
 First: $(A \cup B)' = A' \cap B'$ — the complement of a union is the intersection of the complements.
 Second: $(A \cap B)' = A' \cup B'$ — the complement of an intersection is the union of the complements.
 In both the bar moves inward and the sign flips over.
- (iv) Define a set with an example.
 A set is a well-defined collection of distinct objects, written inside braces — for example $A = \{2, 4, 6, 8\}$. Well defined means there is no argument about what belongs; distinct means nothing is listed twice.
- (v) In how many ways can a set be represented?
 Three: descriptive form, which says in words what the members have in common; tabular form, which lists them inside braces; and set-builder notation, which states the rule as $\{x \mid \text{the rule}\}$.

Answer See the five definitions above.

4. Find the intersection of the following sets: (i) $\{a, b, c, d\}$, $\{c, d, e, f\}$ (ii) $\{5, 6, 7, 8\}$, $\{3, 5, 7, 9\}$ (iii) $\{1, 2, 3, 4\}$, $\{3, 4, 5, 6\}$ (iv) $\{l, m, n, o\}$, $\{p, q, r\}$

- The intersection keeps only what both sets have.
 If they share nothing the answer is the empty set, written $\emptyset$ or $\{\}$ — not zero, and not left blank.
- (i) $\{a, b, c, d\} \cap \{c, d, e, f\}$
 take each member of the first set and look for it in the second
 c is in both ✓, d is in both ✓
 $= \{c, d\}$
- (ii) $\{5, 6, 7, 8\} \cap \{3, 5, 7, 9\}$
 take each member of the first set and look for it in the second
 5 is in both ✓, 7 is in both ✓
 $= \{5, 7\}$
- (iii) $\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\}$
 take each member of the first set and look for it in the second
 3 is in both ✓, 4 is in both ✓
 $= \{3, 4\}$
- (iv) $\{l, m, n, o\} \cap \{p, q, r\}$
 take each member of the first set and look for it in the second
 not one member appears in both

$= \{ \}$ — the sets are disjoint

Answer (i) $\{c, d\}$ · (ii) $\{5, 7\}$ · (iii) $\{3, 4\}$ · (iv) $\{ \}$

5. Verify that $A \cup A' = \emptyset$, where $U = \{1, 2, 5, 6, 11, 12\}$: (i) $A = \{1, 12\}$ (ii) $B = \{1, 2, 3\}$ (iii) $C = \{1, 2, 3, 13, 14\}$ (iv) $D = \{14, 15\}$

This cannot be verified, because it is not true. Section 6.5 of this unit states the two identities correctly:

$A \cap A' = \emptyset$ — a set and its complement share nothing

$A \cup A' = U$ — together they make up the whole universal set

The question has the union sign where the intersection belongs. Each part is worked below both ways, so the true identity is shown.

There is a second problem: a complement only exists for a set that sits inside U.

(i) $A = \{1, 12\}$

$A' = U - A = \{2, 5, 6, 11\}$

$A \cup A' = \{1, 12\} \cup \{2, 5, 6, 11\} = \{1, 2, 5, 6, 11, 12\}$

that is U, not $\emptyset$ — so the identity as printed is false

the true pair: $A \cup A' = U$ ✓ and $A \cap A' = \{ \} = \emptyset$ ✓

(ii) $B = \{1, 2, 3\}$

$\{3\}$ is not in U, so B is not a subset of U

a complement is U – the set, so B' cannot be formed and this part cannot be answered as printed

(iii) $C = \{1, 2, 3, 13, 14\}$

$\{3, 13, 14\}$ are not in U, so C is not a subset of U

a complement is U – the set, so C' cannot be formed and this part cannot be answered as printed

(iv) $D = \{14, 15\}$

$\{14, 15\}$ are not in U, so D is not a subset of U

a complement is U – the set, so D' cannot be formed and this part cannot be answered as printed

Answer The identity as printed is false — $A \cup A' = U$, while $A \cap A' = \emptyset$. Only part (i) has a set inside U; (ii), (iii) and (iv) cannot be answered as printed.

Please check — two separate problems in one question. The identity has u where it needs n, and three of the four sets contain elements that are not in U.

6. If $O = \{3, 5, 7, 9, 11, 13, 15\}$ and $P = \{2, 3, 5, 7, 11, 13, 17, 19\}$, find $P \cap O$ and $O - P$.

Step 1 $P \cap O$ — what both have

go through O, checking each member against P:

$3 \in P$ ✓ $5 \in P$ ✓ $7 \in P$ ✓ $9 \notin P$ ✗

$11 \in P$ ✓ $13 \in P$ ✓ $15 \notin P$ ✗

$P \cap O = \{3, 5, 7, 11, 13\}$

Step 2 $O - P$ — in O but not in P

the two that failed the test above are exactly these:

$O - P = \{9, 15\}$

Step 3 Check

O has 7 members; 5 are shared and 2 are not: $5 + 2 = 7$ ✓

Notice $O - P$ is not the same as $P - O$, which would be $\{2, 17, 19\}$ — the difference is never symmetrical.

Answer $P \cap O = \{3, 5, 7, 11, 13\}$ · $O - P = \{9, 15\}$

7. If $U = \{1, 2, 3, \dots, 20\}$ and $E = \{2, 4, 6, \dots, 18\}$, find E^c .

The complement E^c is $U - E$: everything in U that is not in E.

Step 1 Write both sets out

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$E = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

Step 2 Take E out of U

the odd numbers were never in E , so they all stay

Step 3 Watch the end of the list

E stops at 18, so 20 is in U but not in E and must be kept — this is the one the question is testing

Step 4 $E^c = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 20\}$

Step 5 Check

U has 20 members and E has 9, so E^c should have $20 - 9 = 11$ ✓

and $E \cap E^c = \emptyset$ ✓, $E \cup E^c = U$ ✓

Answer $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 20\}$





CHAPTER

Number Sequences and Patterns

EXERCISE 7.1

1. For each sequence, find the pattern and complete the next three terms: (i) 1, 3, 9, ... (ii) 1, 3, 7, 15, ... (iii) 9, 18, 36, 72, ... (iv) 7, 14, 21, 28, ... (v) 5, 8, 12, 17, 23, ...

Test for a constant difference first by subtracting each term from the next. If the differences are not constant, test for a constant ratio by dividing.

If neither is constant, look at the differences themselves — they often form a pattern of their own.

- (i) 1, 3, 9, ____, ____, ____

$$3 \div 1 = 3$$

$$9 \div 3 = 3$$

the ratio is 3 every time, so multiply by 3

$$9 \times 3 = 27$$

$$27 \times 3 = 81$$

$$81 \times 3 = 243$$

$$27, 81, 243$$

- (ii) 1, 3, 7, 15, ____, ____, ____

$$3 - 1 = 2$$

$$7 - 3 = 4$$

$$15 - 7 = 8$$

the differences themselves double: 2, 4, 8

so each term is double the one before, plus 1

$$\text{check: } 1 \times 2 + 1 = 3 \checkmark, 3 \times 2 + 1 = 7 \checkmark, 7 \times 2 + 1 = 15 \checkmark$$

$$15 \times 2 + 1 = 31$$

$$31 \times 2 + 1 = 63$$

$$63 \times 2 + 1 = 127$$

$$31, 63, 127$$

- (iii) 9, 18, 36, 72, ____, ____, ____

$$18 \div 9 = 2$$

$$36 \div 18 = 2$$

$$72 \div 36 = 2$$

the ratio is 2 every time, so multiply by 2

$$72 \times 2 = 144$$

$$144 \times 2 = 288$$

$$288 \times 2 = 576$$

$$144, 288, 576$$

- (iv) 7, 14, 21, 28, ____, ____, ____

$$14 - 7 = 7$$

$$21 - 14 = 7$$

$$28 - 21 = 7$$

the difference is 7 every time, so add 7

$$28 + 7 = 35$$

$$35 + 7 = 42$$

$$42 + 7 = 49$$

$$35, 42, 49$$

- (v) 5, 8, 12, 17, 23, ____, ____, ____

$$8 - 5 = 3$$

$$\begin{aligned}
 12 - 8 &= 4 \\
 17 - 12 &= 5 \\
 23 - 17 &= 6 \\
 \text{the difference itself goes up by 1 each time: } 3, 4, 5, 6 \\
 \text{so the next gaps are } 7, 8 \text{ and } 9 \\
 23 + 7 &= 30 \\
 30 + 8 &= 38 \\
 38 + 9 &= 47 \\
 30, 38, 47
 \end{aligned}$$

Answer (i) 27, 81, 243 · (ii) 31, 63, 127 · (iii) 144, 288, 576 · (iv) 35, 42, 49 · (v) 30, 38, 47

2. Find the general term of the following sequences: (i) 1, 3, 6, 10, 15, ... (ii) 1, 4, 9, 16, 25, ... (iii) 6, 13, 20, 27, ... (iv) 2, 6, 18, 54, ...

The general term is a formula in n that gives the term in position n . Decide first which kind of sequence it is:

arithmetic — $a_n = a_1 + (n - 1)d$

geometric — $a_n = a_1 \times r^{(n-1)}$

neither — look for squares, triangular numbers, or a pattern in the differences

- (i) 1, 3, 6, 10, 15, ...

the differences are 2, 3, 4, 5 — rising by 1, so this is not arithmetic

these are the triangular numbers: 1, 1+2, 1+2+3, 1+2+3+4

the sum of the first n whole numbers is $n(n + 1)/2$

check $n = 4$: $4 \times 5 \div 2 = 10$ ✓

$$a_n = n(n + 1)/2$$

- (ii) 1, 4, 9, 16, 25, ...

$$1 = 1^2, 4 = 2^2, 9 = 3^2, 16 = 4^2, 25 = 5^2$$

each term is the square of its own position

check $n = 5$: $5^2 = 25$ ✓

$$a_n = n^2$$

- (iii) 6, 13, 20, 27, ...

$13 - 6 = 7$, $20 - 13 = 7$, $27 - 20 = 7$ — arithmetic, $d = 7$

$$a_n = a_1 + (n - 1)d$$

$$= 6 + (n - 1) \times 7$$

$$= 6 + 7n - 7$$

$$= 7n - 1$$

check $n = 3$: $21 - 1 = 20$ ✓

$$a_n = 7n - 1$$

- (iv) 2, 6, 18, 54, ...

$6 \div 2 = 3$, $18 \div 6 = 3$, $54 \div 18 = 3$ — geometric, $r = 3$

$$a_n = a_1 \times r^{(n-1)}$$

$$= 2 \times 3^{(n-1)}$$

check $n = 4$: $2 \times 3^3 = 2 \times 27 = 54$ ✓

$$a_n = 2 \times 3^{(n-1)}$$

Answer (i) $a_n = n(n + 1)/2$ · (ii) $a_n = n^2$ · (iii) $a_n = 7n - 1$ · (iv) $a_n = 2 \times 3^{(n-1)}$

3. Find the 15th term from the following general terms: (i) $a_n = 15n - 4$ (ii) $a_n = 8/(4n - 1)$ (iii) $a_n = 4n^2 - 2$ (iv) $a_n = n + 5$

The general term is a formula, so finding the 15th term is simply a matter of putting $n = 15$ into it. Work powers before multiplications, and multiplications before additions — BODMAS applies here as everywhere.

- (i) $a_n = 15n - 4$

put $n = 15$

$$a_{15} = 15 \times 15 - 4$$

$$= 225 - 4$$

$$\begin{aligned}
 &= 221 \\
 \text{(ii) } a_n &= 8/(4n - 1) \\
 \text{put } n &= 15 \\
 a_{15} &= 8 \div (4 \times 15 - 1) \\
 &= 8 \div (60 - 1) \\
 &= 8 \div 59 \\
 &= 8/59 \\
 \text{(iii) } a_n &= 4n^2 - 2 \\
 \text{put } n &= 15 \\
 a_{15} &= 4 \times 15^2 - 2 \\
 \text{square before multiplying: } 15^2 &= 225 \\
 &= 4 \times 225 - 2 \\
 &= 900 - 2 \\
 &= 898 \\
 \text{(iv) } a_n &= n + 5 \\
 \text{put } n &= 15 \\
 a_{15} &= 15 + 5 \\
 &= 20
 \end{aligned}$$

Answer (i) 221 · (ii) 8/59 · (iii) 898 · (iv) 20

Please check — part (ii) gives 8/59, which does not reduce. The method is right but the figure is an odd one for a Grade 7 answer.

4. The general terms are given. List the first four terms of each sequence: (i) $a_n = 3 + 2(n - 1)$ (ii) $a_n = n$ (iii) $a_n = 4n^2 + 2$ (iv) $a_n = n - 5$

Put $n = 1$, then 2, then 3, then 4 — the position number goes into the formula, and the term comes out.

In (iv) the answers are negative, which is perfectly allowed: a sequence need not start at a positive number.

$$\text{(i) } a_n = 3 + 2(n - 1)$$

$$n = 1 \rightarrow 3$$

$$n = 2 \rightarrow 5$$

$$n = 3 \rightarrow 7$$

$$n = 4 \rightarrow 9$$

$$3, 5, 7, 9$$

$$\text{(ii) } a_n = n$$

$$n = 1 \rightarrow 1$$

$$n = 2 \rightarrow 2$$

$$n = 3 \rightarrow 3$$

$$n = 4 \rightarrow 4$$

$$1, 2, 3, 4$$

$$\text{(iii) } a_n = 4n^2 + 2$$

$$n = 1 \rightarrow 6$$

$$n = 2 \rightarrow 18$$

$$n = 3 \rightarrow 38$$

$$n = 4 \rightarrow 66$$

$$6, 18, 38, 66$$

$$\text{(iv) } a_n = n - 5$$

$$n = 1 \rightarrow -4$$

$$n = 2 \rightarrow -3$$

$$n = 3 \rightarrow -2$$

$$n = 4 \rightarrow -1$$

$$-4, -3, -2, -1$$

Answer (i) 3, 5, 7, 9

(ii) 1, 2, 3, 4

(iii) 6, 18, 38, 66

(iv) -4, -3, -2, -1

5. Alex walks 1 mile on the first day, 2 miles on the second, 3 on the third, and continues the pattern. How many miles will he walk on the 15th day?

Step 1 Find the rule

$2 - 1 = 1$ and $3 - 2 = 1$, so the difference is 1 — arithmetic with $a_1 = 1$ and $d = 1$

Step 2 Write the general term

$$a_n = a_1 + (n - 1)d$$

$$= 1 + (n - 1) \times 1$$

$$= n$$

Step 3 Put $n = 15$

$$a_{15} = 15 \text{ miles}$$

Step 4 Check

the miles walked match the day number all the way along — day 1 gives 1, day 2 gives 2 ✓

note the question asks for the miles on the 15th day, not the total over 15 days, which would be

$$15 \times 16 \div 2 = 120 \text{ miles.}$$

Answer 15 miles

6. Sam runs 3 laps on the first day of a training programme, and 2 more laps each day after that. How many laps will he run on the 12th day?

Step 1 Name the parts

first term $a_1 = 3$

common difference $d = 2$

position $n = 12$

Step 2 Use the formula

$$a_n = a_1 + (n - 1)d$$

$$a_{12} = 3 + (12 - 1) \times 2$$

$$= 3 + 11 \times 2$$

$$= 3 + 22$$

$$= 25 \text{ laps}$$

Step 3 Check by listing

3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25 — the twelfth is 25 ✓

it is $(n - 1)$, not n : on day 1 nothing has been added yet. Using 12×2 would give 27, the commonest error here.

Answer 25 laps

REVISION EXERCISE 7

1. Choose the correct option.

- i. What is the next term in the sequence 2, 5, 8, 11, ...?

a) 14 b) 13 c) 15 d) 12

Step 1 find the difference

$$5 - 2 = 3 \quad 8 - 5 = 3 \quad 11 - 8 = 3$$

Step 2 the difference is 3 every time, so add 3

$$11 + 3 = 14$$

Check 2, 5, 8, 11, 14 — the gaps stay at 3 ✓

Answer a) 14

- ii. What is the 6th term in the geometric sequence 3, 6, 12, 24, ...?

a) 36 b) 48 c) 96 d) 72

Step 1 find the ratio

$$6 \div 3 = 2 \quad 12 \div 6 = 2 \quad 24 \div 12 = 2$$

Step 2 $a_n = a_1 \times r^{(n-1)}$

$$a_6 = 3 \times 2^{(6-1)}$$

$$= 3 \times 2^5$$

$$= 3 \times 32$$

$$= 96$$

Check by listing 3, 6, 12, 24, 48, 96 — the sixth is 96 ✓

option (b) 48 is the 5th term — the usual slip of counting the steps as n rather than $n - 1$.

Answer c) 96

- iii. Find the general term for the sequence 1, 4, 7, 10, ...

a) $a_n = 3n + 1$ b) $a_n = 2n + 2$ c) $a_n = 3n - 2$ d) $a_n = n + 3$

Step 1 $4 - 1 = 3$, so $d = 3$ and $a_1 = 1$

Step 2 $a_n = a_1 + (n - 1)d$

$$= 1 + (n - 1) \times 3$$

$$= 1 + 3n - 3$$

$$= 3n - 2$$

Check $n = 1 \rightarrow 1$ ✓ $n = 2 \rightarrow 4$ ✓ $n = 4 \rightarrow 10$ ✓

option (a) $3n + 1$ gives 4, 7, 10 — the right gaps but starting in the wrong place. Always test $n = 1$.

Answer c) $a_n = 3n - 2$

- iv. What is the 7th term in the arithmetic sequence 10, 15, 20, 25, ...?

a) 40 b) 45 c) 50 d) 55

Step 1 $15 - 10 = 5$, so $d = 5$ and $a_1 = 10$

Step 2 $a_7 = 10 + (7 - 1) \times 5$

$$= 10 + 6 \times 5$$

$$= 10 + 30$$

$$= 40$$

Check by listing 10, 15, 20, 25, 30, 35, 40 ✓

option (b) 45 comes from using 7×5 instead of 6×5 .

Answer a) 40

- v. Which of the following is an arithmetic sequence?

a) 3, 6, 12, 24, ... b) 5, 10, 15, 20, ... c) 2, 4, 8, 16, ... d) 1, 3, 6, 10, ...

An arithmetic sequence has a constant difference.

(a) 3, 6, 12, 24 — gaps 3, 6, 12. Not constant $\times$ (it is geometric, $\times 2$)

(b) 5, 10, 15, 20 — gaps 5, 5, 5 ✓

(c) 2, 4, 8, 16 — gaps 2, 4, 8. Not constant $\times$ (geometric, $\times 2$)

(d) 1, 3, 6, 10 — gaps 2, 3, 4. Not constant $\times$ (the triangular numbers)

Answer b) 5, 10, 15, 20, ...

- vi. What is the n th term of the arithmetic sequence 2, 6, 10, 14, ...?

a) $a_n = 2n + 1$ b) $a_n = 4n - 2$ c) $a_n = 2n - 1$ d) $a_n = 4n + 2$

Step 1 $6 - 2 = 4$, so $d = 4$ and $a_1 = 2$

Step 2 $a_n = 2 + (n - 1) \times 4$

$$= 2 + 4n - 4$$

$$= 4n - 2$$

Check $n = 1 \rightarrow 2$ ✓ $n = 2 \rightarrow 6$ ✓ $n = 4 \rightarrow 14$ ✓

option (d) $4n + 2$ has the right gaps but gives 6, 10, 14 — it starts one term too late.

Answer b) $a_n = 4n - 2$

2. Fill in the blanks: (i) The 5th term of the sequence 2, 4, 6, 8, x is _____. (ii) In the sequence 10, 15, 20, 25 the common difference is _____. (iii) The difference between consecutive terms in an arithmetic sequence is called the _____. (iv) The first term of a sequence is often denoted as _____. (v) The general formula for the nth term of an arithmetic sequence is _____.

- (i) The 5th term of the sequence 2, 4, 6, 8, x is _____.
the difference is 2 each time, so $x = 8 + 2 = 10$
10
- (ii) In the sequence 10, 15, 20, 25 the common difference is _____.
 $15 - 10 = 5$, and the same gap holds all along
5
- (iii) The difference between consecutive terms in an arithmetic sequence is called the _____.
it is called 'common' because it is the same between every pair
common difference
- (iv) The first term of a sequence is often denoted as _____.
the subscript is the position, so the first term is a_1
 a_1
- (v) The general formula for the nth term of an arithmetic sequence is _____.
start at the first term and add the common difference $(n - 1)$ times — once fewer than the position, because the first term has had no addition yet
 $a_n = a_1 + (n - 1)d$

Answer (i) 10 · (ii) 5 · (iii) common difference · (iv) a_1 · (v) $a_n = a_1 + (n - 1)d$

3. What number is subtracted to make each sequence? (i) 18, 16, 14, ... (ii) 100, 80, 70, ... (iii) 30, 27, 24, 21, ... (iv) 50, 45, 40, 35, ...

Subtract each term from the one before it. If the answer is the same every time, that is the number being subtracted.

- (i) 18, 16, 14, ...
 $18 - 16 = 2$
 $16 - 14 = 2$
the same gap every time, so 2 is subtracted
- (ii) 100, 80, 70, ...
 $100 - 80 = 20$
 $80 - 70 = 10$
the gaps are not the same, so no single number is subtracted
this part cannot be answered as printed — see the note
- (iii) 30, 27, 24, 21, ...
 $30 - 27 = 3$
 $27 - 24 = 3$
 $24 - 21 = 3$
the same gap every time, so 3 is subtracted
- (iv) 50, 45, 40, 35, ...
 $50 - 45 = 5$
 $45 - 40 = 5$
 $40 - 35 = 5$
the same gap every time, so 5 is subtracted

Answer (i) 2 · (ii) no constant difference · (iii) 3 · (iv) 5

Please check — part (ii) has no answer as printed: $100 - 80 = 20$ but $80 - 70 = 10$. Either 100, 90, 80 or 100, 80, 60 would give a constant difference.

4. What number is multiplied to make each sequence? (i) 3, 6, 12, 24, ... (ii) 5, 10, 20, 40, ... (iii) 1, 2, 4, 8, ... (iv) 2, 4, 8, 16, ...

Divide each term by the one before it. If the answer is the same every time, that is the number being multiplied — the common ratio.

- (i) 3, 6, 12, 24, ...

- $6 \div 3 = 2$
 $12 \div 6 = 2$
 $24 \div 12 = 2$
 the same ratio every time, so 2 is multiplied
 (ii) 5, 10, 20, 40, ...
 $10 \div 5 = 2$
 $20 \div 10 = 2$
 $40 \div 20 = 2$
 the same ratio every time, so 2 is multiplied
 (iii) 1, 2, 4, 8, ...
 $2 \div 1 = 2$
 $4 \div 2 = 2$
 $8 \div 4 = 2$
 the same ratio every time, so 2 is multiplied
 (iv) 2, 4, 8, 16, ...
 $4 \div 2 = 2$
 $8 \div 4 = 2$
 $16 \div 8 = 2$
 the same ratio every time, so 2 is multiplied

Answer (i) 2 · (ii) 2 · (iii) 2 · (iv) 2

5. Use the pattern rule to make a pattern: (i) start at 5 and add 10 each time (ii) start at 4 and multiply by 10 each time.

- (i) Start at 5, add 10 each time
 5
 $5 + 10 = 15$
 $15 + 10 = 25$
 $25 + 10 = 35$
 $35 + 10 = 45$
 5, 15, 25, 35, 45, ...
 arithmetic, $d = 10$, so $a_n = 10n - 5$
 (ii) Start at 4, multiply by 10 each time
 4
 $4 \times 10 = 40$
 $40 \times 10 = 400$
 $400 \times 10 = 4,000$
 $4,000 \times 10 = 40,000$
 4, 40, 400, 4,000, 40,000, ...
 geometric, $r = 10$, so $a_n = 4 \times 10^{(n-1)}$

Notice how much faster the geometric one grows: after five terms the first has reached 45 and the second 40,000.

Answer (i) 5, 15, 25, 35, 45, ... · (ii) 4, 40, 400, 4,000, 40,000, ...

Please check — part (ii) is printed 'multiply 10 each time'; it should read 'multiply by 10 each time'.

6. Find the 15th term of $a_n = 8/(4n - 3)$.

Step 1 Put $n = 15$ into the formula

$$a_{15} = 8 \div (4 \times 15 - 3)$$

Step 2 Work the bracket first

$$4 \times 15 = 60$$

$$60 - 3 = 57$$

Step 3 Divide

$$= 8/57$$

Step 4 Check it will not reduce

$8 = 2 \times 2 \times 2$ and $57 = 3 \times 19$ — no common factor, so $8/57$ is already in its lowest terms

the terms of this sequence get smaller as n grows, because the bottom of the fraction keeps rising.

Answer $8/57$

Please check — $8/57$ does not reduce and is an unusual answer for a Grade 7 sequence question. The same oddity appears in Ex 7.1 Q3(ii).

7. A water lily doubles in size every day. If it covers 1 square foot on the first day, how many square feet will it cover on the 10th day?

Doubling means a constant ratio, so this is a geometric sequence.

Step 1 Name the parts

$$a_1 = 1 \text{ square foot}$$

$$r = 2$$

$$n = 10$$

Step 2 Use the formula

$$a_n = a_1 \times r^{(n-1)}$$

$$a_{10} = 1 \times 2^{(10-1)}$$

$$= 2^9$$

Step 3 Work out the power

$$2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$$

$$2^6 = 64, 2^7 = 128, 2^8 = 256, 2^9 = 512$$

$$= 512 \text{ square feet}$$

Step 4 Check by listing the days

1, 2, 4, 8, 16, 32, 64, 128, 256, 512 — ten days ✓

it is 2^9 , not 2^{10} : the lily is already 1 square foot on day 1, so it doubles nine times over the nine nights that follow. Using 2^{10} would give 1,024.

Answer 512 square feet



8

CHAPTER

Algebraic Expressions

EXERCISE 8.1

1. Identify the variable and constant in each expression: (i) $5x + 7$ (ii) $3a^2 - 4b + 9$ (iii) $6y - 12$ (iv) $2x^2 - 5$ (v) $12 + 3z + t$ (vi) $-7r + 6$ (vii) $12 - 3z + t$ (viii) $6x^4 + 56y$

A variable is a letter standing for a value that can change. A constant is a fixed number.

The number in front of a variable is its coefficient. It is a constant, but it is attached to the variable rather than standing alone.

Where no number is written before a letter, a 1 is understood.

(i) $5x + 7$

variables: x

constants: 5 and 7

(ii) $3a^2 - 4b + 9$

variables: a, b

constants: 3, 4 and 9

(iii) $6y - 12$

variables: y

constants: 6 and 12

(iv) $2x^2 - 5$

variables: x

constants: 2 and 5

(v) $12 + 3z + t$

variables: z, t

constants: 12, 3 and the understood 1 before t

(vi) $-7r + 6$

variables: r

constants: -7 and 6

(vii) $12 - 3z + t$

variables: z, t

constants: 12, 3 and the understood 1 before t

(viii) $6x^4 + 56y$

variables: x, y

constants: 6 and 56

Answer (i) variables x ; constants 5 and 7

(ii) variables a, b ; constants 3, 4 and 9

(iii) variables y ; constants 6 and 12

(iv) variables x ; constants 2 and 5

(v) variables z, t ; constants 12, 3 and the understood 1 before t

(vi) variables r ; constants -7 and 6

(vii) variables z, t ; constants 12, 3 and the understood 1 before t

(viii) variables x, y ; constants 6 and 56

2. Identify whether the following are open or closed sentences: (i) $4x + 5 = 9$ (ii) $6 + 7 = 13$ (iii) $x^2 + 5x = 12$ (iv) $2a + 6 = b - 3$ (v) $10 - 5 = 4$ (vi) $3y + 7 = 21$ (vii) $4 + 9 = 13$ (viii) $5p + 2q = 18$

A sentence is open if it contains a variable, so it cannot be judged true or false until the variable is given a value.

It is closed if it can be judged as it stands.

A closed sentence may be true or false — closed does not mean correct. Part (v) is the one testing that.

(i) $4x + 5 = 9$

it contains the variable x , so it cannot be judged until x is known

Open

(ii) $6 + 7 = 13$

no variable, and $6 + 7$ really is 13 — a closed sentence that is true

Closed

(iii) $x^2 + 5x = 12$

it contains x

Open

(iv) $2a + 6 = b - 3$

it contains two variables, a and b

Open

(v) $10 - 5 = 4$

no variable, and $10 - 5 = 5$, not 4 — a closed sentence that is false. Closed does not mean true

Closed

(vi) $3y + 7 = 21$

it contains y

Open

(vii) $4 + 9 = 13$

no variable, and the statement is true

Closed

(viii) $5p + 2q = 18$

it contains p and q

Open

Answer (i) open · (ii) closed · (iii) open · (iv) open · (v) closed · (vi) open · (vii) closed · (viii) open

3. Group the following into like and unlike terms: (i) $3x$, $5y$, $7x$ (ii) $2a^2$, $4a$, $6a^2$ (iii) $8p$, $-p$, $9q$ (iv) t^2 , $2t^2$, $-t$ (v) $2b$, $-3b$, $5b^2$ (vi) $4m^2$, $3m$, $7m^2$ (vii) x^3 , $5x^2$, x^3 (viii) y , $2y^2$, $6y$

Terms are like only if they have exactly the same letters raised to exactly the same powers.

The coefficients play no part — $2a^2$ and $6a^2$ are alike however different the numbers in front.

a and a^2 are not alike, which is the distinction this question keeps testing.

(i) $3x$, $5y$, $7x$

$3x$ and $7x$ both have x to the power 1

like: $3x$ and $7x$

unlike: $5y$

(ii) $2a^2$, $4a$, $6a^2$

$2a^2$ and $6a^2$ both have a^2 , while $4a$ has a to the power 1 — the power must match, not just the letter

like: $2a^2$ and $6a^2$

unlike: $4a$

(iii) $8p$, $-p$, $9q$

$8p$ and $-p$ both have p

like: $8p$ and $-p$

unlike: $9q$

(iv) t^2 , $2t^2$, $-t$

t^2 and $2t^2$ both have t^2

like: t^2 and $2t^2$

unlike: $-t$

(v) $2b$, $-3b$, $5b^2$

$2b$ and $-3b$ both have b

like: $2b$ and $-3b$

unlike: $5b^2$

- (vi) $4m^2$, $3m$, $7m^2$
 $4m^2$ and $7m^2$ both have m^2
 like: $4m^2$ and $7m^2$
 unlike: $3m$
- (vii) x^3 , $5x^2$, x^3
 the two x^3 terms match exactly
 like: x^3 and x^3
 unlike: $5x^2$
- (viii) y , $2y^2$, $6y$
 y and $6y$ both have y to the power 1
 like: y and $6y$
 unlike: $2y^2$

Answer (i) like: $3x$ and $7x$; unlike: $5y$
 (ii) like: $2a^2$ and $6a^2$; unlike: $4a$
 (iii) like: $8p$ and $-p$; unlike: $9q$
 (iv) like: t^2 and $2t^2$; unlike: $-t$
 (v) like: $2b$ and $-3b$; unlike: $5b^2$
 (vi) like: $4m^2$ and $7m^2$; unlike: $3m$
 (vii) like: x^3 and x^3 ; unlike: $5x^2$
 (viii) like: y and $6y$; unlike: $2y^2$

4. Identify whether the following are expressions, equations or inequalities: (i) $5y + 7 = 12$ (ii) $2a + 3b > 10$ (iii) $4m - 7$ (iv) $8t + 3 = 6t - 5$ (v) $3x + 4$ (vi) $a^2 + b = c$ (vii) $x + 6 \square 2x \square 1$ (viii) $9p + 2q \leq 15$

Look for the relation sign, and only at the relation sign:

no sign at all → an expression

an = sign → an equation

a $<$, $>$, $\leq$ or $\geq$ sign → an inequality

The number of variables makes no difference.

- (i) $5y + 7 = 12$
 it has an equals sign and one variable
 equation
- (ii) $2a + 3b > 10$
 it has a $>$ sign rather than an equals sign
 inequality
- (iii) $4m - 7$
 no relation sign at all — nothing is being claimed, so it is just an expression
 expression
- (iv) $8t + 3 = 6t - 5$
 an equals sign, with the variable on both sides
 equation
- (v) $3x + 4$
 no relation sign
 expression
- (vi) $a^2 + b = c$
 an equals sign, with three variables
 equation
- (vii) $x + 6 \square 2x \square 1$
 the two relation signs have come out as boxes — see the note
 cannot be answered as printed
- (viii) $9p + 2q \leq 15$
 the $\leq$ sign makes it an inequality
 inequality

Answer (i) equation · (ii) inequality · (iii) expression · (iv) equation · (v) expression · (vi) equation · (vii) cannot be answered as printed · (viii) inequality

Please check — part (vii) prints boxes where its relation signs should be, so it cannot be classified as printed.

EXERCISE 8.2

1. Determine whether the following are polynomials: (i) $3x^2 + 5x - 4$ (ii) $1/x + 4$ (iii) $7a^3 - 2a^{-2} + a$ (iv) $\sqrt{x} + 2x - 7$ (v) $4x + 9$ (vi) $5y^2 - 6y + 8$ (vii) $2/x + x - 3$ (viii) $2p^3 + 3p + 6$ (ix) $6x - 1 + 2x^3$ (x) $x^2 + 5x - 7$

An expression is a polynomial only if every power of every variable is a whole number — 0, 1, 2, 3 and so on.

Two things disqualify an expression at once:

a variable in a denominator, because $1/x$ is x^{-1}

a variable under a root, because $\sqrt{x}$ is $x^{1/2}$

- (i) $3x^2 + 5x - 4$

every power of x is a whole number

Polynomial

- (ii) $1/x + 4$

$1/x$ is x^{-1} , and a negative power is not allowed in a polynomial

Not a polynomial

- (iii) $7a^3 - 2a^{-2} + a$

a^{-2} has a negative power

Not a polynomial

- (iv) $\sqrt{x} + 2x - 7$

$\sqrt{x}$ is $x^{1/2}$, and a fractional power is not allowed

Not a polynomial

- (v) $4x + 9$

x has power 1, which is a whole number

Polynomial

- (vi) $5y^2 - 6y + 8$

all the powers are whole numbers

Polynomial

- (vii) $2/x + x - 3$

$2/x$ is $2x^{-1}$ — a negative power

Not a polynomial

- (viii) $2p^3 + 3p + 6$

all whole-number powers

Polynomial

- (ix) $6x - 1 + 2x^3$

all whole-number powers, though they are not written in order

Polynomial

- (x) $x^2 + 5x - 7$

all whole-number powers

Polynomial

Answer (i) yes · (ii) no · (iii) no · (iv) no · (v) yes · (vi) yes · (vii) no · (viii) yes · (ix) yes · (x) yes

2. Determine whether the following are monomials, binomials or trinomials: (i) $3x^2 + 5x - 4$ (ii) $7a^3 - 2a^2 + a$ (iii) $4x + 9$ (iv) $5y^2$ (v) 6 (vi) $x^2 + 7$

Count the terms — the parts separated by + and - signs.

one term → monomial

two terms → binomial

three terms → trinomial

- (i) $3x^2 + 5x - 4$

3 terms

- Trinomial
 (ii) $7a^3 - 2a^2 + a$
 3 terms
 Trinomial
 (iii) $4x + 9$
 2 terms
 Binomial
 (iv) $5y^2$
 1 term
 Monomial
 (v) 6
 1 term
 Monomial
 (vi) $x^2 + 7$
 2 terms
 Binomial

Answer (i) trinomial · (ii) trinomial · (iii) binomial · (iv) monomial · (v) monomial · (vi) binomial

3. Add the following polynomials: (i) $(3x^2 + 2x + 4)$ and $(5x^2 - 3x + 6)$ (ii) $(x^3 + 2x^2 + 4)$ and $(2x^3 - x^2 + 3)$ (iii) $(4a + 2b + 3)$ and $(5a - b + 6)$ (iv) $(6m^2 + 3m - 2)$ and $(-3m^2 + 2m + 5)$

Adding polynomials means collecting like terms. Deal with one kind of term at a time, highest power first.

- (i) $3x^2 + 2x + 4 + (5x^2 - 3x + 6)$
 x^2 : $3x^2 + 5x^2 = 8x^2$
 x : $2x - 3x = -x$
 the constants: $4 + 6 = 10$
 $= 8x^2 - x + 10$
 (ii) $x^3 + 2x^2 + 4 + (2x^3 - x^2 + 3)$
 x^3 : $x^3 + 2x^3 = 3x^3$
 x^2 : $2x^2 - x^2 = x^2$
 the constants: $4 + 3 = 7$
 $= 3x^3 + x^2 + 7$
 (iii) $4a + 2b + 3 + (5a - b + 6)$
 a : $4a + 5a = 9a$
 b : $2b - b = b$
 the constants: $3 + 6 = 9$
 $= 9a + b + 9$
 (iv) $6m^2 + 3m - 2 + (-3m^2 + 2m + 5)$
 m^2 : $6m^2 - 3m^2 = 3m^2$
 m : $3m + 2m = 5m$
 the constants: $-2 + 5 = 3$
 $= 3m^2 + 5m + 3$

Answer (i) $8x^2 - x + 10$ · (ii) $3x^3 + x^2 + 7$ · (iii) $9a + b + 9$ · (iv) $3m^2 + 5m + 3$

4. Subtract the second polynomial from the first: (i) $(7x^2 + 4x - 5)$ and $(3x^2 - x + 2)$ (ii) $(8y^3 - 2y^2 + 7)$ and $(4y^3 + 3y^2 - 5)$ (iii) $(5a - 3b + 9)$ and $(2a + b - 4)$ (iv) $(9p^2 + 4p - 3)$ and $(2p^2 + 5p + 1)$

A minus sign in front of a bracket changes the sign of every term inside it. That one step accounts for almost every mark lost here.

- (i) $7x^2 + 4x - 5 - (3x^2 - x + 2)$
 change every sign inside the bracket being taken away, then add:
 $7x^2 + 4x - 5 - 3x^2 + x - 2$
 x^2 : $7x^2 - 3x^2 = 4x^2$
 x : $4x + x = 5x$
 the constants: $-5 - 2 = -7$

$$= 4x^2 + 5x - 7$$

(ii) $8y^3 - 2y^2 + 7 - (4y^3 + 3y^2 - 5)$
 change every sign inside the bracket being taken away, then add:
 $8y^3 - 2y^2 + 7 - 4y^3 - 3y^2 + 5$
 y^3 : $8y^3 - 4y^3 = 4y^3$
 y^2 : $-2y^2 - 3y^2 = -5y^2$
 the constants: $7 + 5 = 12$
 $= 4y^3 - 5y^2 + 12$

(iii) $5a - 3b + 9 - (2a + b - 4)$
 change every sign inside the bracket being taken away, then add:
 $5a - 3b + 9 - 2a - b + 4$
 a : $5a - 2a = 3a$
 b : $-3b - b = -4b$
 the constants: $9 + 4 = 13$
 $= 3a - 4b + 13$

(iv) $9p^2 + 4p - 3 - (2p^2 + 5p + 1)$
 change every sign inside the bracket being taken away, then add:
 $9p^2 + 4p - 3 - 2p^2 - 5p - 1$
 p^2 : $9p^2 - 2p^2 = 7p^2$
 p : $4p - 5p = -p$
 the constants: $-3 - 1 = -4$
 $= 7p^2 - p - 4$

Answer (i) $4x^2 + 5x - 7$ · (ii) $4y^3 - 5y^2 + 12$ · (iii) $3a - 4b + 13$ · (iv) $7p^2 - p - 4$

5. Perform the indicated operations: (i) $(2x^2 + 5x - 3) + (x^2 - 4x + 7)$ (ii) $(3m^3 + 2m^2 - m) - (m^3 - m^2 + 2)$ (iii) $(4a^2 + 3a - 1) - (2a^2 - a + 5) + (a^2 + 4)$

Work left to right, changing the signs of any bracket that follows a minus.

(i) $2x^2 + 5x - 3 + (x^2 - 4x + 7)$
 x^2 : $2x^2 + x^2 = 3x^2$
 x : $5x - 4x = x$
 the constants: $-3 + 7 = 4$
 $= 3x^2 + x + 4$

(ii) $3m^3 + 2m^2 - m - (m^3 - m^2 + 2)$
 change every sign inside the bracket being taken away, then add:
 $3m^3 + 2m^2 - m - m^3 + m^2 - 2$
 m^3 : $3m^3 - m^3 = 2m^3$
 m^2 : $2m^2 + m^2 = 3m^2$
 m : $-m$ stands alone
 the constants: -2 stands alone
 $= 2m^3 + 3m^2 - m - 2$

(iii) $4a^2 + 3a - 1 - (2a^2 - a + 5) + (a^2 + 4)$
 change every sign inside the bracket being taken away, then add:
 $4a^2 + 3a - 1 - 2a^2 + a - 5 + a^2 + 4$
 a^2 : $4a^2 - 2a^2 + a^2 = 3a^2$
 a : $3a + a = 4a$
 the constants: $-1 - 5 + 4 = -2$
 $= 3a^2 + 4a - 2$

Answer (i) $3x^2 + x + 4$ · (ii) $2m^3 + 3m^2 - m - 2$ · (iii) $3a^2 + 4a - 2$

6. If $A = x^2 + 5 - 4x$, $B = 2x^2 - 3x + 6$ and $C = 4x - 7$, find: (i) $A - B$ (ii) $C - A$ (iii) $A - B - C$ (iv) $A + B - C$ (v) $B - C$ (vi) $B - A$

Write the three polynomials out first and keep them in descending powers, so like terms line up:

$$A = x^2 - 4x + 5$$

$$B = 2x^2 - 3x + 6$$

$$C = 4x - 7$$

Note A is printed as $x^2 + 5 - 4x$, out of order — rewriting it as $x^2 - 4x + 5$ first makes every part below easier.

(i) $A - B$

write each letter out: $(x^2 - 4x + 5) - (2x^2 - 3x + 6)$

change the signs inside any bracket being taken away, then collect like terms

$$= -x^2 - x - 1$$

(ii) $C - A$

write each letter out: $(4x - 7) - (x^2 - 4x + 5)$

change the signs inside any bracket being taken away, then collect like terms

$$= -x^2 + 8x - 12$$

(iii) $A - B - C$

write each letter out: $(x^2 - 4x + 5) - (2x^2 - 3x + 6) - (4x - 7)$

change the signs inside any bracket being taken away, then collect like terms

$$= -x^2 - 5x + 6$$

(iv) $A + B - C$

write each letter out: $(x^2 - 4x + 5) + (2x^2 - 3x + 6) - (4x - 7)$

change the signs inside any bracket being taken away, then collect like terms

$$= 3x^2 - 11x + 18$$

(v) $B - C$

write each letter out: $(2x^2 - 3x + 6) - (4x - 7)$

change the signs inside any bracket being taken away, then collect like terms

$$= 2x^2 - 7x + 13$$

(vi) $B - A$

write each letter out: $(2x^2 - 3x + 6) - (x^2 - 4x + 5)$

change the signs inside any bracket being taken away, then collect like terms

$$= x^2 + x + 1$$

Worth noticing: $A - B$ and $B - A$ come out as exact opposites, every sign reversed. Subtraction is never symmetrical.

Answer (i) $-x^2 - x - 1$ · (ii) $-x^2 + 8x - 12$ · (iii) $-x^2 - 5x + 6$ · (iv) $3x^2 - 11x + 18$ · (v) $2x^2 - 7x + 13$ · (vi) $x^2 + x + 1$

7. What must be added to $6p^2 + 2p - 5$ to get $9p^2 + 7p - 1$?

Call the missing polynomial M. Then

$$(6p^2 + 2p - 5) + M = 9p^2 + 7p - 1$$

Step 1 Make M the subject

$$M = (9p^2 + 7p - 1) - (6p^2 + 2p - 5)$$

Step 2 Change the signs in the bracket

$$M = 9p^2 + 7p - 1 - 6p^2 - 2p + 5$$

Step 3 Collect like terms

$$p^2: 9p^2 - 6p^2 = 3p^2$$

$$p: 7p - 2p = 5p$$

$$\text{the constants: } -1 + 5 = 4$$

$$M = 3p^2 + 5p + 4$$

Step 4 Check by adding it back

$$(6p^2 + 2p - 5) + (3p^2 + 5p + 4) = 9p^2 + 7p - 1 \checkmark$$

Answer $3p^2 + 5p + 4$

8. What must be subtracted from $8y^2 + 6y - 9$ to get $y^2 + 2y + 4$?

Call the polynomial to be taken away S. Then

$$(8y^2 + 6y - 9) - S = y^2 + 2y + 4$$

Step 1 Make S the subject

$$S = (8y^2 + 6y - 9) - (y^2 + 2y + 4)$$

note the order: it is the first minus the second, not the other way round

Step 2 Change the signs in the bracket

$$S = 8y^2 + 6y - 9 - y^2 - 2y - 4$$

Step 3 Collect like terms

$$y^2: 8y^2 - y^2 = 7y^2$$

$$y: 6y - 2y = 4y$$

$$\text{the constants: } -9 - 4 = -13$$

$$S = 7y^2 + 4y - 13$$

Step 4 Check by subtracting it

$$(8y^2 + 6y - 9) - (7y^2 + 4y - 13) = y^2 + 2y + 4 \checkmark$$

Answer $7y^2 + 4y - 13$

9. Subtract $2x^2 - 4x + 3$ from the sum of $3x^2 + 2x - 5$ and $x^2 - 6x + 7$.

Two operations, and the order matters: the sum is formed first, then the subtraction.

Step 1 Find the sum

$$(3x^2 + 2x - 5) + (x^2 - 6x + 7)$$

$$x^2: 3x^2 + x^2 = 4x^2$$

$$x: 2x - 6x = -4x$$

$$\text{the constants: } -5 + 7 = 2$$

$$\text{sum} = 4x^2 - 4x + 2$$

Step 2 Subtract from that sum

$$(4x^2 - 4x + 2) - (2x^2 - 4x + 3)$$

$$\text{change the signs: } 4x^2 - 4x + 2 - 2x^2 + 4x - 3$$

Step 3 Collect like terms

$$x^2: 4x^2 - 2x^2 = 2x^2$$

$$x: -4x + 4x = 0 \text{ — the } x \text{ terms cancel exactly}$$

$$\text{the constants: } 2 - 3 = -1$$

$$= 2x^2 - 1$$

Step 4 Check

$$(2x^2 - 1) + (2x^2 - 4x + 3) = 4x^2 - 4x + 2, \text{ the sum } \checkmark$$

Answer $2x^2 - 1$

EXERCISE 8.3

1. Find the following products: (i) $(2x + 3)(x - 4)$ (ii) $(y + 5)(2y + 3)$ (iii) $(x + 1)(x^2 - x + 4)$ (iv) $(3x + 1)(2x - 5)$ (v) $(4x)(3x^2)$ (vi) $(2y^3)(5y)$ (vii) $(6a^2)(7a^4)$ (viii) $(8m^2n)(2mn^2)$ (ix) $(4y^2)(3y - 2)$ (x) $5a(2a^2 + 3a + 4)$ (xi) $2n(5n^2 + 3n + 1)$ (xii) $(15/3 \text{ pq})(5/2 \text{ pq})$

Multiplying a monomial by a monomial: multiply the numbers, then add the powers of each letter — $a^m \times a^n = a^{m+n}$.

Multiplying by a bracket: every term inside the bracket must be multiplied, not just the first.

Multiplying two brackets: every term of one times every term of the other, then collect like terms.

(i) $(2x + 3)(x - 4)$

multiply every term of the first by every term of the second:

$$(3) \times (-4) = -12$$

$$(3) \times (x) = 3x$$

$$(2x) \times (-4) = -8x$$

$$(2x) \times (x) = 2x^2$$

add them and collect like terms

$$= 2x^2 - 5x - 12$$

(ii) $(y + 5)(2y + 3)$

multiply every term of the first by every term of the second:

$$(5) \times (3) = 15$$

$$(5) \times (2y) = 10y$$

$$(y) \times (3) = 3y$$

$$(y) \times (2y) = 2y^2$$

add them and collect like terms

$$= 2y^2 + 13y + 15$$

$$(iii) (x + 1)(x^2 - x + 4)$$

multiply every term of the first by every term of the second:

$$(1) \times (4) = 4$$

$$(1) \times (x^2) = x^2$$

$$(1) \times (-x) = -x$$

$$(x) \times (4) = 4x$$

$$(x) \times (x^2) = x^3$$

$$(x) \times (-x) = -x^2$$

add them and collect like terms

$$= x^3 + 3x + 4$$

$$(iv) (3x + 1)(2x - 5)$$

multiply every term of the first by every term of the second:

$$(1) \times (-5) = -5$$

$$(1) \times (2x) = 2x$$

$$(3x) \times (-5) = -15x$$

$$(3x) \times (2x) = 6x^2$$

add them and collect like terms

$$= 6x^2 - 13x - 5$$

$$(v) (4x)(3x^2)$$

multiply the numbers, then add the powers of each letter

$$= 12x^3$$

$$(vi) (2y^3)(5y)$$

multiply the numbers, then add the powers of each letter

$$= 10y^4$$

$$(vii) (6a^2)(7a^4)$$

multiply the numbers, then add the powers of each letter

$$= 42a^6$$

$$(viii) (8m^2n)(2mn^2)$$

multiply the numbers, then add the powers of each letter

$$= 16m^3n^3$$

$$(ix) (4y^2)(3y - 2)$$

multiply every term of the first by every term of the second:

$$(4y^2) \times (-2) = -8y^2$$

$$(4y^2) \times (3y) = 12y^3$$

add them and collect like terms

$$= 12y^3 - 8y^2$$

$$(x) 5a(2a^2 + 3a + 4)$$

multiply every term of the first by every term of the second:

$$(5a) \times (4) = 20a$$

$$(5a) \times (2a^2) = 10a^3$$

$$(5a) \times (3a) = 15a^2$$

add them and collect like terms

$$= 10a^3 + 15a^2 + 20a$$

$$(xi) 2n(5n^2 + 3n + 1)$$

multiply every term of the first by every term of the second:

$$(2n) \times (1) = 2n$$

$$(2n) \times (3n) = 6n^2$$

$$(2n) \times (5n^2) = 10n^3$$

add them and collect like terms

$$= 10n^3 + 6n^2 + 2n$$

$$\begin{aligned} & \text{(xii) } (15/3 \text{ pq})(5/2 \text{ pq}) \\ & \text{multiply the numbers, then add the powers of each letter} \\ & = 25p^2q^2/2 \end{aligned}$$

Answer (i) $2x^2 - 5x - 12$ · (ii) $2y^2 + 13y + 15$ · (iii) $x^3 + 3x + 4$ · (iv) $6x^2 - 13x - 5$ · (v) $12x^3$ · (vi) $10y^4$ · (vii) $42a^6$ · (viii) $16m^3n^3$ · (ix) $12y^3 - 8y^2$ · (x) $10a^3 + 15a^2 + 20a$ · (xi) $10n^3 + 6n^2 + 2n$ · (xii) $25p^2q^2/2$

Please check — in part (xii) the $15/3$ reduces to 5, so the question is really $(5pq)(5/2 \text{ pq})$. Writing $15/3$ rather than 5 serves no purpose here.

2. Divide the following polynomials: (i) $3x^3 + 6x^2 - 9x$ by $3x$ (ii) $5y^2 + 10y - 15$ by 5 (iii) $4a^3b + 8a^2b^3 - 12ab^3$ by $4ab$ (iv) $9x^2y + 6xy^2 - 3y^3$ by $3xy$ (v) $2x + 8$ by 2 (vi) $5y - 10$ by 5 (vii) $15x^2 + 30x$ by $15x$ (viii) $7xy + 14x$ by $7x$ (ix) $8a^4b^2$ by $2a^2b$

Divide each term of the polynomial by the monomial, one at a time — the division bar carries over every term.

For the letters, subtract the powers: $a^m \div a^n = a^{m-n}$.

Where a letter cancels entirely its power becomes 0, and anything to the power 0 is 1 — that is why some terms lose their letter altogether.

$$\text{(i) } (3x^3 + 6x^2 - 9x) \div 3x$$

divide each term of the top by $3x$ separately:

$$-9x \div 3x = -3$$

$$3x^3 \div 3x = x^2$$

$$6x^2 \div 3x = 2x$$

$$= x^2 + 2x - 3$$

$$\text{check: } 3x \times (x^2 + 2x - 3) = 3x^3 + 6x^2 - 9x \checkmark$$

$$\text{(ii) } (5y^2 + 10y - 15) \div 5$$

divide each term of the top by 5 separately:

$$-15 \div 5 = -3$$

$$5y^2 \div 5 = y^2$$

$$10y \div 5 = 2y$$

$$= y^2 + 2y - 3$$

$$\text{check: } 5 \times (y^2 + 2y - 3) = 5y^2 + 10y - 15 \checkmark$$

$$\text{(iii) } (4a^3b + 8a^2b^3 - 12ab^3) \div 4ab$$

divide each term of the top by $4ab$ separately:

$$-12ab^3 \div 4ab = -3b^2$$

$$4a^3b \div 4ab = a^2$$

$$8a^2b^3 \div 4ab = 2ab^2$$

$$= a^2 + 2ab^2 - 3b^2$$

$$\text{check: } 4ab \times (a^2 + 2ab^2 - 3b^2) = 4a^3b + 8a^2b^3 - 12ab^3 \checkmark$$

$$\text{(iv) } (9x^2y + 6xy^2 - 3y^3) \div 3xy$$

divide each term of the top by $3xy$ separately:

$$-3y^3 \div 3xy = -y^2/x$$

$$6xy^2 \div 3xy = 2y$$

$$9x^2y \div 3xy = 3x$$

$$= 3x + 2y - y^2/x$$

$$\text{check: } 3xy \times (3x + 2y - y^2/x) = 9x^2y + 6xy^2 - 3y^3 \checkmark$$

$$\text{(v) } (2x + 8) \div 2$$

divide each term of the top by 2 separately:

$$8 \div 2 = 4$$

$$2x \div 2 = x$$

$$= x + 4$$

$$\text{check: } 2 \times (x + 4) = 2x + 8 \checkmark$$

$$\text{(vi) } (5y - 10) \div 5$$

divide each term of the top by 5 separately:

$$-10 \div 5 = -2$$

$$5y \div 5 = y$$

$$= y - 2$$

check: $5 \times (y - 2) = 5y - 10$ ✓

(vii) $(15x^2 + 30x) \div 15x$
 divide each term of the top by $15x$ separately:
 $15x^2 \div 15x = x$
 $30x \div 15x = 2$
 $= x + 2$
 check: $15x \times (x + 2) = 15x^2 + 30x$ ✓

(viii) $(7xy + 14x) \div 7x$
 divide each term of the top by $7x$ separately:
 $14x \div 7x = 2$
 $7xy \div 7x = y$
 $= y + 2$
 check: $7x \times (y + 2) = 7xy + 14x$ ✓

(ix) $(8a^4b^2) \div 2a^2b$
 divide each term of the top by $2a^2b$ separately:
 $8a^4b^2 \div 2a^2b = 4a^2b$
 $= 4a^2b$
 check: $2a^2b \times (4a^2b) = 8a^4b^2$ ✓

Answer (i) $x^2 + 2x - 3$ · (ii) $y^2 + 2y - 3$ · (iii) $a^2 + 2ab^2 - 3b^2$ · (iv) $3x + 2y - y^2/x$ · (v) $x + 4$ · (vi) $y - 2$ · (vii) $x + 2$ · (viii) $y + 2$ · (ix) $4a^2b$

Please check — part (iv) does not divide exactly: $3y^3$ has no x , so dividing by $3xy$ leaves y^2/x and the answer is not a polynomial. If the term were $3xy^3$ the answer would be $3x + 2y - y^2$.

EXERCISE 8.4

1. Simplify the following: (i) $5[2\{3(x + 2) - 4\} + 6]$ (ii) $4[2(x - 1) + 3] - 2[5 + 3\{2(x + 1) - 4\}]$ (iii) $6[3\{2(x - 3)\} + 4] + 2[5 - 3(x + 1)]$ (iv) $3[2(x + 2) + 4] - 5[3\{2(x - 1)\} - 2]$ (v) $7[4 + 3\{2(x - 1) + 1\}] - 5[x + 2]$ (vi) $5[3\{x + 2\} + 4] - 2[4(x - 1) + 3]$ (vii) $3[2(x + 4) + 1] + 5[6 - 3\{2(x + 1)\}]$ (viii) $8[5 + 2\{3(x - 2)\}] - 4[3 + 2(x + 3)]$ (ix) $7[3\{x + 1\} - 2] - 25(x - 3)$ (x) $4[2(x - 1) + 3] + 6(x + 2)$

These are BODMAS questions with letters in them. Work from the innermost bracket outward:
 round brackets () first
 then braces { }

then square brackets []

Multiply the number outside a bracket into every term inside it, and watch the sign when that number is negative.

Collect the x terms and the constants separately at the end.

(i) $5[2\{3(x + 2) - 4\} + 6]$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= 30x + 50$$

(ii) $4[2(x - 1) + 3] - 2[5 + 3\{2(x + 1) - 4\}]$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= -4x + 6$$

(iii) $6[3\{2(x - 3)\} + 4] + 2[5 - 3(x + 1)]$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= 30x - 80$$

(iv) $3[2(x + 2) + 4] - 5[3\{2(x - 1)\} - 2]$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= -24x + 64$$

$$(v) \quad 7[4 + 3\{2(x - 1) + 1\}] - 5[x + 2]$$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= 37x - 3$$

$$(vi) \quad 5[3\{x + 2\} + 4] - 2\{4(x - 1) + 3\}$$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= 7x + 52$$

$$(vii) \quad 3[2(x + 4) + 1] + 5[6 - 3\{2(x + 1)\}]$$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= -24x + 27$$

$$(viii) \quad 8[5 + 2\{3(x - 2)\}] - 4[3 + 2(x + 3)]$$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= 40x - 92$$

$$(ix) \quad 7[3\{x + 1\} - 2] - 25(x - 3)$$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= -4x + 82$$

$$(x) \quad 4[2(x - 1) + 3] + 6(x + 2)$$

work from the innermost bracket outward — round brackets first, then braces, then square brackets

$$= 14x + 16$$

Answer (i) $30x + 50$ · (ii) $-4x + 6$ · (iii) $30x - 80$ · (iv) $-24x + 64$ · (v) $37x - 3$ · (vi) $7x + 52$ · (vii) $-24x + 27$ · (viii) $40x - 92$ · (ix) $-4x + 82$ · (x) $14x + 16$

EXERCISE 8.6

1. Factorise the following: (i) $14y - 28$ (ii) $20p^3 + 15p^2$ (iii) $9b + 6b^2$ (iv) $8a^3b^2 - 12a^2b^2$ (v) $6x^2 + 3x + 2y + y^2$ (vi) $4(x + 2) + 8$ (vii) $6(m - 2) + 9(m - 2)$ (viii) $8(a + 5) - 4(a + 5)$ (ix) $2x^2 + 6x + 3(x + 2)$ (x) $3(y - 1) + 6(y - 1)$ (xi) $x^2 + 3x + 2y + 6$ (xii) $7x^2 + 14x + 3(x + 4)$

Factorising is the reverse of expanding: look for what the terms have in common and take it outside a bracket.

Where there are four terms, regroup them in pairs, take the common factor out of each pair, and hope the same bracket appears twice — if it does, that bracket itself comes out.

Always check by multiplying back out.

$$(i) \quad 14y - 28$$

look for what every term has in common

$$= 14(y - 2)$$

check by multiplying back out: $14y - 28$ ✓

$$(ii) \quad 20p^3 + 15p^2$$

look for what every term has in common

$$= 5p^2(4p + 3)$$

check by multiplying back out: $20p^3 + 15p^2$ ✓

$$(iii) \quad 9b + 6b^2$$

look for what every term has in common

$$= 3b(2b + 3)$$

check by multiplying back out: $6b^2 + 9b$ ✓

$$(iv) \quad 8a^3b^2 - 12a^2b^2$$

look for what every term has in common

$$= 4a^2b^2(2a - 3)$$

check by multiplying back out: $8a^3b^2 - 12a^2b^2$ ✓

(v) $6x^2 + 3x + 2y + y^2$

group the terms in pairs and take out what is common from each:

$3x(2x + 1) + y(2 + y)$ — the two brackets differ, so this one does not regroup as printed cannot be factorised by regrouping as printed — see the note

(vi) $4(x + 2) + 8$

expand first if there are brackets: $4x + 16$

$= 4(x + 4)$

check by multiplying back out: $4x + 16$ ✓

(vii) $6(m - 2) + 9(m - 2)$

expand first if there are brackets: $15m - 30$

$= 15(m - 2)$

check by multiplying back out: $15m - 30$ ✓

(viii) $8(a + 5) - 4(a + 5)$

expand first if there are brackets: $4a + 20$

$= 4(a + 5)$

check by multiplying back out: $4a + 20$ ✓

(ix) $2x^2 + 6x + 3(x + 2)$

group the terms in pairs and take out what is common from each:

$2x(x + 3) + 3(x + 2)$ — the two brackets differ. With $4x$ in place of $6x$ it would give $(x + 2)(2x + 3)$

cannot be factorised by regrouping as printed — see the note

(x) $3(y - 1) + 6(y - 1)$

expand first if there are brackets: $9y - 9$

$= 9(y - 1)$

check by multiplying back out: $9y - 9$ ✓

(xi) $x^2 + 3x + 2y + 6$

group the terms in pairs and take out what is common from each:

$x(x + 3) + 2(y + 3)$ — the two brackets differ. With $2x$ in place of $2y$ it would give $(x + 3)(x + 2)$

cannot be factorised by regrouping as printed — see the note

(xii) $7x^2 + 14x + 3(x + 4)$

group the terms in pairs and take out what is common from each:

$7x(x + 2) + 3(x + 4)$ — the two brackets differ. With $28x$ in place of $14x$ it would give $(x + 4)(7x + 3)$

3)

cannot be factorised by regrouping as printed — see the note

Answer (i) $14(y - 2)$

(ii) $5p^2(4p + 3)$

(iii) $3b(2b + 3)$

(iv) $4a^2b^2(2a - 3)$

(v) does not regroup as printed

(vi) $4(x + 4)$

(vii) $15(m - 2)$

(viii) $4(a + 5)$

(ix) does not regroup as printed

(x) $9(y - 1)$

(xi) does not regroup as printed

(xii) does not regroup as printed

Please check — parts (v), (ix), (xi) and (xii) do not regroup as printed — the two pairs give different brackets. The corrections that would make them work are given above.

2. Factorise by middle-term breaking: (i) $x^2 + 5x + 6$ (ii) $x^2 + 7x + 10$ (iii) $x^2 + 8x + 12$ (iv) $2x^2 + 9x + 10$ (v) $3x^2 + 11x + 10$ (vi) $x^2 - 9x + 20$ (vii) $2x^2 - 7x - 15$ (viii) $6x^2 + 13x - 5$ (ix) $4x^2 - 12x + 9$

For $ax^2 + bx + c$, split the middle term bx into two terms whose coefficients multiply to $a \times c$ and add to b .

Then group the four terms in pairs and take out the common factor of each. The same bracket will appear twice, and that bracket comes out.

Multiplying to $a \times c$, not just to c , is what makes the method work when a is not 1.

(i) $x^2 + 5x + 6$

$a = 1, b = 5, c = 6$

find two numbers multiplying to $a \times c = 1 \times 6 = 6$ and adding to $b = 5$

the pair is 2 and 3, since $2 \times 3 = 6$ and $2 + 3 = 5$

break the middle term: $x^2 + 2x + 3x + 6$

group in pairs and take out the common factor of each

$= (x + 2)(x + 3)$

check by multiplying back out: $x^2 + 5x + 6$ ✓

(ii) $x^2 + 7x + 10$

$a = 1, b = 7, c = 10$

find two numbers multiplying to $a \times c = 1 \times 10 = 10$ and adding to $b = 7$

the pair is 2 and 5, since $2 \times 5 = 10$ and $2 + 5 = 7$

break the middle term: $x^2 + 2x + 5x + 10$

group in pairs and take out the common factor of each

$= (x + 2)(x + 5)$

check by multiplying back out: $x^2 + 7x + 10$ ✓

(iii) $x^2 + 8x + 12$

$a = 1, b = 8, c = 12$

find two numbers multiplying to $a \times c = 1 \times 12 = 12$ and adding to $b = 8$

the pair is 2 and 6, since $2 \times 6 = 12$ and $2 + 6 = 8$

break the middle term: $x^2 + 2x + 6x + 12$

group in pairs and take out the common factor of each

$= (x + 2)(x + 6)$

check by multiplying back out: $x^2 + 8x + 12$ ✓

(iv) $2x^2 + 9x + 10$

$a = 2, b = 9, c = 10$

find two numbers multiplying to $a \times c = 2 \times 10 = 20$ and adding to $b = 9$

the pair is 4 and 5, since $4 \times 5 = 20$ and $4 + 5 = 9$

break the middle term: $2x^2 + 4x + 5x + 10$

group in pairs and take out the common factor of each

$= (x + 2)(2x + 5)$

check by multiplying back out: $2x^2 + 9x + 10$ ✓

(v) $3x^2 + 11x + 10$

$a = 3, b = 11, c = 10$

find two numbers multiplying to $a \times c = 3 \times 10 = 30$ and adding to $b = 11$

the pair is 5 and 6, since $5 \times 6 = 30$ and $5 + 6 = 11$

break the middle term: $3x^2 + 5x + 6x + 10$

group in pairs and take out the common factor of each

$= (x + 2)(3x + 5)$

check by multiplying back out: $3x^2 + 11x + 10$ ✓

(vi) $x^2 - 9x + 20$

$a = 1, b = -9, c = 20$

find two numbers multiplying to $a \times c = 1 \times 20 = 20$ and adding to $b = -9$

the pair is -4 and -5, since $-4 \times -5 = 20$ and $-4 + -5 = -9$

break the middle term: $x^2 - 4x - 5x + 20$

group in pairs and take out the common factor of each

$= (x - 5)(x - 4)$

check by multiplying back out: $x^2 - 9x + 20$ ✓

(vii) $2x^2 - 7x - 15$

$a = 2, b = -7, c = -15$

find two numbers multiplying to $a \times c = 2 \times -15 = -30$ and adding to $b = -7$

the pair is 3 and -10, since $3 \times -10 = -30$ and $3 + -10 = -7$

break the middle term: $2x^2 + 3x - 10x - 15$
 group in pairs and take out the common factor of each
 $= (x - 5)(2x + 3)$
 check by multiplying back out: $2x^2 - 7x - 15$ ✓

(viii) $6x^2 + 13x - 5$
 $a = 6, b = 13, c = -5$
 find two numbers multiplying to $a \times c = 6 \times -5 = -30$ and adding to $b = 13$
 the pair is -2 and 15 , since $-2 \times 15 = -30$ and $-2 + 15 = 13$
 break the middle term: $6x^2 - 2x + 15x - 5$
 group in pairs and take out the common factor of each
 $= (2x + 5)(3x - 1)$
 check by multiplying back out: $6x^2 + 13x - 5$ ✓

(ix) $4x^2 - 12x + 9$
 $a = 4, b = -12, c = 9$
 find two numbers multiplying to $a \times c = 4 \times 9 = 36$ and adding to $b = -12$
 the pair is -6 and -6 , since $-6 \times -6 = 36$ and $-6 + -6 = -12$
 break the middle term: $4x^2 - 6x - 6x + 9$
 group in pairs and take out the common factor of each
 $= (2x - 3)^2$
 check by multiplying back out: $4x^2 - 12x + 9$ ✓

Answer (i) $(x + 2)(x + 3)$

(ii) $(x + 2)(x + 5)$

(iii) $(x + 2)(x + 6)$

(iv) $(x + 2)(2x + 5)$

(v) $(x + 2)(3x + 5)$

(vi) $(x - 5)(x - 4)$

(vii) $(x - 5)(2x + 3)$

(viii) $(2x + 5)(3x - 1)$

(ix) $(2x - 3)^2$

REVISION EXERCISE 8

1. Choose the correct option.

i. Which of the following is a polynomial?

a) $3x + 2$ b) $x^2 + 5x - 4$ c) -7 d) All of the above

A polynomial needs every power of its variables to be a whole number.

(a) $3x + 2$ — x has power 1 ✓

(b) $x^2 + 5x - 4$ — powers 2 and 1 ✓

(c) -7 — a constant is a polynomial of degree 0 ✓

All three qualify, so (d) all of the above.

Part (c) is the one most often rejected: a lone number counts, because -7 can be written as $-7x^0$.

Answer d) All of the above

ii. Identify the like terms in $5x + 3y - 2x + 4$.

a) $5x$ and $3y$ b) $5x$ and $-2x$ c) $3y$ and 4 d) None of the above

Like terms have the same letters to the same powers.

(a) $5x$ and $3y$ — different letters ✗

(b) $5x$ and $-2x$ — both have x ✓

(c) $3y$ and 4 — one has y , the other no letter ✗

They can be collected: $5x - 2x = 3x$, so the expression simplifies to $3x + 3y + 4$.

Answer b) $5x$ and $-2x$

iii. Who is known as the founding father of algebra?

- a) Euclid b) Isaac Newton c) Muhammad bin Musa Al-Khwarizmi d) Pythagoras

Al-Khwarizmi, a ninth-century mathematician, wrote the book from whose title the word algebra comes — al-jabr.

The word algorithm comes from his name.

Euclid is remembered for geometry, Newton for calculus and the laws of motion, and Pythagoras for the theorem about right-angled triangles.

Answer c) Muhammad bin Musa Al-Khwarizmi

iv. What is the result of $(3x^2 + 2x) + (4x^2 - 3x)$?

- a) $7x^2 - x$ b) $7x^2 + x$ c) $x^2 + 5x$ d) $x^2 - 5x$

Step 1 collect the x^2 terms

$$3x^2 + 4x^2 = 7x^2$$

Step 2 collect the x terms

$$2x - 3x = -x$$

$$= 7x^2 - x$$

Option (b) $7x^2 + x$ comes from adding $2x$ and $3x$ instead of subtracting — the minus belongs to the $3x$.

Answer a) $7x^2 - x$

v. $(98)^2 =$

- a) 9604 b) 960 c) 96 d) 964

Use the identity $(a - b)^2 = a^2 - 2ab + b^2$ with $a = 100$ and $b = 2$ — far quicker than long multiplication.

$$\text{Step 1 } 98 = 100 - 2$$

$$\text{Step 2 } (100 - 2)^2 = 100^2 - 2 \times 100 \times 2 + 2^2$$

$$\text{Step 3 } = 10,000 - 400 + 4$$

$$\text{Step 4 } = 9,604$$

$$\text{Check } 98 \times 98 = 9,604 \checkmark$$

The other options have too few digits: a number near 100 squared must be near 10,000.

Answer a) 9604

2. Fill in the blanks: (i) A ____ is a fixed value that does not change. (ii) In the expression $3x + 5$, the number ____ is a constant. (iii) A ____ is a symbol that represents a quantity that can change. (iv) The middle term breaking method is used to factor ____ expressions. (v) In $2a + 4$, a is a ____ because it can take different values, while 4 is a _____. (vi) A ____ is a mathematical statement that shows the equality of two expressions. (vii) The product of a monomial and a binomial is found using the ____ property.

(i) A ____ is a fixed value that does not change.

a fixed number is a constant; a letter that can change is a variable
constant

(ii) In the expression $3x + 5$, the number ____ is a constant.

the 5 stands alone. The 3 is a coefficient — also a fixed number, but attached to x
5

(iii) A ____ is a symbol that represents a quantity that can change.

usually a letter such as x , y or a
variable

(iv) The middle term breaking method is used to factor ____ expressions.

those of the form $ax^2 + bx + c$
quadratic

(v) In $2a + 4$, a is a ____ because it can take different values, while 4 is a ____.

a can change; 4 cannot
variable ... constant

(vi) A ____ is a mathematical statement that shows the equality of two expressions.

it must contain an equals sign
equation

- (vii) The product of a monomial and a binomial is found using the ____ property.
 $a(b + c) = ab + ac$ — the monomial is shared over every term of the bracket
 distributive

Answer (i) constant · (ii) 5 · (iii) variable · (iv) quadratic · (v) variable ... constant · (vi) equation · (vii) distributive

3. Short answer questions: (i) Define a variable and give an example. (ii) Difference between an open sentence and a closed sentence. (iii) Difference between a constant and a variable. (iv) Is $4x^3 - 2x + 5$ a monomial, binomial or trinomial?

- (i) Define a variable and give an example.

A variable is a symbol, usually a letter, standing for a quantity whose value can change. In $3x + 5$ the x is a variable: it may be 1, 2, 100 or anything else.

- (ii) Difference between an open sentence and a closed sentence.

An open sentence contains a variable, so it cannot be judged true or false until the variable is given a value — $4x + 5 = 9$ is open. A closed sentence has no variable and can be judged at once — $6 + 7 = 13$ is closed and true, $10 - 5 = 4$ is closed and false.

- (iii) Difference between a constant and a variable.

A constant has one fixed value that never changes, such as 7 or -3. A variable stands for a value that may change, such as x . A coefficient like the 3 in $3x$ is a constant attached to a variable.

- (iv) Is $4x^3 - 2x + 5$ a monomial, binomial or trinomial?

It has three terms — $4x^3$, $-2x$ and $+5$ — so it is a trinomial.

Answer See the four answers above.

4. Add the following polynomials: (i) $4a^3 - 2a + 6$, $5a^3 + 3a - 1$ (ii) $7y^2 + 4y + 1$, $2y^3 - 5y + 3$ (iii) $6m^3 - 3m + 2$, $4m^3 + 5m - 7$ (iv) $9p - 5$, $3p^3 + 4p + 1$

Collect like terms, highest power first. Where one polynomial has a power the other lacks, that term simply carries down unchanged.

- (i) $4a^3 - 2a + 6 + (5a^3 + 3a - 1)$

$$a^3: 4a^3 + 5a^3 = 9a^3$$

$$a: -2a + 3a = a$$

$$\text{the constants: } 6 - 1 = 5$$

$$= 9a^3 + a + 5$$

- (ii) $7y^2 + 4y + 1 + (2y^3 - 5y + 3)$

$$y^3: 2y^3 \text{ stands alone}$$

$$y^2: 7y^2 \text{ stands alone}$$

$$y: 4y - 5y = -y$$

$$\text{the constants: } 1 + 3 = 4$$

$$= 2y^3 + 7y^2 - y + 4$$

- (iii) $6m^3 - 3m + 2 + (4m^3 + 5m - 7)$

$$m^3: 6m^3 + 4m^3 = 10m^3$$

$$m: -3m + 5m = 2m$$

$$\text{the constants: } 2 - 7 = -5$$

$$= 10m^3 + 2m - 5$$

- (iv) $9p - 5 + (3p^3 + 4p + 1)$

$$p^3: 3p^3 \text{ stands alone}$$

$$p: 9p + 4p = 13p$$

$$\text{the constants: } -5 + 1 = -4$$

$$= 3p^3 + 13p - 4$$

Answer (i) $9a^3 + a + 5$ · (ii) $2y^3 + 7y^2 - y + 4$ · (iii) $10m^3 + 2m - 5$ · (iv) $3p^3 + 13p - 4$

5. Subtract the second polynomial from the first: (i) $4a^3 - 2a + 6$, $5a^3 + 3a - 1$ (ii) $9p - 5$, $3p^3 + 4p + 1$ (iii) $6m^3 - 3m + 2$, $4m^3 + 5m - 7$ (iv) $7y^2 + 4y + 1$, $2y^3 - 5y + 3$

Change every sign in the second polynomial, then add. The first polynomial listed is the one being subtracted from.

(i) $4a^3 - 2a + 6 - (5a^3 + 3a - 1)$

change every sign inside the bracket being taken away, then add:

$$4a^3 - 2a + 6 - 5a^3 - 3a + 1$$

$$a^3: 4a^3 - 5a^3 = -a^3$$

$$a: -2a - 3a = -5a$$

$$\text{the constants: } 6 + 1 = 7$$

$$= -a^3 - 5a + 7$$

(ii) $9p - 5 - (3p^3 + 4p + 1)$

change every sign inside the bracket being taken away, then add:

$$9p - 5 - 3p^3 - 4p - 1$$

$$p^3: -3p^3 \text{ stands alone}$$

$$p: 9p - 4p = 5p$$

$$\text{the constants: } -5 - 1 = -6$$

$$= -3p^3 + 5p - 6$$

(iii) $6m^3 - 3m + 2 - (4m^3 + 5m - 7)$

change every sign inside the bracket being taken away, then add:

$$6m^3 - 3m + 2 - 4m^3 - 5m + 7$$

$$m^3: 6m^3 - 4m^3 = 2m^3$$

$$m: -3m - 5m = -8m$$

$$\text{the constants: } 2 + 7 = 9$$

$$= 2m^3 - 8m + 9$$

(iv) $7y^2 + 4y + 1 - (2y^3 - 5y + 3)$

change every sign inside the bracket being taken away, then add:

$$7y^2 + 4y + 1 - 2y^3 + 5y - 3$$

$$y^3: -2y^3 \text{ stands alone}$$

$$y^2: 7y^2 \text{ stands alone}$$

$$y: 4y + 5y = 9y$$

$$\text{the constants: } 1 - 3 = -2$$

$$= -2y^3 + 7y^2 + 9y - 2$$

Answer (i) $-a^3 - 5a + 7$ · (ii) $-3p^3 + 5p - 6$ · (iii) $2m^3 - 8m + 9$ · (iv) $-2y^3 + 7y^2 + 9y - 2$

6. Factorise by regrouping: (i) $2x^3 + 4x^2 + 3x + 6$ (ii) $3a^3 + 6a^2 + 2a + 4$ (iii) $5m^2 + 10m + 2n + 4$ (iv) $4y^3 - 8y^2 + 2y - 4$

Group the four terms into two pairs, take the common factor out of each pair, and look for the same bracket appearing twice. If it does, that bracket comes out.

If the two brackets differ, the regrouping has failed — either the pairing was wrong or the question is at fault.

(i) $2x^3 + 4x^2 + 3x + 6$

$$\text{group in pairs: } (2x^3 + 4x^2) + (3x + 6)$$

take out $2x^2$ from the first and 3 from the second

$$2x^2(x + 2) + 3(x + 2)$$

the bracket $(x + 2)$ appears twice, so take it out

$$= (x + 2)(2x^2 + 3)$$

check by multiplying back out: $2x^3 + 4x^2 + 3x + 6$ ✓

(ii) $3a^3 + 6a^2 + 2a + 4$

$$\text{group in pairs: } (3a^3 + 6a^2) + (2a + 4)$$

$$3a^2(a + 2) + 2(a + 2)$$

the bracket $(a + 2)$ appears twice

$$= (a + 2)(3a^2 + 2)$$

check by multiplying back out: $3a^3 + 6a^2 + 2a + 4$ ✓

(iii) $5m^2 + 10m + 2n + 4$

$$\text{group in pairs: } (5m^2 + 10m) + (2n + 4)$$

$$5m(m + 2) + 2(n + 2)$$

the two brackets differ — $(m + 2)$ against $(n + 2)$ — so this one cannot be regrouped as printed with $2m$ in place of $2n$ it would give $5m(m + 2) + 2(m + 2) = (m + 2)(5m + 2)$

$$(iv) \quad 4y^3 - 8y^2 + 2y - 4$$

group in pairs: $(4y^3 - 8y^2) + (2y - 4)$

$$4y^2(y - 2) + 2(y - 2)$$

the bracket $(y - 2)$ appears twice

$$= 2(y - 2)(2y^2 + 1)$$

check by multiplying back out: $4y^3 - 8y^2 + 2y - 4$ ✓

Answer (i) $(x + 2)(2x^2 + 3)$ · (ii) $(a + 2)(3a^2 + 2)$ · (iii) does not regroup as printed · (iv) $2(y - 2)(2y^2 + 1)$

Please check — part (iii) cannot be regrouped as printed. With $2m$ in place of $2n$ it factorises to $(m + 2)(5m + 2)$.



9

CHAPTER

Linear Equations

EXERCISE 9.1

1. Solve the following equations: (i) $2x + 3 = 7$ (ii) $x - 5 = 10$ (iii) $4x = 16$ (iv) $3 + x = 8$ (v) $5x - 2 = 3x + 10$ (vi) $7 + 2x = 3x + 1$ (vii) $4(x - 3) = 2(x + 5)$ (viii) $3x + 5 = 2(x + 4)$ (ix) $2(x - 1) + 3 = 5(x + 1)$ (x) $5(x + 2) - 3(x - 1) = 8$ (xi) $4(x + 1) = 2(3x - 2) + 14$ (xii) $(3x + 4)/2 - 1 = (x + 6)/3$

The rule throughout: whatever is done to one side must be done to the other, so the two sides stay equal.

Clear any brackets first, then gather the x terms on one side and the numbers on the other, then divide.

Every answer below is put back into the original equation as a check — that is the only way to be certain.

(i) $2x + 3 = 7$

take 3 from both sides: $2x = 7 - 3 = 4$

divide both sides by 2

$x = 2$

check by putting it back into the original equation:

left side = 7

right side = 7

the two sides agree ✓

(ii) $x - 5 = 10$

add 5 to both sides

$x = 15$

check by putting it back into the original equation:

left side = 10

right side = 10

the two sides agree ✓

(iii) $4x = 16$

divide both sides by 4

$x = 4$

check by putting it back into the original equation:

left side = 16

right side = 16

the two sides agree ✓

(iv) $3 + x = 8$

take 3 from both sides

$x = 5$

check by putting it back into the original equation:

left side = 8

right side = 8

the two sides agree ✓

(v) $5x - 2 = 3x + 10$

gather the x terms on one side: $5x - 3x = 10 + 2$

$2x = 12$

divide both sides by 2

$x = 6$

check by putting it back into the original equation:

left side = 28

right side = 28

the two sides agree ✓

(vi) $7 + 2x = 3x + 1$

gather the x terms: $7 - 1 = 3x - 2x$

$$6 = x$$

$$x = 6$$

check by putting it back into the original equation:

left side = 19

right side = 19

the two sides agree ✓

(vii) $4(x - 3) = 2(x + 5)$

expand both brackets: $4x - 12 = 2x + 10$

gather the x terms: $4x - 2x = 10 + 12$

$$2x = 22$$

$$x = 11$$

check by putting it back into the original equation:

left side = 32

right side = 32

the two sides agree ✓

(viii) $3x + 5 = 2(x + 4)$

expand the bracket: $3x + 5 = 2x + 8$

gather the x terms: $3x - 2x = 8 - 5$

$$x = 3$$

check by putting it back into the original equation:

left side = 14

right side = 14

the two sides agree ✓

(ix) $2(x - 1) + 3 = 5(x + 1)$

expand both sides: $2x - 2 + 3 = 5x + 5$

$$2x + 1 = 5x + 5$$

gather the x terms: $1 - 5 = 5x - 2x$

$$-4 = 3x$$

$$x = -4/3$$

check by putting it back into the original equation:

left side = $-5/3$

right side = $-5/3$

the two sides agree ✓

(x) $5(x + 2) - 3(x - 1) = 8$

expand, watching the sign on the second bracket: $5x + 10 - 3x + 3 = 8$

$$2x + 13 = 8$$

$$2x = 8 - 13 = -5$$

$$x = -5/2$$

check by putting it back into the original equation:

left side = 8

right side = 8

the two sides agree ✓

(xi) $4(x + 1) = 2(3x - 2) + 14$

expand both sides: $4x + 4 = 6x - 4 + 14$

$$4x + 4 = 6x + 10$$

gather the x terms: $4 - 10 = 6x - 4x$

$$-6 = 2x$$

$$x = -3$$

check by putting it back into the original equation:

left side = -8

right side = -8

the two sides agree ✓

$$(xii) \quad (3x + 4)/2 - 1 = (x + 6)/3$$

read with the equals sign restored — see the note

multiply every term by 6, the LCM of 2 and 3:

$$3(3x + 4) - 6 = 2(x + 6)$$

$$9x + 12 - 6 = 2x + 12$$

$$9x + 6 = 2x + 12$$

$$7x = 6$$

$$x = 6/7$$

check by putting it back into the original equation:

left side = 16/7

right side = 16/7

the two sides agree ✓

Answer (i) $x = 2$ · (ii) $x = 15$ · (iii) $x = 4$ · (iv) $x = 5$ · (v) $x = 6$ · (vi) $x = 6$ · (vii) $x = 11$ · (viii) $x = 3$ · (ix) $x = -4/3$ · (x) $x = -5/2$ · (xi) $x = -3$ · (xii) $x = 6/7$

Please check — part (xii) is printed without an equals sign and with a stray 1. It is read here as $(3x + 4)/2 - 1 = (x + 6)/3$.

2. Find the solution of the following and verify the results: (i) $x/2 + 3 = 7$ (ii) $3x/4 = 9$ (iii) $(x - 1)/5 = 2$ (iv) $3(x + 1)/5 = (2x - 2)/2$ (v) $2((x + 5)/3) = (7 - x)/2$ (vi) $(5x - 1)/2 + 4 = 7$

These all carry fractions. Clear the fractions first, by multiplying through by the denominator — or, where there is a fraction on each side, by cross-multiplying.

The question asks for the result to be verified, so each answer is substituted back below.

$$(i) \quad x/2 + 3 = 7$$

take 3 from both sides: $x/2 = 4$

multiply both sides by 2

$$x = 8$$

check by putting it back into the original equation:

left side = 7

right side = 7

the two sides agree ✓

$$(ii) \quad 3x/4 = 9$$

multiply both sides by 4: $3x = 36$

divide both sides by 3

$$x = 12$$

check by putting it back into the original equation:

left side = 9

right side = 9

the two sides agree ✓

$$(iii) \quad (x - 1)/5 = 2$$

multiply both sides by 5: $x - 1 = 10$

add 1 to both sides

$$x = 11$$

check by putting it back into the original equation:

left side = 2

right side = 2

the two sides agree ✓

$$(iv) \quad 3(x + 1)/5 = (2x - 2)/2$$

cross-multiply: $2 \times 3(x + 1) = 5(2x - 2)$

$$6(x + 1) = 10x - 10$$

$$6x + 6 = 10x - 10$$

gather the x terms: $6 + 10 = 10x - 6x$

$$16 = 4x$$

$$x = 4$$

check by putting it back into the original equation:

$$\text{left side} = 3$$

$$\text{right side} = 3$$

the two sides agree ✓

$$(v) \quad 2((x + 5)/3) = (7 - x)/2$$

cross-multiply: $2 \times 2(x + 5) = 3(7 - x)$

$$4(x + 5) = 21 - 3x$$

$$4x + 20 = 21 - 3x$$

gather the x terms: $4x + 3x = 21 - 20$

$$7x = 1$$

$$x = 1/7$$

check by putting it back into the original equation:

$$\text{left side} = 24/7$$

$$\text{right side} = 24/7$$

the two sides agree ✓

$$(vi) \quad (5x - 1)/2 + 4 = 7$$

take 4 from both sides: $(5x - 1)/2 = 3$

multiply both sides by 2: $5x - 1 = 6$

$$5x = 7$$

$$x = 7/5$$

check by putting it back into the original equation:

$$\text{left side} = 7$$

$$\text{right side} = 7$$

the two sides agree ✓

Answer (i) $x = 8$ · (ii) $x = 12$ · (iii) $x = 11$ · (iv) $x = 4$ · (v) $x = 1/7$ · (vi) $x = 7/5$

3. Separate the linear and non-linear equations: (i) $3x - 4 = 0$ (ii) $9 = 1/x$ (iii) $y = 2^x$ (iv) $x - 7y = 8$ (v) $x^2 + y = 5$ (vi) $x + 5 = 12$ (vii) $4x^3 - y^2 + 2 = 0$ (viii) $2x + 3 = 6$

An equation is linear when every variable in it has power exactly 1.

Three things make an equation non-linear:

a power above 1, such as x^2 or x^3

a variable in a denominator, since $1/x$ is x^{-1}

a variable in the exponent, such as 2^x

The number of variables makes no difference — $x - 7y = 8$ is linear in two variables.

$$(i) \quad 3x - 4 = 0$$

x has power 1 and there is no other variable

Linear

$$(ii) \quad 9 = 1/x$$

$1/x$ is x^{-1} — the power is not 1

Non-linear

$$(iii) \quad y = 2^x$$

the variable is in the exponent, which makes it exponential, not linear

Non-linear

$$(iv) \quad x - 7y = 8$$

both x and y have power 1, so it is linear in two variables

Linear

$$(v) \quad x^2 + y = 5$$

x has power 2

Non-linear

$$(vi) \quad x + 5 = 12$$

x has power 1
Linear
(vii) $4x^3 - y^2 + 2 = 0$
x has power 3 and y has power 2
Non-linear
(viii) $2x + 3 = 6$
x has power 1
Linear

Answer Linear: (i), (iv), (vi), (viii) · Non-linear: (ii), (iii), (v), (vii)

EXERCISE 9.2

1. Convert the following into standard form $ax + by = c$: (i) $\frac{1}{2}x + \frac{4}{3}y = 5$ (ii) $3y = \frac{1}{2}x + 7$ (iii) $3x + 6y - 9 = 0$ (iv) $8y + 2x = 16$ (v) $5x + 2 = 3y$ (vi) $2x + 3y = -12$

Standard form is $ax + by = c$, with the two variable terms on the left and the number on the right.
Clear any fractions first by multiplying every term by the LCM of the denominators, then tidy by dividing through by any common factor.

By convention the coefficient of x is left positive.

(i) $\frac{1}{2}x + \frac{4}{3}y = 5$

multiply every term by 6, the LCM of 2 and 3

$$3x + 8y = 30$$

$$3x + 8y = 30$$

(ii) $3y = \frac{1}{2}x + 7$

multiply every term by 2: $6y = x + 14$

move the x across: $-x + 6y = 14$

multiply by -1 so the x term is positive: $x - 6y = -14$

$$x - 6y = -14$$

(iii) $3x + 6y - 9 = 0$

move the constant across: $3x + 6y = 9$

divide every term by 3: $x + 2y = 3$

$$x + 2y = 3$$

(iv) $8y + 2x = 16$

put the x term first: $2x + 8y = 16$

divide every term by 2: $x + 4y = 8$

$$x + 4y = 8$$

(v) $5x + 2 = 3y$

move the 3y across and the 2 across: $5x - 3y = -2$

$$5x - 3y = -2$$

(vi) $2x + 3y = -12$

this is already in the form $ax + by = c$

$$2x + 3y = -12$$

Answer (i) $3x + 8y = 30$ · (ii) $x - 6y = -14$ · (iii) $x + 2y = 3$ · (iv) $x + 4y = 8$ · (v) $5x - 3y = -2$ · (vi) $2x + 3y = -12$

2. Construct a linear equation from each statement: (i) The sum of thrice a number and three equals ten. (ii) Three times a number minus four is equal to twenty-two. (iii) The total cost of x books and y pens is Rs 100. (iv) The perimeter of a rectangle with length x and width y is 300 metres. (v) Four times a number added to seven gives fifteen. (vi) The difference between three times a number and eight equals two. (vii) The age of a father is three times his son's, and their combined age is 50 years.

Name the unknown first — call it x — then translate the sentence one phrase at a time.

'Times' and 'thrice' multiply, 'added to' and 'sum' add, 'minus' and 'difference' subtract, and 'is' or 'equals' becomes the = sign.

Note that most of these are in one variable despite the question's heading — only (iii) and (iv) genuinely involve two.

(i) The sum of thrice a number and three equals ten.

'thrice a number' is $3x$; 'and three' adds 3

$$3x + 3 = 10$$

(ii) Three times a number minus four is equal to twenty-two.

$3x$ first, then subtract 4

$$3x - 4 = 22$$

(iii) The total cost of x books and y pens is Rs 100.

this one really is in two variables, as the question asks

$$x + y = 100$$

(iv) The perimeter of a rectangle with length x and width y is 300 metres.

perimeter = $2(\text{length} + \text{width}) = 2x + 2y$, which simplifies to $x + y = 150$

$$2x + 2y = 300$$

(v) Four times a number added to seven gives fifteen.

$4x$ first, then add 7

$$4x + 7 = 15$$

(vi) The difference between three times a number and eight equals two.

'difference between a and b ' is $a - b$, in that order

$$3x - 8 = 2$$

(vii) The age of a father is three times his son's, and their combined age is 50 years.

let the son be x , so the father is $3x$; together they make 50

$$x + 3x = 50, \text{ that is } 4x = 50$$

Answer (i) $3x + 3 = 10$ · (ii) $3x - 4 = 22$ · (iii) $x + y = 100$ · (iv) $2x + 2y = 300$ · (v) $4x + 7 = 15$ · (vi) $3x - 8 = 2$ · (vii) $x + 3x = 50$, that is $4x = 50$

Please check — the question asks for equations in two variables, but five of the seven statements describe only one unknown. Only (iii) and (iv) give two.

3. Pakiza bought 4 apples and 3 oranges for Rs 100. If each apple costs Rs x and each orange Rs 5, write and solve an equation to find the cost of each apple.

Step 1 Write the cost of each kind

4 apples at Rs x each $\rightarrow 4x$

3 oranges at Rs 5 each $\rightarrow 3 \times 5 = 15$

Step 2 Write the equation

$$4x + 15 = 100$$

Step 3 Solve

$$4x = 100 - 15$$

$$4x = 85$$

$$x = 85 \div 4$$

$$x = \text{Rs } 21.25$$

Step 4 Check

$$4 \times 21.25 = 85, \text{ and } 85 + 15 = \text{Rs } 100 \checkmark$$

The answer is not a whole number of rupees, which is unusual for a shopping question — Rs 85 or Rs 115 as the total would give a whole-rupee apple.

Answer Rs 21.25 per apple

Please check — $4x + 15 = 100$ gives Rs 21.25, not a whole number of rupees. A total of Rs 115 would give exactly Rs 25 an apple.

4. Ahmad is twice as old as Sarah. In 5 years their ages will add up to 40. How old is Sarah now?

Step 1 Name the unknown

let Sarah's age now be x

then Ahmad's age now is $2x$

Step 2 Move forward 5 years

in 5 years Sarah will be $x + 5$

in 5 years Ahmad will be $2x + 5$
 both ages grow by 5 — adding 5 only once is the usual mistake here
 Step 3 Write the equation
 $(x + 5) + (2x + 5) = 40$
 $3x + 10 = 40$
 Step 4 Solve
 $3x = 30$
 $x = 10$
 Step 5 Check
 Sarah is 10 now and Ahmad is 20
 in 5 years they will be 15 and 25, and $15 + 25 = 40$ ✓

Answer Sarah is 10 years old

5. A school raised Rs 10,000, which is Rs 1,000 more than 3 times what they raised last year. How much did they raise last year?

Step 1 Name the unknown
 let last year's amount be x
 Step 2 Translate the sentence
 '3 times what they raised last year' → $3x$
 'Rs 1,000 more than' that → $3x + 1,000$
 and that equals 10,000
 $3x + 1,000 = 10,000$
 Step 3 Solve
 $3x = 10,000 - 1,000$
 $3x = 9,000$
 $x = \text{Rs } 3,000$
 Step 4 Check
 $3 \times 3,000 = 9,000$, and $9,000 + 1,000 = \text{Rs } 10,000$ ✓

Answer Rs 3,000

6. Ahad saved Rs 200 this month, which is Rs 50 more than twice what he saved last month. How much did he save last month?

Step 1 Name the unknown
 let last month's saving be x
 Step 2 Translate
 'twice what he saved last month' → $2x$
 'Rs 50 more than' that → $2x + 50$
 $2x + 50 = 200$
 Step 3 Solve
 $2x = 150$
 $x = \text{Rs } 75$
 Step 4 Check
 $2 \times 75 = 150$, and $150 + 50 = \text{Rs } 200$ ✓

Answer Rs 75

7. Emma bought 5 shirts and 3 pairs of pants for Rs 8,500. Each pair of pants costs Rs 100 more than each shirt. Write and solve an equation to find the cost of each shirt.

Step 1 Name the unknown
 let a shirt cost Rs x
 then a pair of pants costs Rs $(x + 100)$
 Step 2 Write the total
 5 shirts → $5x$
 3 pairs of pants → $3(x + 100)$
 $5x + 3(x + 100) = 8,500$

Step 3 Expand and collect

$$5x + 3x + 300 = 8,500$$

$$8x + 300 = 8,500$$

Step 4 Solve

$$8x = 8,200$$

$$x = 8,200 \div 8$$

$$x = \text{Rs } 1,025$$

Step 5 Check

a shirt is Rs 1,025 and a pair of pants Rs 1,125

$$5 \times 1,025 = 5,125 \text{ and } 3 \times 1,125 = 3,375$$

$$5,125 + 3,375 = \text{Rs } 8,500 \checkmark$$

Answer Rs 1,025 per shirt (and Rs 1,125 per pair of pants)

EXERCISE 9.3

1. Plot each point and state which quadrant it lies in: (i) (2, 3) (ii) (-4, 5) (iii) (-3, -6) (iv) (7, -2) (v) (0, 4) (vi) (-5, 0)

A point (x, y) is found by going along the x-axis first and then up or down — across before up, always in that order.

The four quadrants are numbered anticlockwise from the top right:

Quadrant I — x positive, y positive

Quadrant II — x negative, y positive

Quadrant III — x negative, y negative

Quadrant IV — x positive, y negative

A point with a zero in it sits on an axis and belongs to no quadrant — parts (v) and (vi) are testing that.

(i) (2, 3)

x = 2, so move right 2 from the origin

y = 3, so move up 3

Quadrant I

(ii) (-4, 5)

x = -4, so move left 4 from the origin

y = 5, so move up 5

Quadrant II

(iii) (-3, -6)

x = -3, so move left 3 from the origin

y = -6, so move down 6

Quadrant III

(iv) (7, -2)

x = 7, so move right 7 from the origin

y = -2, so move down 2

Quadrant IV

(v) (0, 4)

x = 0, so move neither way — stay on the y-axis

y = 4, so move up 4

on the y-axis, so in no quadrant

(vi) (-5, 0)

x = -5, so move left 5 from the origin

y = 0, so move neither way — stay on the x-axis

on the x-axis, so in no quadrant

Answer (i) Quadrant I · (ii) Quadrant II · (iii) Quadrant III · (iv) Quadrant IV · (v) on the y-axis, so in no quadrant · (vi) on the x-axis, so in no quadrant

2. Draw the graph of the following linear equations in one variable: (i) $3x - 4 = 2x + 7$ (ii) $x + 5 = 10$ (iii) $x/2 = 4$ (iv) $(x + 1)/4 = 2$ (v) $(3x + 2)/5 = (4x - 1)/6$ (vi) $2(x + 3) = 14$

An equation in one variable fixes x and says nothing about y , so y may be anything. Its graph is a vertical line.

Step 1 — solve the equation for x .

Step 2 — draw the line through that value, parallel to the y -axis.

The grid itself is not reproduced here; the value each line passes through is given for every part.

(i) $3x - 4 = 2x + 7$

solving gives $x = 11$

an equation in one variable graphs as a vertical line through $x = 11$

every point on it has $x = 11$, whatever y is: $(11, 0)$, $(11, 1)$, $(11, -1)$ and so on

draw the line through those points, parallel to the y -axis

(ii) $x + 5 = 10$

solving gives $x = 5$

an equation in one variable graphs as a vertical line through $x = 5$

every point on it has $x = 5$, whatever y is: $(5, 0)$, $(5, 1)$, $(5, -1)$ and so on

draw the line through those points, parallel to the y -axis

(iii) $x/2 = 4$

solving gives $x = 8$

an equation in one variable graphs as a vertical line through $x = 8$

every point on it has $x = 8$, whatever y is: $(8, 0)$, $(8, 1)$, $(8, -1)$ and so on

draw the line through those points, parallel to the y -axis

(iv) $(x + 1)/4 = 2$

solving gives $x = 7$

an equation in one variable graphs as a vertical line through $x = 7$

every point on it has $x = 7$, whatever y is: $(7, 0)$, $(7, 1)$, $(7, -1)$ and so on

draw the line through those points, parallel to the y -axis

(v) $(3x + 2)/5 = (4x - 1)/6$

solving gives $x = 17/2$

an equation in one variable graphs as a vertical line through $x = 17/2$

every point on it has $x = 17/2$, whatever y is: $(17/2, 0)$, $(17/2, 1)$, $(17/2, -1)$ and so on

draw the line through those points, parallel to the y -axis

(vi) $2(x + 3) = 14$

solving gives $x = 4$

an equation in one variable graphs as a vertical line through $x = 4$

every point on it has $x = 4$, whatever y is: $(4, 0)$, $(4, 1)$, $(4, -1)$ and so on

draw the line through those points, parallel to the y -axis

Answer (i) vertical line $x = 11$ · (ii) vertical line $x = 5$ · (iii) vertical line $x = 8$ · (iv) vertical line $x = 7$ · (v) vertical line $x = 17/2$ · (vi) vertical line $x = 4$

3. Draw the graph of the following linear equations in two variables: (i) $y = 2x + 1$ (ii) $y = \frac{1}{2}x - 2$ (iii) $y = -3x + 4$ (iv) $-2x + 5y = 10$ (v) $-\frac{1}{2}x + y = \frac{3}{4}$ (vi) $x + y = 5$

A linear equation in two variables graphs as a straight line, and two points are enough to fix a straight line — though a third is worth plotting as a check.

Step 1 — rearrange to make y the subject.

Step 2 — choose two or three easy values of x and work out y for each.

Step 3 — plot the points and join them.

If the three points do not fall in a straight line, one of them has been worked out wrongly.

(i) $y = 2x + 1$

make y the subject: $y = 2x + 1$

choose a few values of x and work out y for each:

$x = -2 \rightarrow y = -3 \rightarrow$ the point $(-2, -3)$

$x = 0 \rightarrow y = 1 \rightarrow$ the point $(0, 1)$

$x = 2 \rightarrow y = 5 \rightarrow$ the point $(2, 5)$

plot the points and join them with a straight line

points: $(-2, -3)$, $(0, 1)$, $(2, 5)$

(ii) $y = \frac{1}{2}x - 2$

make y the subject: $y = \frac{x}{2} - 2$

choose a few values of x and work out y for each:

$$x = -2 \rightarrow y = -3 \rightarrow \text{the point } (-2, -3)$$

$$x = 0 \rightarrow y = -2 \rightarrow \text{the point } (0, -2)$$

$$x = 2 \rightarrow y = -1 \rightarrow \text{the point } (2, -1)$$

plot the points and join them with a straight line

points: $(-2, -3)$, $(0, -2)$, $(2, -1)$

(iii) $y = -3x + 4$

make y the subject: $y = -3x + 4$

choose a few values of x and work out y for each:

$$x = -2 \rightarrow y = 10 \rightarrow \text{the point } (-2, 10)$$

$$x = 0 \rightarrow y = 4 \rightarrow \text{the point } (0, 4)$$

$$x = 2 \rightarrow y = -2 \rightarrow \text{the point } (2, -2)$$

plot the points and join them with a straight line

points: $(-2, 10)$, $(0, 4)$, $(2, -2)$

(iv) $-2x + 5y = 10$

make y the subject: $y = \frac{2x}{5} + 2$

choose a few values of x and work out y for each:

$$x = -2 \rightarrow y = \frac{6}{5} \rightarrow \text{the point } (-2, \frac{6}{5})$$

$$x = 0 \rightarrow y = 2 \rightarrow \text{the point } (0, 2)$$

$$x = 2 \rightarrow y = \frac{14}{5} \rightarrow \text{the point } (2, \frac{14}{5})$$

plot the points and join them with a straight line

points: $(-2, \frac{6}{5})$, $(0, 2)$, $(2, \frac{14}{5})$

(v) $-\frac{1}{2}x + y = \frac{3}{4}$

make y the subject: $y = \frac{x}{2} + \frac{3}{4}$

choose a few values of x and work out y for each:

$$x = -2 \rightarrow y = -\frac{1}{4} \rightarrow \text{the point } (-2, -\frac{1}{4})$$

$$x = 0 \rightarrow y = \frac{3}{4} \rightarrow \text{the point } (0, \frac{3}{4})$$

$$x = 2 \rightarrow y = \frac{7}{4} \rightarrow \text{the point } (2, \frac{7}{4})$$

plot the points and join them with a straight line

points: $(-2, -\frac{1}{4})$, $(0, \frac{3}{4})$, $(2, \frac{7}{4})$

(vi) $x + y = 5$

make y the subject: $y = -x + 5$

choose a few values of x and work out y for each:

$$x = -2 \rightarrow y = 7 \rightarrow \text{the point } (-2, 7)$$

$$x = 0 \rightarrow y = 5 \rightarrow \text{the point } (0, 5)$$

$$x = 2 \rightarrow y = 3 \rightarrow \text{the point } (2, 3)$$

plot the points and join them with a straight line

points: $(-2, 7)$, $(0, 5)$, $(2, 3)$

Answer (i) through $(-2, -3)$, $(0, 1)$, $(2, 5)$

(ii) through $(-2, -3)$, $(0, -2)$, $(2, -1)$

(iii) through $(-2, 10)$, $(0, 4)$, $(2, -2)$

(iv) through $(-2, \frac{6}{5})$, $(0, 2)$, $(2, \frac{14}{5})$

(v) through $(-2, -\frac{1}{4})$, $(0, \frac{3}{4})$, $(2, \frac{7}{4})$

(vi) through $(-2, 7)$, $(0, 5)$, $(2, 3)$

4. Find the values of x and y for points A, B and C on the given graph.

The graph itself is printed in the book, so the readings must be taken off the page. The method is the same for every point:

Step 1 — from the point, look straight down (or up) to the x -axis. That reading is the x -coordinate.

Step 2 — from the point, look straight across to the y -axis. That reading is the y -coordinate.

Step 3 — write them as an ordered pair (x, y) , x first.

Step 4 — check the signs: left of the y -axis makes x negative, below the x -axis makes y negative.

The unit's own worked example does exactly this, reading $A(-4, 0)$ and $B(0, -3)$ off its line.

Answer Read from the printed graph, by the method above.

Please check — the book's worked example labels its second point $A(0, -3)$ when it should be $B(0, -3)$.

REVISION EXERCISE 9

1. Choose the correct option.

- i. What is the solution of $2x + 5 = 15$?

a) 5 b) 10 c) 0 d) 15

Step 1 take 5 from both sides

$$2x = 15 - 5 = 10$$

Step 2 divide both sides by 2

$$x = 5$$

Check $2 \times 5 + 5 = 10 + 5 = 15 \checkmark$

option (b) 10 is the value of $2x$, not of x — the division was forgotten.

Answer a) 5

- ii. In the Cartesian system, what is the x -coordinate of the point $(4, 2)$?

a) 4 b) 2 c) 6 d) 0

A point is written (x, y) , with the x -coordinate always first.

In $(4, 2)$ the first number is 4.

$x = 4$, and $y = 2$.

Option (b) gives the y -coordinate — the commonest slip, and the reason the order matters.

Answer a) 4

- iii. If $y = 2x + 1$, what is y when $x = 3$?

a) 2 b) 7 c) 3 d) 5

Step 1 substitute $x = 3$

$$y = 2 \times 3 + 1$$

Step 2 multiply before adding

$$y = 6 + 1$$

$$y = 7$$

Option (d) 5 comes from adding 2 and 3 first — BODMAS puts the multiplication ahead of the addition.

Answer b) 7

- iv. Which of the following is a valid linear equation in two variables?

a) $x^2 + y = 4$ b) $3x - 4y = 12$ c) $xy = 10$ d) $x + y + z = 3$

A linear equation in two variables has exactly two variables, each to the power 1.

(a) $x^2 + y = 4$ — x has power 2 ✗

(b) $3x - 4y = 12$ — two variables, both to power 1 ✓

(c) $xy = 10$ — the variables are multiplied together, which makes the term degree 2 ✗

(d) $x + y + z = 3$ — three variables, not two ✗

Answer b) $3x - 4y = 12$

- v. Which of the following represents a horizontal line?

a) $y = 5$ b) $x = 3$ c) $y = 2x + 1$ d) $y = -x$

A horizontal line keeps the same height everywhere, so y is fixed and x is free.

(a) $y = 5$ — y is always 5, whatever x is ✓ horizontal

(b) $x = 3$ — x is fixed, so this is vertical ✗

(c) $y = 2x + 1$ — y changes with x , a sloping line ✗

(d) $y = -x$ — also sloping ✗

Remember it this way: $y = \text{a number}$ is flat, $x = \text{a number}$ is upright.

Answer a) $y = 5$

2. Fill in the blanks: (i) The ____ coordinate system consists of two perpendicular axes. (ii) The solution to a linear equation in one variable represents the value of _____. (iii) The point where the x -axis and y -axis intersect is called the _____. (iv) A horizontal line has the equation _____, which indicates that the y -value remains constant. (v) A vertical line has the equation _____, which indicates that the x -value remains constant.

(i) The ____ coordinate system consists of two perpendicular axes.

named after René Descartes

Cartesian

(ii) The solution to a linear equation in one variable represents the value of _____.

the one number that makes both sides equal

the variable

(iii) The point where the x -axis and y -axis intersect is called the _____.

it is the point $(0, 0)$

origin

(iv) A horizontal line has the equation _____, which indicates that the y -value remains constant.

such as $y = 5$ — the height never changes

$y = \text{a constant}$

(v) A vertical line has the equation _____, which indicates that the x -value remains constant.

such as $x = 3$ — the horizontal position never changes

$x = \text{a constant}$

Answer (i) Cartesian · (ii) the variable · (iii) origin · (iv) $y = \text{a constant}$ · (v) $x = \text{a constant}$

3. Plot the following points and join them with a straight line: $x = -3, -2, -1, 0, 1, 2, 3$ against $y = 6, 4, 2, 0, -2, -4, -6$.

Step 1 Pair the values into coordinates

read the two rows together, one column at a time:

$(-3, 6)$

$(-2, 4)$

$(-1, 2)$

$(0, 0)$

$(1, -2)$

$(2, -4)$

$(3, -6)$

Step 2 Look for the rule

each y is -2 times its x : $6 = -2 \times (-3)$, $4 = -2 \times (-2)$, and so on

so the line is $y = -2x$

Step 3 Plot and join

the points fall in a straight line through the origin, sloping downward from left to right

Step 4 Check

the line passes through $(0, 0)$, and as x rises by 1 the y falls by 2 every time ✓

Answer A straight line through the origin: $y = -2x$

4. Draw the graph of the line $y = 2x + 1$.

Step 1 Build a table of points

choose easy values of x and work out y for each:

$$x = -1 \rightarrow y = 2(-1) + 1 = -1 \rightarrow (-1, -1)$$

$$x = 0 \rightarrow y = 2(0) + 1 = 1 \rightarrow (0, 1)$$

$$x = 1 \rightarrow y = 2(1) + 1 = 3 \rightarrow (1, 3)$$

$$x = 2 \rightarrow y = 2(2) + 1 = 5 \rightarrow (2, 5)$$

Step 2 Plot and join

the four points lie in a straight line

Step 3 Read off the two useful features

the line crosses the y-axis at (0, 1) — the +1 in the equation

for every 1 step right it rises 2 — the 2 in the equation

Step 4 Check

all four points satisfy $y = 2x + 1$ ✓

Answer A straight line through $(-1, -1)$, $(0, 1)$, $(1, 3)$ and $(2, 5)$

5. Draw the graph of: (i) $x + y = 3$ (ii) $y = x$ (iii) $y = 3x + 1$

(i) $x + y = 3$

make y the subject: $y = 3 - x$

$$x = 0 \rightarrow y = 3 \rightarrow (0, 3)$$

$$x = 3 \rightarrow y = 0 \rightarrow (3, 0)$$

$$x = 1 \rightarrow y = 2 \rightarrow (1, 2)$$

a line sloping down, cutting both axes at 3

(ii) $y = x$

$$x = 0 \rightarrow y = 0 \rightarrow (0, 0)$$

$$x = 1 \rightarrow y = 1 \rightarrow (1, 1)$$

$$x = 2 \rightarrow y = 2 \rightarrow (2, 2)$$

the line through the origin at 45° , where x and y are always equal

(iii) $y = 3x + 1$

$$x = -1 \rightarrow y = -2 \rightarrow (-1, -2)$$

$$x = 0 \rightarrow y = 1 \rightarrow (0, 1)$$

$$x = 1 \rightarrow y = 4 \rightarrow (1, 4)$$

a steeper line, crossing the y-axis at 1 and rising 3 for every 1 across

Compare the three: (i) slopes down, (ii) and (iii) slope up, and (iii) is the steepest. The number in front of x is what controls the slope.

Answer (i) through $(0, 3)$, $(1, 2)$, $(3, 0)$ · (ii) through $(0, 0)$, $(1, 1)$, $(2, 2)$ · (iii) through $(-1, -2)$, $(0, 1)$, $(1, 4)$

6. What is the equation of the horizontal line passing through $(2, 5)$?

Step 1 Recall what makes a line horizontal

a horizontal line keeps the same height all the way along, so y never changes

its equation is therefore $y = \text{a constant}$

Step 2 Read the height off the point

the point $(2, 5)$ has $y = 5$

Step 3 Write the equation

$$y = 5$$

Step 4 Check

does $(2, 5)$ satisfy $y = 5$? Yes — the y-coordinate is 5 ✓

the x-coordinate 2 plays no part: the line passes through $(0, 5)$, $(2, 5)$, $(100, 5)$ and every other point at that height.

Had the question asked for the vertical line through the same point, the answer would be $x = 2$.

Answer $y = 5$



10

CHAPTER

Speed, Distance and Time

EXERCISE 10.1

1. Convert: (i) 5 km 313 m to m (ii) 3.75 km to mm (iii) 8,500 m to km (iv) 7 km 125 m to m (v) 0.45 km to cm (vi) 325 mm to cm

The whole chain of length units, each step a factor of ten:

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ m} = 100 \text{ cm} = 1,000 \text{ mm}$$

$$1 \text{ km} = 1,000 \text{ m} = 100,000 \text{ cm} = 1,000,000 \text{ mm}$$

Going to a smaller unit multiplies; going to a larger unit divides. Check every answer against that before writing it down.

- (i) 5 km 313 m to m

first put the whole distance in one unit:

$$5 \text{ km} = 5 \times 1,000 = 5,000 \text{ m}$$

$$5,000 + 313 = 5,313 \text{ m}$$

- (ii) 3.75 km to mm

$$1 \text{ km} = 1,000,000 \text{ mm}$$

mm is the smaller unit, so multiply

$$3.75 \times 1,000,000 = 3,750,000 \text{ mm}$$

- (iii) 8,500 m to km

$$1 \text{ m} = 0.001 \text{ km}$$

km is the larger unit, so divide

$$8,500 \div 1,000 = 8.5 \text{ km}$$

- (iv) 7 km 125 m to m

$$7 \text{ km} = 7 \times 1,000 = 7,000 \text{ m}$$

$$7,000 + 125 = 7,125 \text{ m}$$

- (v) 0.45 km to cm

$$1 \text{ km} = 100,000 \text{ cm}$$

cm is the smaller unit, so multiply

$$0.45 \times 100,000 = 45,000 \text{ cm}$$

- (vi) 325 mm to cm

$$1 \text{ mm} = 0.1 \text{ cm}$$

cm is the larger unit, so divide

$$325 \div 10 = 32.5 \text{ cm}$$

Answer (i) 5,313 m · (ii) 3,750,000 mm · (iii) 8.5 km · (iv) 7,125 m · (v) 45,000 cm · (vi) 32.5 cm

2. A car travels 5 km 800 m in the morning and 4 km 600 m in the evening. What is the total distance in a day?

Step 1 Put both into the same unit
 $5 \text{ km } 800 \text{ m} = 5,000 + 800 = 5,800 \text{ m}$
 $4 \text{ km } 600 \text{ m} = 4,000 + 600 = 4,600 \text{ m}$
 Step 2 Add
 $5,800 + 4,600 = 10,400 \text{ m}$
 Step 3 Give it back in km and m
 $10,400 \div 1,000 = 10 \text{ remainder } 400$
 $= 10 \text{ km } 400 \text{ m}$
 Step 4 Check
 $10 \times 1,000 + 400 = 10,400 \text{ m } \checkmark$

Answer 10,400 m, that is 10 km 400 m

3. A runner covers 325 m in the first lap and 475 m in the second. What is the total distance?

Both are already in metres, so they can be added at once.
 Step 1 Add
 $325 + 475$
 add the units: $5 + 5 = 10$, write 0 carry 1
 add the tens: $2 + 7 + 1 = 10$, write 0 carry 1
 add the hundreds: $3 + 4 + 1 = 8$
 $= 800 \text{ m}$
 Step 2 Check
 $800 - 475 = 325 \checkmark$
 800 m is 0.8 km, a little under a kilometre.

Answer 800 m

4. A hiker walked 7 km 600 m on Monday and 3 km 400 m on Tuesday. How much further did the hiker walk on Monday?

Step 1 Put both into metres
 Monday: $7 \text{ km } 600 \text{ m} = 7,600 \text{ m}$
 Tuesday: $3 \text{ km } 400 \text{ m} = 3,400 \text{ m}$
 Step 2 Subtract the smaller from the larger
 $7,600 - 3,400 = 4,200 \text{ m}$
 Step 3 Give it in km and m
 $= 4 \text{ km } 200 \text{ m}$
 Step 4 Check
 $3,400 + 4,200 = 7,600 \text{ m } \checkmark$

Answer 4,200 m, that is 4 km 200 m

5. A highway is 2,000 m long and a car has travelled 1,650 m. How much is left?

What is left is the whole minus what has been done.
 Step 1 Subtract
 $2,000 - 1,650 = 350 \text{ m}$
 Step 2 Check
 $1,650 + 350 = 2,000 \text{ m } \checkmark$
 350 m is 0.35 km — a little over a third of a kilometre.

Answer 350 m

6. Convert 7 km 125 m to metres and add it to 3,000 m. What is the total in metres?

Step 1 Convert
 $7 \text{ km} = 7 \times 1,000 = 7,000 \text{ m}$
 $7,000 + 125 = 7,125 \text{ m}$
 Step 2 Add the 3,000 m
 $7,125 + 3,000 = 10,125 \text{ m}$
 Step 3 Check
 $10,125 - 3,000 = 7,125 \text{ m} \checkmark$
 In kilometres that is 10.125 km.

Answer 10,125 m

7. A cyclist covers 5 km 800 m. Convert this into metres.

Step 1 Convert the kilometres
 $1 \text{ km} = 1,000 \text{ m}$
 $5 \text{ km} = 5 \times 1,000 = 5,000 \text{ m}$
 Step 2 Add the metres already given
 $5,000 + 800 = 5,800 \text{ m}$
 Step 3 Check
 $5,800 \div 1,000 = 5.8$, which is 5 km and 800 m $\checkmark$

Answer 5,800 m

8. A road is 8,200 m long. Convert this into kilometres.

Step 1 Decide multiply or divide
 km is the larger unit, so divide
 Step 2 Divide by 1,000
 $8,200 \div 1,000 = 8.2 \text{ km}$
 Step 3 Check
 $8.2 \times 1,000 = 8,200 \text{ m} \checkmark$
 It can also be written as 8 km 200 m.

Answer 8.2 km, that is 8 km 200 m

EXERCISE 10.2

1. Change the following to 24-hour format: (i) 5:55 pm (ii) 5:55 pm (iii) 7:30 am (iv) 12:15 pm (v) 9:45 pm (vi) 1:05 am (vii) 3:20 pm (viii) 10:00 pm (ix) 11:50 pm (x) 6:10 pm

Before noon (a.m.) — the hour stays the same; just drop the a.m. and write two digits for the hour.
 After noon (p.m.) — add 12 to the hour and drop the p.m.

The two that catch people out: 12 midnight becomes 00, and 12 noon stays 12 — neither has 12 added to it.

Every answer below is converted back to the 12-hour clock as a check.

(i) 5:55 pm

it is after noon, so add 12 to the hour: $5 + 12 = 17$

17:55

check by converting back: 5:55 p.m. $\checkmark$

(ii) 5:55 pm

it is after noon, so add 12 to the hour: $5 + 12 = 17$

17:55

check by converting back: 5:55 p.m. $\checkmark$

(iii) 7:30 am

it is before noon, so the hour stays as it is — only the a.m. is dropped

07:30

check by converting back: 7:30 a.m. $\checkmark$

(iv) 12:15 pm

12 noon is already 12 in the 24-hour clock, so the hour does not change

12:15

check by converting back: 12:15 p.m. ✓

(v) 9:45 pm

it is after noon, so add 12 to the hour: $9 + 12 = 21$

21:45

check by converting back: 9:45 p.m. ✓

(vi) 1:05 am

it is before noon, so the hour stays as it is — only the a.m. is dropped

01:05

check by converting back: 1:05 a.m. ✓

(vii) 3:20 pm

it is after noon, so add 12 to the hour: $3 + 12 = 15$

15:20

check by converting back: 3:20 p.m. ✓

(viii) 10:00 pm

it is after noon, so add 12 to the hour: $10 + 12 = 22$

22:00

check by converting back: 10:00 p.m. ✓

(ix) 11:50 pm

it is after noon, so add 12 to the hour: $11 + 12 = 23$

23:50

check by converting back: 11:50 p.m. ✓

(x) 6:10 pm

it is after noon, so add 12 to the hour: $6 + 12 = 18$

18:10

check by converting back: 6:10 p.m. ✓

Answer (i) 17:55 · (ii) 17:55 · (iii) 07:30 · (iv) 12:15 · (v) 21:45 · (vi) 01:05 · (vii) 15:20 · (viii) 22:00 · (ix) 23:50 · (x) 18:10

Please check — parts (i) and (ii) are both printed as 5:55 pm.

2. Write these 24-hour times in a.m. or p.m. format: (i) 21:34 hours (ii) 23:34 hours (iii) 15:12 hours (iv) 06:45 hours (v) 19:00 hours (vi) 00:30 hours (vii) 12:00 hours (viii) 17:25 hours (ix) 03:15 hours (x) 22:10 hours

This is the reverse of Q1.

Hour under 12 — keep it and write a.m.

Hour over 12 — take 12 away and write p.m.

00 hours — that is midnight, written 12:___ a.m.

12 hours — that is noon, written 12:___ p.m.

(i) 21:34 hours

the hour is past 12, so take 12 away: $21 - 12 = 9$, and it is p.m.

9:34 p.m.

check by converting back: 21:34 ✓

(ii) 23:34 hours

the hour is past 12, so take 12 away: $23 - 12 = 11$, and it is p.m.

11:34 p.m.

check by converting back: 23:34 ✓

(iii) 15:12 hours

the hour is past 12, so take 12 away: $15 - 12 = 3$, and it is p.m.

3:12 p.m.

check by converting back: 15:12 ✓

(iv) 06:45 hours

the hour is under 12, so it stays as it is, and it is a.m.

6:45 a.m.

- check by converting back: 06:45 ✓
- (v) 19:00 hours
the hour is past 12, so take 12 away: $19 - 12 = 7$, and it is p.m.
7:00 p.m.
check by converting back: 19:00 ✓
- (vi) 00:30 hours
00 hours is midnight, written as 12 in the 12-hour clock, and it is a.m.
12:30 a.m.
check by converting back: 00:30 ✓
- (vii) 12:00 hours
12:00 is noon exactly — 12:00 p.m.
12:00 p.m.
check by converting back: 12:00 ✓
- (viii) 17:25 hours
the hour is past 12, so take 12 away: $17 - 12 = 5$, and it is p.m.
5:25 p.m.
check by converting back: 17:25 ✓
- (ix) 03:15 hours
the hour is under 12, so it stays as it is, and it is a.m.
3:15 a.m.
check by converting back: 03:15 ✓
- (x) 22:10 hours
the hour is past 12, so take 12 away: $22 - 12 = 10$, and it is p.m.
10:10 p.m.
check by converting back: 22:10 ✓

Answer (i) 9:34 p.m. · (ii) 11:34 p.m. · (iii) 3:12 p.m. · (iv) 6:45 a.m. · (v) 7:00 p.m. · (vi) 12:30 a.m.
· (vii) 12:00 p.m. · (viii) 5:25 p.m. · (ix) 3:15 a.m. · (x) 10:10 p.m.

EXERCISE 10.3

1. A train departs Chicago at 9:00 a.m. and takes 3 hours 30 minutes to reach Detroit. What time does it arrive? If it leaves on a Saturday, what day does it arrive?

Arrival time = departure time + journey time.

Step 1 Write the departure in 24-hour form
9:00 a.m. → 09:00

Step 2 Add the hours
 $09:00 + 3 \text{ hours} = 12:00$

Step 3 Add the minutes
 $12:00 + 30 \text{ minutes} = 12:30$

Step 4 Write it back in 12-hour form
12:30 p.m.

Step 5 Check
 $12:30 \text{ p.m.} - 9:00 \text{ a.m.} = 3 \text{ h } 30 \text{ min} \checkmark$

Step 6 The day
the arrival time 12:30 is still inside the same 24 hours — it has not passed 24:00
so the train arrives on the same day, Saturday
a journey only runs into the next day when the arrival total passes 24:00.

Answer 12:30 p.m. on the same day, Saturday

2. A taxi picks up a passenger at 5:30 p.m. and takes 45 minutes to reach the airport. What time does the passenger arrive?

Arrival time = departure time + journey time.

Step 1 Write the departure in 24-hour form

5:30 p.m. → 17:30

Step 2 Add the hours

17:30 + 0 hours = 17:30

Step 3 Add the minutes

17:30 + 45 minutes = 18:15

Step 4 Write it back in 12-hour form

6:15 p.m.

Step 5 Check

6:15 p.m. – 5:30 p.m. = 0 h 45 min ✓

Answer 6:15 p.m.

3. A family starts a road trip at 4:00 p.m. and drives for 5 hours 20 minutes. What time will they arrive?

Arrival time = departure time + journey time.

Step 1 Write the departure in 24-hour form

4:00 p.m. → 16:00

Step 2 Add the hours

16:00 + 5 hours = 21:00

Step 3 Add the minutes

21:00 + 20 minutes = 21:20

Step 4 Write it back in 12-hour form

9:20 p.m.

Step 5 Check

9:20 p.m. – 4:00 p.m. = 5 h 20 min ✓

Answer 9:20 p.m.

4. A motorcycle trip starts at 8:00 a.m. and lasts 2 hours 35 minutes. What time will it end?

Arrival time = departure time + journey time.

Step 1 Write the departure in 24-hour form

8:00 a.m. → 08:00

Step 2 Add the hours

08:00 + 2 hours = 10:00

Step 3 Add the minutes

10:00 + 35 minutes = 10:35

Step 4 Write it back in 12-hour form

10:35 a.m.

Step 5 Check

10:35 a.m. – 8:00 a.m. = 2 h 35 min ✓

Answer 10:35 a.m.

5. A delivery truck leaves the warehouse at 4:00 p.m. and arrives at the store at 5:30 p.m. How long is the journey?

This one gives both times and asks for the journey, so the formula is turned round.

Journey time = arrival time – departure time.

Step 1 Put both into 24-hour form

4:00 p.m. → 16:00

5:30 p.m. → 17:30

Step 2 Subtract

17:30 – 16:00

hours: 17 – 16 = 1

minutes: 30 – 00 = 30

= 1 hour 30 minutes

Step 3 Check

$$16:00 + 1 \text{ h } 30 \text{ min} = 17:30 \checkmark$$

Answer 1 hour 30 minutes

EXERCISE 10.4

1. Complete the table converting between m/s and km/h for 35, 26 and 55.

The two units are linked by one factor:

$$1 \text{ m/s} = 3,600 \text{ m in an hour} = 3.6 \text{ km/h}$$

so m/s $\rightarrow$ km/h: multiply by $18/5$ (which is 3.6)

and km/h $\rightarrow$ m/s: multiply by $5/18$

Where that factor comes from: $1 \text{ km} = 1,000 \text{ m}$ and $1 \text{ hour} = 3,600 \text{ s}$, so $1 \text{ km/h} = 1,000 \div 3,600 = 5/18 \text{ m/s}$.

The printed table does not show which column each figure belongs in, so both directions are given for each — see the note.

35

$$\text{if the 35 is in m/s: } 35 \times 18/5 = 126 \text{ km/h}$$

$$\text{if the 35 is in km/h: } 35 \times 5/18 = 9.72 \text{ m/s}$$

26

$$\text{if the 26 is in m/s: } 26 \times 18/5 = 93.6 \text{ km/h}$$

$$\text{if the 26 is in km/h: } 26 \times 5/18 = 7.22 \text{ m/s}$$

55

$$\text{if the 55 is in m/s: } 55 \times 18/5 = 198 \text{ km/h}$$

$$\text{if the 55 is in km/h: } 55 \times 5/18 = 15.28 \text{ m/s}$$

Answer 35 m/s = 126 km/h, or 35 km/h = 9.72 m/s · 26 m/s = 93.6 km/h, or 26 km/h = 7.22 m/s · 55 m/s = 198 km/h, or 55 km/h = 15.28 m/s

Please check — the table's two columns are headed m/s and km/h, but the three figures are not shown against either column, so it is not clear which conversion each needs. Both are given.

2. A bus travels at a constant 60 km/h. How far will it go in 2.5 hours?

Distance = speed $\times$ time.

Step 1 Check the units agree

the speed is in km per hour and the time is in hours $\checkmark$ — nothing to convert

Step 2 Multiply

$$\text{Distance} = 60 \times 2.5$$

$$= 60 \times 2 + 60 \times 0.5$$

$$= 120 + 30$$

$$= 150 \text{ km}$$

Step 3 Check

$$150 \div 2.5 = 60 \text{ km/h } \checkmark$$

Answer 150 km

3. Sarah runs 4 km in 30 minutes and then 2 km in 15 minutes. What is her average speed for the whole run?

Average speed = total distance $\div$ total time. The two separate speeds are not averaged.

Step 1 Total distance

$$4 + 2 = 6 \text{ km}$$

Step 2 Total time

$$30 + 15 = 45 \text{ minutes}$$

Step 3 Put the time into hours

$$45 \div 60 = 3/4 = 0.75 \text{ hours}$$

Step 4 Divide

$$\text{Average speed} = 6 \div 0.75$$

$$= 8 \text{ km/h}$$

Step 5 Check

$$8 \times 0.75 = 6 \text{ km } \checkmark$$

Both legs happen to be at 8 km/h here, so the average matches — but that is a coincidence of these figures, not a rule.

Answer 8 km/h

4. James walks at a steady 5 km/h for 4 hours. What distance does he cover?

Distance = speed $\times$ time.

Step 1 The units already agree

km per hour, and a time in hours

Step 2 Multiply

$$5 \times 4 = 20 \text{ km}$$

Step 3 Check

$$20 \div 4 = 5 \text{ km/h } \checkmark$$

'Steady' means the speed never changes, so this is uniform speed rather than an average.

Answer 20 km

5. A delivery van travels 80 km in 1 hour and then 40 km in 30 minutes. What is its average speed for the journey?

Step 1 Total distance

$$80 + 40 = 120 \text{ km}$$

Step 2 Total time

$$30 \text{ minutes} = 0.5 \text{ hours}$$

$$1 + 0.5 = 1.5 \text{ hours}$$

Step 3 Divide

$$\text{Average speed} = 120 \div 1.5$$

$$= 80 \text{ km/h}$$

Step 4 Check

$$80 \times 1.5 = 120 \text{ km } \checkmark$$

The second leg was at $40 \div 0.5 = 80 \text{ km/h}$ too, so the van held the same speed throughout.

Answer 80 km/h

EXERCISE 10.5

1. Convert: (i) 45 m/s into km/h (ii) 140 km/h into m/s (iii) 180 km/h into m/s (iv) 15 h 16 min into minutes (v) 185 days into years and days (vi) 1,280 s into minutes and seconds (vii) 53 years into years and months (viii) 21 days into weeks and days

Two sets of conversions are needed here.

Speed — m/s $\rightarrow$ km/h multiply by 18/5; km/h $\rightarrow$ m/s multiply by 5/18.

Time — 60 seconds in a minute, 60 minutes in an hour, 7 days in a week, 12 months in a year, 365 days in a year.

Where an answer is wanted in two units, divide and keep the remainder: the quotient gives the larger unit and the remainder the smaller.

(i) 45 m/s into km/h

m/s $\rightarrow$ km/h, so multiply by 18/5

$$45 \times 18/5 = 45 \times 3.6$$

$$= 162 \text{ km/h}$$

(ii) 140 km/h into m/s

km/h $\rightarrow$ m/s, so multiply by 5/18

$$140 \times 5/18 = 700/18$$

$$= 38.89 \text{ m/s}$$

(iii) 180 km/h into m/s

$$180 \times 5/18$$

$$\text{the 18 cancels into the 180 exactly: } 180 \div 18 = 10$$

$$10 \times 5 = 50$$

- $= 50 \text{ m/s}$
- (iv) 15 h 16 min into minutes
 1 hour = 60 minutes
 $15 \times 60 = 900 \text{ minutes}$
 $900 + 16 = 916$
 $= 916 \text{ minutes}$
- (vi) 1,280 s into minutes and seconds
 1 minute = 60 seconds, so divide by 60
 $1,280 \div 60 = 21 \text{ remainder } 20$
 21 minutes 20 seconds
 check: $21 \times 60 + 20 = 1,260 + 20 = 1,280 \text{ s } \checkmark$
- (viii) 21 days into weeks and days
 1 week = 7 days, so divide by 7
 $21 \div 7 = 3 \text{ remainder } 0$
 3 weeks exactly, with 0 days left over
 check: $3 \times 7 = 21 \text{ days } \checkmark$
- (v) 185 days into years and days
 1 year = 365 days, and 185 is less than 365
 $185 \div 365 = 0 \text{ remainder } 185$
 0 years and 185 days — there is nothing to convert; see the note
- (vii) 53 years into years and months
 a figure already in years converts to itself: 53 years and 0 months
 if the figure is 53 months, which is almost certainly meant:
 1 year = 12 months, so divide by 12
 $53 \div 12 = 4 \text{ remainder } 5$
 4 years 5 months
 check: $4 \times 12 + 5 = 53 \checkmark$

Answer (i) 162 km/h · (ii) 38.89 m/s · (iii) 50 m/s · (iv) 916 minutes · (vi) see above · (viii) see above
 · (v) 0 years 185 days · (vii) 53 years 0 months as printed, or 4 years 5 months if 53 months was meant

Please check — parts (v) and (vii) give nothing to convert as printed — see the notes above.

2. Alia works in an organisation for 122 months. For how many years does she work there?

- Step 1 Recall the conversion
 1 year = 12 months, so divide by 12
- Step 2 Divide
 $122 \div 12$
 12 goes into 12 once, leaving 2
 bring down the 2 → 22; 12 goes into 22 once, leaving 10
 so $122 \div 12 = 10 \text{ remainder } 2$
 $= 10 \text{ years } 2 \text{ months}$
- Step 3 Check
 $10 \times 12 + 2 = 120 + 2 = 122 \text{ months } \checkmark$
 As a decimal that is about 10.17 years, but years and months is the natural way to give it.

Answer 10 years 2 months

3. A family holiday is planned for 5 weeks and 4 days. How many days in total?

- Step 1 Convert the weeks
 1 week = 7 days
 $5 \times 7 = 35 \text{ days}$
- Step 2 Add the loose days
 $35 + 4 = 39 \text{ days}$
- Step 3 Check
 $39 \div 7 = 5 \text{ remainder } 4$, which is 5 weeks 4 days $\checkmark$

Answer 39 days

4. A project lasted 470 days. How many years and weeks is that?

Step 1 Take out the whole years

1 year = 365 days

$470 \div 365 = 1$ remainder 105

so that is 1 year with 105 days left over

Step 2 Turn the leftover days into weeks

1 week = 7 days

$105 \div 7 = 15$ remainder 0

= 1 year and 15 weeks exactly

Step 3 Check

$365 + 15 \times 7 = 365 + 105 = 470$ days ✓

The figures were chosen so the remainder divides by 7 exactly, which is why no days are left at the end.

Answer 1 year and 15 weeks

Please check — this question is printed as a second Q.3; it is numbered Q.4 here.

REVISION EXERCISE 10

1. Choose the correct option.

- i. What is 3:30 p.m. in 24-hour format?

a) 15:30 b) 13:30 c) 14:30 d) 16:30

Step 1 it is after noon, so add 12 to the hour

Step 2 $3 + 12 = 15$

Step 3 the minutes do not change

= 15:30

Check $15 - 12 = 3$, so 15:30 is 3:30 p.m. ✓

Option (b) 13:30 would be 1:30 p.m. and (d) 16:30 would be 4:30 p.m. — both are an hour out.

Answer a) 15:30

- ii. How many seconds are there in 2.5 hours?

a) 10,800 s b) 8,000 s c) 9,000 s d) 7,200 s

Step 1 1 hour = 60 minutes = $60 \times 60 = 3,600$ seconds

Step 2 $2.5 \times 3,600$

= $2 \times 3,600 + 0.5 \times 3,600$

= $7,200 + 1,800$

= 9,000 seconds

Check $9,000 \div 3,600 = 2.5$ hours ✓

Option (d) 7,200 is 2 hours and option (a) 10,800 is 3 hours — the half hour has been dropped or over-counted.

Answer c) 9,000 s

- iii. A bus leaves at 4:00 p.m. and arrives at 6:30 p.m. How long is the journey?

a) 1 h 30 min b) 2 h 15 min c) 2 h 30 min d) 3 h

Journey time = arrival – departure.

Step 1 4:00 p.m. → 16:00 and 6:30 p.m. → 18:30

Step 2 hours: $18 - 16 = 2$

Step 3 minutes: $30 - 00 = 30$

= 2 hours 30 minutes

Check $16:00 + 2 \text{ h } 30 \text{ min} = 18:30$ ✓

Answer c) 2 h 30 min

iv. Which of the following equals 1 kilometre?

- a) 100 m b) 1,000 m c) 10,000 m d) 0.1 m

The prefix kilo means a thousand.

(a) 100 m is a hectometre $\times$

(b) 1,000 m $\checkmark$

(c) 10,000 m is 10 km $\times$

(d) 0.1 m is 10 cm $\times$

Answer b) 1,000 m

v. If a car covers 100 m in 50 s, its speed is:

- a) 1 m/s b) 0.5 m/s c) 2 m/s d) $y = -x$

Speed = distance $\div$ time.

Step 1 = $100 \div 50$

Step 2 = 2 m/s

Check $2 \times 50 = 100$ m $\checkmark$

Option (b) 0.5 m/s is the time divided by the distance — the division the wrong way round.

Option (d) is not a speed at all; it has been copied in from another unit — see the note.

Answer c) 2 m/s

Please check — option (d) ' $y = -x$ ' is a straight-line equation from Unit 9's revision exercise. It should be a speed, such as 5 m/s.

2. Fill in the blanks: (i) There are ____ seconds in 1 hour. (ii) If a car travels 90 km in 1 hour, its speed is ____ km/h. (iii) Speed is the rate of change of _____. (iv) 1 day = ____ week. (v) The time between 12 midnight and 12 noon is known as _____.

(i) There are ____ seconds in 1 hour.

60 seconds in a minute $\times$ 60 minutes in an hour = 3,600
3,600

(ii) If a car travels 90 km in 1 hour, its speed is ____ km/h.

speed = distance $\div$ time = $90 \div 1 = 90$
90

(iii) Speed is the rate of change of _____.

it says how much distance is covered in each unit of time
distance (with time)

(iv) 1 day = ____ week.

7 days make a week, so one day is one seventh of it. The question reads more naturally the other way round — see the note
 $1/7$

(v) The time between 12 midnight and 12 noon is known as _____.

the hours after noon are p.m., post meridiem
a.m. (ante meridiem, 'before midday')

Answer (i) 3,600 · (ii) 90 · (iii) distance (with time) · (iv) $1/7$ · (v) a.m. (ante meridiem, 'before midday')

Please check — part (iv) asks for a day as a fraction of a week, which gives $1/7$. Asked the other way round — '1 week = ____ days' — it would give 7. Part (v) is printed without the word 'time'.

3. A car travels 150 km in 3 hours. What is its average speed in km/h?

Average speed = total distance $\div$ total time.

Step 1 The units already agree
km and hours, and km/h is wanted

Step 2 Divide
 $150 \div 3 = 50$ km/h

Step 3 Check
 $50 \times 3 = 150$ km $\checkmark$

Answer 50 km/h

4. If a runner completes a 10 km race in 40 minutes, what is their speed in km/h?

The answer is wanted in km per hour, but the time is given in minutes — so the time must be converted first.

Step 1 Put the time into hours

$$40 \div 60 = 2/3 \text{ hour}$$

Step 2 Divide

$$\text{Speed} = 10 \div 2/3$$

dividing by a fraction means multiplying by its reciprocal

$$= 10 \times 3/2$$

$$= 15 \text{ km/h}$$

Step 3 Check

$$15 \times 2/3 = 10 \text{ km } \checkmark$$

Dividing 10 by 40 without converting would give 0.25, which is km per minute — the commonest error here.

Answer 15 km/h

5. How many seconds are in 2 hours and 15 minutes?

Step 1 Put everything into minutes

$$2 \text{ hours} = 2 \times 60 = 120 \text{ minutes}$$

$$120 + 15 = 135 \text{ minutes}$$

Step 2 Turn minutes into seconds

$$1 \text{ minute} = 60 \text{ seconds}$$

$$135 \times 60$$

$$= 135 \times 6 \times 10$$

$$= 810 \times 10$$

$$= 8,100 \text{ seconds}$$

Step 3 Check

$$8,100 \div 60 = 135 \text{ minutes, and } 135 \div 60 = 2 \text{ h } 15 \text{ min } \checkmark$$

Answer 8,100 seconds

6. Complete the table of 12-hour and 24-hour times: 5:30 a.m.; 23:10; Midnight; 4:20 p.m.; 00:40.

The same two rules as Exercise 10.2:

12-hour → 24-hour: add 12 to a p.m. hour, keep an a.m. hour as it is

24-hour → 12-hour: take 12 off an hour above 12 and mark it p.m.

Midnight is the one to watch: it is 00:00 in the 24-hour clock and 12:00 a.m. in the 12-hour clock.

1. given: 5:30 a.m.

12-hour: 5:30 a.m.

24-hour: 05:30

2. given: 23:10 hours

12-hour: 11:10 p.m.

24-hour: 23:10

3. given: Midnight

12-hour: 12:00 a.m.

24-hour: 00:00

4. given: 4:20 p.m.

12-hour: 4:20 p.m.

24-hour: 16:20

5. given: 00:40 hours

12-hour: 12:40 a.m.

24-hour: 00:40

Answer 1 5:30 a.m. = 05:30 · 2 11:10 p.m. = 23:10 · 3 12:00 a.m. = 00:00 · 4 4:20 p.m. = 16:20 · 5 12:40 a.m. = 00:40





CHAPTER

Perimeter and Area

EXERCISE 11.1

1. Find the area and perimeter of the composite shape shown.

A composite shape is made of simpler ones joined together, so the method is always the same three moves.

Step 1 Cut the shape up

draw lines to split it into rectangles, triangles and parts of circles — whichever fit

Step 2 Find each piece separately

rectangle: area = length $\times$ breadth

triangle: area = $\frac{1}{2} \times$ base $\times$ height

half circle: area = $\frac{1}{2} \times \pi r^2$

Step 3 Add the areas

For the perimeter, a different rule

go round the outside only, adding the edges you meet

the cutting lines drawn in Step 1 are inside the shape, so they are not part of the perimeter — adding them is the commonest mistake in this exercise

where part of a circle is on the outside, its curved edge counts: a half circle contributes $\frac{1}{2} \times 2\pi r = \pi r$

The shape and its measurements are printed in the book, so the figures must be read off the page.

Answer Worked from the printed figure by the method above.

2. Find the area of the shaded region of the given shape.

A shaded region is nearly always the difference between two areas.

Step 1 Find the area of the whole outer shape

Step 2 Find the area of the hole or inner shape

Step 3 Subtract

shaded area = outer area – inner area

Step 4 Check it is sensible

the answer must be smaller than the outer area and larger than zero

If the inner shape is a circle, remember to halve the diameter before squaring — using d where r belongs makes the area four times too big.

The figure is printed in the book.

Answer Outer area – inner area, from the printed figure.

EXERCISE 11.2

1. Find the area and circumference of the following circles: (i) $r = 9$ cm (ii) $r = 13$ m (iii) $r = 6$ cm (iv) $d = 25$ cm (v) $d = 6$ m (vi) $d = 19$ ft

Two formulas serve the whole exercise:

Area $A = \pi r^2$ — in square units

Circumference $C = 2\pi r$ — in ordinary length units

Where the diameter is given, halve it first. Using d where r belongs makes the area four times too large, because the radius is squared.

π is taken as 3.14 throughout, and answers are given to 2 decimal places.

(i) $r = 9$ cm

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.14 \times 9^2 \\ &= 3.14 \times 81 \\ &= 254.34 \text{ cm}^2 \\ \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 9 \\ &= 56.52 \text{ cm}\end{aligned}$$

(ii) $r = 13 \text{ m}$

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.14 \times 13^2 \\ &= 3.14 \times 169 \\ &= 530.66 \text{ m}^2 \\ \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 13 \\ &= 81.64 \text{ m}\end{aligned}$$

(iii) $r = 6 \text{ cm}$

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.14 \times 6^2 \\ &= 3.14 \times 36 \\ &= 113.04 \text{ cm}^2 \\ \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 6 \\ &= 37.68 \text{ cm}\end{aligned}$$

(iv) $d = 25 \text{ cm}$

the diameter is given, so halve it first: $r = d \div 2$

$$r = 25 \div 2 = 12.5 \text{ cm}$$

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.14 \times 12.5^2 \\ &= 3.14 \times 156.25 \\ &= 490.63 \text{ cm}^2 \\ \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 12.5 \\ &= 78.5 \text{ cm}\end{aligned}$$

(v) $d = 6 \text{ m}$

the diameter is given, so halve it first: $r = d \div 2$

$$r = 6 \div 2 = 3 \text{ m}$$

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.14 \times 3^2 \\ &= 3.14 \times 9 \\ &= 28.26 \text{ m}^2 \\ \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 3 \\ &= 18.84 \text{ m}\end{aligned}$$

(vi) $d = 19 \text{ ft}$

the diameter is given, so halve it first: $r = d \div 2$

$$r = 19 \div 2 = 9.5 \text{ ft}$$

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= 3.14 \times 9.5^2 \\ &= 3.14 \times 90.25 \\ &= 283.39 \text{ ft}^2 \\ \text{Circumference} &= 2\pi r \\ &= 2 \times 3.14 \times 9.5 \\ &= 59.66 \text{ ft}\end{aligned}$$

Answer (i) $A = 254.34 \text{ cm}^2$, $C = 56.52 \text{ cm}$

(ii) $A = 530.66 \text{ m}^2$, $C = 81.64 \text{ m}$

(iii) $A = 113.04 \text{ cm}^2$, $C = 37.68 \text{ cm}$

(iv) $A = 490.63 \text{ cm}^2$, $C = 78.5 \text{ cm}$

(v) $A = 28.26 \text{ m}^2$, $C = 18.84 \text{ m}$

(vi) $A = 283.39 \text{ ft}^2$, $C = 59.66 \text{ ft}$

2. Find the radius of the circle whose circumference is: (i) 32 cm (ii) 144 mm (iii) 88 m (iv) 242 cm (v) 53 m (vi) 48 cm

This is the circumference formula used backwards.

$C = 2\pi r$, so dividing both sides by 2π gives $r = C \div 2\pi$.

With $\pi = 3.14$ that means dividing by 6.28 every time.

(i) $C = 32 \text{ cm}$

$C = 2\pi r$, so $r = C \div 2\pi$

$r = 32 \div (2 \times 3.14)$

$= 32 \div 6.28$

$= 5.1 \text{ cm}$

check: $2 \times 3.14 \times 5.1 = 32.03 \text{ cm} \checkmark$

(ii) $C = 144 \text{ mm}$

$C = 2\pi r$, so $r = C \div 2\pi$

$r = 144 \div (2 \times 3.14)$

$= 144 \div 6.28$

$= 22.93 \text{ mm}$

check: $2 \times 3.14 \times 22.93 = 144 \text{ mm} \checkmark$

(iii) $C = 88 \text{ m}$

$C = 2\pi r$, so $r = C \div 2\pi$

$r = 88 \div (2 \times 3.14)$

$= 88 \div 6.28$

$= 14.01 \text{ m}$

check: $2 \times 3.14 \times 14.01 = 87.98 \text{ m} \checkmark$

with $\pi = 22/7$ this comes out exactly: $r = 88 \times 7 \div 44 = 14 \text{ m}$ — see the note

(iv) $C = 242 \text{ cm}$

$C = 2\pi r$, so $r = C \div 2\pi$

$r = 242 \div (2 \times 3.14)$

$= 242 \div 6.28$

$= 38.54 \text{ cm}$

check: $2 \times 3.14 \times 38.54 = 242.03 \text{ cm} \checkmark$

with $\pi = 22/7$ this comes out exactly: $r = 242 \times 7 \div 44 = 38.5 \text{ cm}$ — see the note

(v) $C = 53 \text{ m}$

$C = 2\pi r$, so $r = C \div 2\pi$

$r = 53 \div (2 \times 3.14)$

$= 53 \div 6.28$

$= 8.44 \text{ m}$

check: $2 \times 3.14 \times 8.44 = 53 \text{ m} \checkmark$

(vi) $C = 48 \text{ cm}$

$C = 2\pi r$, so $r = C \div 2\pi$

$r = 48 \div (2 \times 3.14)$

$= 48 \div 6.28$

$= 7.64 \text{ cm}$

check: $2 \times 3.14 \times 7.64 = 47.98 \text{ cm} \checkmark$

Answer (i) 5.1 cm · (ii) 22.93 mm · (iii) 14.01 m · (iv) 38.54 cm · (v) 8.44 m · (vi) 7.64 cm

Please check — parts (iii) and (iv) come out exactly with $\pi = 22/7$ — 14 m and 38.5 cm — which strongly suggests that is the value intended. The question does not say which to use.

3. Find the area of each circle, taking $\pi = 3.14$: (i) $C = 15$ ft (ii) $C = 20$ yd (iii) $C = 35$ m

The area formula needs the radius, but the circumference is what is given — so this takes two steps rather than one.

Step 1 — find r from $C = 2\pi r$, that is $r = C \div 2\pi$.

Step 2 — put that r into $A = \pi r^2$.

Going straight from C to A without finding r first is where this question is usually lost.

- (i) $C = 15$ ft

the area needs the radius, so find that first

$$\text{Step 1 } r = C \div 2\pi$$

$$r = 15 \div 6.28 = 2.39 \text{ ft}$$

$$\text{Step 2 } A = \pi r^2$$

$$A = 3.14 \times 2.39^2$$

$$= 3.14 \times 5.71$$

$$= 17.91 \text{ ft}^2$$

$$\text{check: } 2 \times 3.14 \times 2.39 = 15 \text{ ft, the circumference given } \checkmark$$

- (ii) $C = 20$ yd

the area needs the radius, so find that first

$$\text{Step 1 } r = C \div 2\pi$$

$$r = 20 \div 6.28 = 3.18 \text{ yd}$$

$$\text{Step 2 } A = \pi r^2$$

$$A = 3.14 \times 3.18^2$$

$$= 3.14 \times 10.14$$

$$= 31.85 \text{ yd}^2$$

$$\text{check: } 2 \times 3.14 \times 3.18 = 20 \text{ yd, the circumference given } \checkmark$$

- (iii) $C = 35$ m

the area needs the radius, so find that first

$$\text{Step 1 } r = C \div 2\pi$$

$$r = 35 \div 6.28 = 5.57 \text{ m}$$

$$\text{Step 2 } A = \pi r^2$$

$$A = 3.14 \times 5.57^2$$

$$= 3.14 \times 31.06$$

$$= 97.53 \text{ m}^2$$

$$\text{check: } 2 \times 3.14 \times 5.57 = 35 \text{ m, the circumference given } \checkmark$$

Answer (i) 17.91 ft^2 · (ii) 31.85 yd^2 · (iii) 97.53 m^2

4. A park is designing a circular bicycle track with a radius of 50 m. Find the circumference of the track.

The distance round a circle is its circumference.

Step 1 Write the formula

$$C = 2\pi r$$

Step 2 Put in the figures

$$C = 2 \times 3.14 \times 50$$

Step 3 Work it out

$$2 \times 50 = 100$$

$$100 \times 3.14 = 314 \text{ m}$$

Step 4 Check

$$314 \div (2 \times 3.14) = 50 \text{ m, the radius given } \checkmark$$

A cyclist going once round the track covers 314 m, a little over three times the diameter — which is what π means.

Answer 314 m

5. A round dining table has a radius of 4 ft. What are the area and circumference of the tabletop?

Area — the space on the tabletop

$$A = \pi r^2$$

$$= 3.14 \times 4^2$$

$$= 3.14 \times 16$$

$$= 50.24 \text{ ft}^2$$

Circumference — the distance round the edge

$$C = 2\pi r$$

$$= 2 \times 3.14 \times 4$$

$$= 25.12 \text{ ft}$$

Check

$$25.12 \div 6.28 = 4 \text{ ft, the radius } \checkmark$$

Note the units: the area is in square feet because two lengths were multiplied; the circumference is in plain feet because it is a distance.

Answer Area 50.24 ft^2 , circumference 25.12 ft

6. Find the area of the figure below.

The figure is printed in the book, but the method for any shape of this kind is the same:

Step 1 — split it into circles, part-circles, rectangles and triangles.

Step 2 — work out each piece with its own formula.

Step 3 — add the pieces that are included and subtract any that are cut out.

Step 4 — give the answer in square units.

For a part of a circle, take the matching fraction of πr^2 : half a circle is $\frac{1}{2}\pi r^2$ and a quarter is $\frac{1}{4}\pi r^2$.

Answer Worked from the printed figure by the method above.

REVISION EXERCISE 11

1. Choose the correct option.

- i. What is the formula for the circumference of a circle?

- a) πr^2 b) $2\pi r^2$ c) $r^2 + h^2$ d) πd^2

The circumference is the distance round a circle: $C = 2\pi r$, or $C = \pi d$ using the diameter.

Test each option against that:

(a) πr^2 — this is the area **X**

(b) $2\pi r^2$ — twice the area, which is nothing in particular **X**

(c) $r^2 + h^2$ — part of Pythagoras, not a circle formula **X**

(d) πd^2 — not the circumference either; πd would be **X**

None of the four options is correct — see the note. The same question is asked again as part (v), where $C = 2\pi r$ is offered.

Answer No correct option is offered

Please check — none of the four options is the circumference of a circle. The answer should be $C = 2\pi r$ or $C = \pi d$, neither of which is offered.

ii. The area of a circle with radius 7 cm (use $\pi = 3.14$) is:

- a) 154 cm^2 b) 44 cm^2 c) 308 cm^2 d) 77 cm^2

$$A = \pi r^2$$

Using the π the question gives

$$A = 3.14 \times 7^2$$

$$= 3.14 \times 49$$

$$= 153.86 \text{ cm}^2 \text{ — which is not one of the options}$$

Using $\pi = 22/7$ instead

$$A = 22/7 \times 49$$

the 7 cancels into the 49, leaving 22×7

$$= 154 \text{ cm}^2, \text{ which is option (a)}$$

So the intended answer is (a), reached with $22/7$ rather than the 3.14 the question asks for — see the note.

Option (b) 44 cm^2 is the circumference $2 \times 22/7 \times 7$, not the area.

Answer a) 154 cm^2

Please check — the question says to use $\pi = 3.14$, which gives 153.86 cm^2 — not among the options. Option (a) 154 cm^2 comes from $\pi = 22/7$.

iii. Which one of the following is correct?

- a) A major arc is always greater than a semi-circumference. b) A minor arc is equal to a semi-circumference.
c) A chord of the circle does not pass through the centre. d) The secant is called a chord.

Take the four one at a time.

(a) a circle is split by two points into a major arc and a minor arc. The major is the longer of the two, so it must be more than half the circumference ✓

(b) the minor arc is the shorter piece, so it is less than half the circumference, not equal to it ✗

(c) a chord joins any two points on the circle. The diameter is a chord and it does pass through the centre, so the statement is not always true ✗

(d) a secant is a line that cuts the circle and carries on past it; a chord stops at the circle. They are not the same ✗

Answer a) A major arc is always greater than a semi-circumference.

iv. This is a picture of:

- a) Minor arc b) Major arc c) Semi-circle d) Chord

The picture is printed in the book, so the figure must be looked at. The four options are told apart like this:

a chord is a straight line joining two points on the circle

a minor arc is the shorter curved piece between two points

a major arc is the longer curved piece between the same two points

a semi-circle is exactly half, cut off by a diameter

So: is the marked part straight or curved? If curved, is it less than half, exactly half, or more than half?

Answer Read from the printed figure, by the method above

v. What is the formula for the circumference of a circle?

- a) $C = 2\pi r$ b) $C = \pi r^2$ c) $C = 2r$ d) $C = r^2\pi$

The circumference is the distance round the circle.

(a) $C = 2\pi r$ ✓

(b) $C = \pi r^2$ — the area ✗

(c) $C = 2r$ — the diameter ✗

(d) $C = r^2\pi$ — the area again, written the other way round ✗

A quick way to tell them apart: a circumference is a length, so its formula has r to the power 1. An area has r squared.

This repeats part (i), which had no correct option.

Answer a) $C = 2\pi r$

2. Fill in the blanks: (i) To find the area of a shaded region in a composite shape, subtract the area of the ____ shape from the area of the ____ shape. (ii) If the radius of a circle is doubled, its area becomes ____ times larger. (iii) The radius of a circle is the distance from the ____ to any point on the circle. (iv) If the radius of a circle is r cm, its circumference is _____. (v) The area of a circle with radius r is _____.

(i) To find the area of a shaded region in a composite shape, subtract the area of the ____ shape from the area of the ____ shape.

what is left after the hole is taken away is the shaded part

inner ... outer

(ii) If the radius of a circle is doubled, its area becomes ____ times larger.

$A = \pi r^2$, so doubling r gives $\pi(2r)^2 = \pi \times 4r^2$ — four times, not twice, because the radius is squared

4

(iii) The radius of a circle is the distance from the ____ to any point on the circle.

and the diameter is twice it

centre

(iv) If the radius of a circle is r cm, its circumference is _____.

or equally πd , since $d = 2r$

$C = 2\pi r$

(v) The area of a circle with radius r is _____.

in square units, because two lengths have been multiplied

$A = \pi r^2$

Answer (i) inner ... outer · (ii) 4 · (iii) centre · (iv) $C = 2\pi r$ · (v) $A = \pi r^2$

3. A rectangular garden is $10 \text{ m} \times 8 \text{ m}$. Inside it is a circular pond of radius 3 m . Find (a) the area of the garden (b) the area of the pond (c) the area of the garden without the pond (d) the perimeter of the garden.

(a) Area of the garden

a rectangle: $A = \text{length} \times \text{breadth}$

$= 10 \times 8$

$= 80 \text{ m}^2$

(b) Area of the pond

a circle: $A = \pi r^2$

$= 3.14 \times 3^2$

$= 3.14 \times 9$

$= 28.26 \text{ m}^2$

(c) The garden without the pond

subtract the pond from the whole garden

$= 80 - 28.26$

$= 51.74 \text{ m}^2$

(d) Perimeter of the garden

the distance round the rectangle: $P = 2(l + b)$

$= 2 \times (10 + 8)$

$= 2 \times 18$

$= 36 \text{ m}$

the pond is inside the garden, so it does not touch the perimeter at all

Check

$51.74 + 28.26 = 80 \text{ m}^2$, the whole garden ✓

Answer (a) 80 m^2 · (b) 28.26 m^2 · (c) 51.74 m^2 · (d) 36 m

4. Find the circumference and area of a circle with (a) radius 5 cm (b) diameter 12 m.

(a) Radius 5 cm

$$C = 2\pi r = 2 \times 3.14 \times 5 = 31.4 \text{ cm}$$

$$A = \pi r^2 = 3.14 \times 25 = 78.5 \text{ cm}^2$$

(b) Diameter 12 m

halve the diameter first: $r = 12 \div 2 = 6 \text{ m}$

$$C = 2 \times 3.14 \times 6 = 37.68 \text{ m}$$

$$A = 3.14 \times 6^2 = 3.14 \times 36 = 113.04 \text{ m}^2$$

Check on (b)

the circumference can also be found straight from the diameter: $C = \pi d = 3.14 \times 12 = 37.68 \text{ m}$

✓ — the same answer

Forgetting to halve the diameter would give an area of $3.14 \times 144 = 452.16 \text{ m}^2$, four times too large.

Answer (a) $C = 31.4 \text{ cm}$, $A = 78.5 \text{ cm}^2$ · (b) $C = 37.68 \text{ m}$, $A = 113.04 \text{ m}^2$



12

CHAPTER

Volume and Surface Area

EXERCISE 12.1

1. Convert the following surface areas: (i) 250 cm^2 to m^2 (ii) 3.5 m^2 to mm^2 (iii) $45,000 \text{ mm}^2$ to cm^2

An area is a length times a length, so the conversion factor is the length factor squared.

$$1 \text{ cm} = 10 \text{ mm, so } 1 \text{ cm}^2 = 10^2 = 100 \text{ mm}^2$$

$$1 \text{ m} = 100 \text{ cm, so } 1 \text{ m}^2 = 100^2 = 10,000 \text{ cm}^2$$

$$1 \text{ m} = 1,000 \text{ mm, so } 1 \text{ m}^2 = 1,000^2 = 1,000,000 \text{ mm}^2$$

Using the length factor instead of its square is the one mistake this question is testing.

- (i) 250 cm^2 to m^2

$$1 \text{ cm} = 0.01 \text{ m, so } 1 \text{ cm}^2 = 0.01^2 = 0.0001 \text{ m}^2$$

m^2 is the larger unit, so divide

$$250 \div 10,000 = 0.025 \text{ m}^2$$

- (ii) 3.5 m^2 to mm^2

$$1 \text{ m} = 1,000 \text{ mm, so } 1 \text{ m}^2 = 1,000^2 = 1,000,000 \text{ mm}^2$$

mm^2 is the smaller unit, so multiply

$$3.5 \times 1,000,000 = 3,500,000 \text{ mm}^2$$

- (iii) $45,000 \text{ mm}^2$ to cm^2

$$1 \text{ mm} = 0.1 \text{ cm, so } 1 \text{ mm}^2 = 0.1^2 = 0.01 \text{ cm}^2$$

cm^2 is the larger unit, so divide

$$45,000 \div 100 = 450 \text{ cm}^2$$

Answer (i) 0.025 m^2 · (ii) $3,500,000 \text{ mm}^2$ · (iii) 450 cm^2

2. Convert the following volumes: (i) $1,200 \text{ cm}^3$ to m^3 (ii) 0.002 m^3 to cm^3 (iii) $150,000 \text{ mm}^3$ to cm^3

A volume is three lengths multiplied, so the factor is the length factor cubed.

$$1 \text{ cm}^3 = 10^3 = 1,000 \text{ mm}^3$$

$$1 \text{ m}^3 = 100^3 = 1,000,000 \text{ cm}^3$$

$$1 \text{ m}^3 = 1,000^3 = 1,000,000,000 \text{ mm}^3$$

Notice how much larger these are than the area factors — a cubic metre holds a million cubic centimetres, not ten thousand.

- (i) $1,200 \text{ cm}^3$ to m^3

$$1 \text{ cm} = 0.01 \text{ m, so } 1 \text{ cm}^3 = 0.01^3 = 0.000001 \text{ m}^3$$

m^3 is the larger unit, so divide

$$1,200 \div 1,000,000 = 0.0012 \text{ m}^3$$

- (ii) 0.002 m^3 to cm^3

$$1 \text{ m} = 100 \text{ cm, so } 1 \text{ m}^3 = 100^3 = 1,000,000 \text{ cm}^3$$

cm^3 is the smaller unit, so multiply

$$0.002 \times 1,000,000 = 2,000 \text{ cm}^3$$

- (iii) $150,000 \text{ mm}^3$ to cm^3

$$1 \text{ mm} = 0.1 \text{ cm, so } 1 \text{ mm}^3 = 0.1^3 = 0.001 \text{ cm}^3$$

cm^3 is the larger unit, so divide

$$150,000 \div 1,000 = 150 \text{ cm}^3$$

Answer (i) 0.0012 m^3 · (ii) $2,000 \text{ cm}^3$ · (iii) 150 cm^3

3. A triangular prism has a base of 5 cm, a height of 4 cm and a length of 12 cm. Find (a) its surface area (b) its volume.

A prism has two identical ends and rectangular sides joining them.

- (a) Surface area

$$\begin{aligned} &\text{by the formula this unit gives, } SA = 3lb + bh \\ &= 3 \times 12 \times 5 + 5 \times 4 \\ &= 180 + 20 \\ &= 200 \text{ cm}^2 \end{aligned}$$

That formula assumes the triangular end is equilateral. Worked properly:

$$\begin{aligned} &\text{the two triangular ends: } 2 \times \frac{1}{2} \times 5 \times 4 = 20 \text{ cm}^2 \\ &\text{the sloping sides are } \sqrt{((5 \div 2)^2 + 4^2)} = 4.72 \text{ cm each} \\ &\text{the three rectangles: } 12 \times (5 + 4.72 + 4.72) = 173.21 \text{ cm}^2 \\ &\text{true surface area} = 193.21 \text{ cm}^2 \text{ — see the note} \end{aligned}$$

(b) Volume

$$\begin{aligned} \text{Volume} &= \text{area of the triangular end} \times \text{length} \\ &= \frac{1}{2} \times b \times h \times l \\ &= \frac{1}{2} \times 5 \times 4 \times 12 \\ &= 10 \times 12 \\ &= 120 \text{ cm}^3 \end{aligned}$$

this formula is right for any triangular prism — only the surface-area one is at fault

Answer (a) 200 cm^2 by the book's formula (394.33 cm^2 worked properly) · (b) 120 cm^3

Please check — the formula this unit gives for a triangular prism's surface area only holds for an equilateral prism. A triangle with base 5 and height 4 is not equilateral, so the two figures differ.

4. A farmer uses a cylindrical container of radius 50 cm and height 120 cm to collect rainwater. Find (a) its surface area (b) how much it holds in litres, given $1 \text{ cm}^3 = 0.001$ litres.

(a) Surface area

a cylinder has two circular ends and one curved side

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh, \text{ which factorises to } 2\pi r(r + h) \\ &= 2 \times 3.14 \times 50 \times (50 + 120) \\ &= 2 \times 3.14 \times 50 \times 170 \\ &= 314 \times 170 \\ &= 53,380 \text{ cm}^2 \end{aligned}$$

(b) Volume, then litres

$$\begin{aligned} V &= \pi r^2 h \\ &= 3.14 \times 50^2 \times 120 \\ &= 3.14 \times 2,500 \times 120 \\ &= 7,850 \times 120 \\ &= 942,000 \text{ cm}^3 \\ \text{now into litres: } 1 \text{ cm}^3 &= 0.001 \text{ litres} \\ 942,000 \times 0.001 &= 942 \text{ litres} \end{aligned}$$

Check

$$\begin{aligned} 942 \text{ litres} \div 0.001 &= 942,000 \text{ cm}^3 \checkmark \\ \text{a tank half a metre across and 1.2 m tall holding most of a cubic metre is about right.} \end{aligned}$$

Answer (a) $53,380 \text{ cm}^2$ · (b) 942 litres

5. A rectangular swimming pool is 10 m long, 6 m wide and 2 m deep, to be tiled on all inner surfaces. Find (a) the area to be tiled (b) how many tiles are needed if each covers 0.25 m^2 .

(a) The area to be tiled

a pool is open at the top, so there are five surfaces, not six

$$\begin{aligned} \text{the floor: } 10 \times 6 &= 60 \text{ m}^2 \\ \text{the two long walls: } 2 \times (10 \times 2) &= 40 \text{ m}^2 \\ \text{the two short walls: } 2 \times (6 \times 2) &= 24 \text{ m}^2 \\ \text{total} &= 60 + 40 + 24 = 124 \text{ m}^2 \end{aligned}$$

(b) The number of tiles

$$\begin{aligned} \text{tiles} &= \text{area} \div \text{area of one tile} \\ &= 124 \div 0.25 \\ \text{dividing by 0.25 is the same as multiplying by 4} \end{aligned}$$

$$= 496 \text{ tiles}$$

If the closed-box formula is wanted instead

$$SA = 2(lb + bh + hl) = 2(60 + 12 + 20)$$

$$= 184 \text{ m}^2, \text{ needing } 736 \text{ tiles}$$

but that counts the water surface as something to be tiled — see the note

Check

$$496 \times 0.25 = 124 \text{ m}^2 \checkmark$$

Answer (a) 124 m^2 · (b) 496 tiles

Please check — a pool has no lid, so only five faces are tiled. Treating it as a closed box gives 184 m^2 and 736 tiles; the question should say which is wanted.

6. A triangular prism has a base of 6 cm, a height of 8 cm and a length of 15 cm. Find (a) its surface area (b) its volume.

The same two formulas as Q3, with different figures.

(a) Surface area

by the formula this unit gives, $SA = 3lb + bh$

$$= 3 \times 15 \times 6 + 6 \times 8$$

$$= 270 + 48$$

$$= 318 \text{ cm}^2$$

That formula assumes the triangular end is equilateral. Worked properly:

$$\text{the two triangular ends: } 2 \times \frac{1}{2} \times 6 \times 8 = 48 \text{ cm}^2$$

$$\text{the sloping sides are } \sqrt{((6 \div 2)^2 + 8^2)} = 8.54 \text{ cm each}$$

$$\text{the three rectangles: } 15 \times (6 + 8.54 + 8.54) = 346.32 \text{ cm}^2$$

$$\text{true surface area} = 394.32 \text{ cm}^2 \text{ — see the note}$$

(b) Volume

Volume = area of the triangular end $\times$ length

$$= \frac{1}{2} \times b \times h \times l$$

$$= \frac{1}{2} \times 6 \times 8 \times 15$$

$$= 24 \times 15$$

$$= 360 \text{ cm}^3$$

this formula is right for any triangular prism — only the surface-area one is at fault

Answer (a) 318 cm^2 by the book's formula (394.33 cm^2 worked properly) · (b) 360 cm^3

Please check — base 6 and height 8 describe an isosceles triangle with sloping sides of $\sqrt{73} \approx 8.54 \text{ cm}$, not an equilateral one, so the book's $3lb + bh$ does not apply.

7. A water tank holds 2,500 litres. Convert its volume to m^3 and to cm^3 .

Two conversions from the same starting figure.

Step 1 Litres to cubic centimetres

$$1 \text{ litre} = 1,000 \text{ cm}^3$$

$$2,500 \times 1,000 = 2,500,000 \text{ cm}^3$$

Step 2 Cubic centimetres to cubic metres

$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$

$$2,500,000 \div 1,000,000 = 2.5 \text{ m}^3$$

Step 3 Check the short way

$$1,000 \text{ litres} = 1 \text{ m}^3, \text{ so } 2,500 \div 1,000 = 2.5 \text{ m}^3 \checkmark$$

A cubic metre of water is a thousand litres — worth remembering, since it links the two systems in one step.

Answer 2.5 m^3 , which is $2,500,000 \text{ cm}^3$

■ REVISION EXERCISE 12

1. Choose the correct option.

i. The surface area of a cube of side 5 cm is:

- a) 100 cm^2 b) 150 cm^2 c) 125 cm^2 d) 150 cm^3

A cube has 6 identical square faces.

Step 1 area of one face = $5 \times 5 = 25 \text{ cm}^2$

Step 2 all six faces = $6 \times 25 = 150 \text{ cm}^2$

Check the options

(a) 100 cm^2 is only four faces ✗

(c) 125 cm^2 is 5^3 , which is the volume ✗

(d) 150 cm^3 has the right number but a cubic unit — an area must be in square units ✗

Option (b) is the trap worth pausing on: the figure is right and the answer is still wrong.

Answer b) 150 cm^2

ii. What is the volume of a rectangular prism $4 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm}$?

- a) 24 cm^3 b) 48 cm^3 c) 24 cm^2 d) 48 cm^2

Volume = length $\times$ breadth $\times$ height.

Step 1 $4 \times 3 = 12$

Step 2 $12 \times 2 = 24 \text{ cm}^3$

Check the options

(b) 48 cm^3 is twice too big — it is the surface-area sum $2(lb + bh + hl)$ misapplied ✗

(c) 24 cm^2 has the right number but a square unit; three lengths multiplied give a cubic unit ✗

(d) 48 cm^2 is wrong in both ✗

Answer a) 24 cm^3

iii. To convert 1 m^2 into cm^2 , you multiply by:

- a) 10 b) 100 c) 1,000 d) 10,000

$1 \text{ m} = 100 \text{ cm}$ is the length factor.

An area is a length times a length, so the factor must be squared.

$100 \times 100 = 10,000$

So $1 \text{ m}^2 = 10,000 \text{ cm}^2$.

Option (b) 100 is the trap: that is the length factor, used without squaring it.

Check a square metre is a square 100 cm by 100 cm, which really does hold 10,000 little centimetre squares ✓

Answer d) 10,000

iv. The formula for the volume of a cylinder is:

- a) πrh b) $2\pi r^2 + 2\pi rh$ c) $2\pi rh$ d) $\pi r^2 h$

A volume is three lengths multiplied, so the formula must have $r \times r \times h$ in it somewhere.

(a) πrh — only two lengths, so this is an area, not a volume ✗

(b) $2\pi r^2 + 2\pi rh$ — the surface area of a cylinder ✗

(c) $2\pi rh$ — the curved side only, again an area ✗

(d) $\pi r^2 h$ ✓ — the area of the circular end, πr^2 , multiplied by the height

Counting the lengths is a quick way to tell an area formula from a volume one.

Answer d) $\pi r^2 h$

v. Which of the following is true about a cube?

- a) All faces are rectangles. b) All faces are squares of equal size. c) Opposite faces are identical. d) It has a circular base.

Take the four in turn.

- (a) all faces are rectangles — true, because every square is a rectangle
 (b) all faces are squares of equal size — true, and the most exact description ✓
 (c) opposite faces are identical — true of a cube
 (d) it has a circular base — false; a cube's faces are all flat and square ✗

Three of the four statements are true, so the question has three correct answers as printed — see the note. (b) is the intended one because it is the only description that fits a cube and nothing else.

Answer b) All faces are squares of equal size.

Please check — options (a) and (c) are true of a cube as well, so three of the four are correct. Only (d) is false.

2. Fill in the blanks: (i) The total space occupied by a 3D object is called its _____. (ii) A cube has _____ faces, all squares of the same size. (iii) The unit of volume in the metric system is _____. (iv) A triangular prism has _____ bases. (v) The total area of all the outer faces of a 3D object is called its _____.

(i) The total space occupied by a 3D object is called its _____.
 measured in cubic units — cm^3 , m^3 and so on
 volume

(ii) A cube has _____ faces, all squares of the same size.
 4 round the sides, 1 top and 1 bottom
 6

(iii) The unit of volume in the metric system is _____.
 with the litre used for liquids: 1,000 litres = 1 m^3
 the cubic metre (m^3)

(iv) A triangular prism has _____ bases.
 the two triangular ends; the three rectangles between them are faces, not bases
 2

(v) The total area of all the outer faces of a 3D object is called its _____.
 measured in square units, unlike the volume
 surface area

Answer (i) volume · (ii) 6 · (iii) the cubic metre (m^3) · (iv) 2 · (v) surface area



13

CHAPTER

Practical Geometry and Angle Properties of Polygons

EXERCISE 13.1

1. Use compass and ruler to construct the following angles, and write the construction steps.

Every angle in this question is built from one of three starting points — 60° , 90° , or a bisection — so these are the steps worth knowing by heart.

To construct 60°

1. draw a line and mark a point A on it
2. open the compass to any width, put the point at A and draw an arc crossing the line at B
3. keeping the same width, put the point at B and draw an arc crossing the first at C
4. join A to C — angle CAB is 60°

why it works: A, B and C are all the same distance apart, so ABC is equilateral, and every angle of an equilateral triangle is 60°

To construct 90°

1. mark a point A on a line
2. from A, draw arcs cutting the line at equal distances on both sides, at P and Q
3. widen the compass, and from P and then Q draw arcs that cross above the line at R
4. join A to R — angle RAP is 90°

To bisect any angle

1. with the point at the vertex, draw an arc crossing both arms, at P and Q
2. from P and from Q, draw arcs of equal radius that cross at R
3. join the vertex to R — the angle is now halved

Putting them together

- $30^\circ = \text{bisect } 60^\circ$
- $45^\circ = \text{bisect } 90^\circ$
- $15^\circ = \text{bisect } 30^\circ$
- $120^\circ = \text{two } 60^\circ \text{ angles side by side}$
- $75^\circ = 60^\circ + 15^\circ$
- $105^\circ = 90^\circ + 15^\circ$
- $135^\circ = 90^\circ + 45^\circ$

Always finish by measuring

check the finished angle with a protractor. A construction is only right if it measures right, and the commonest fault is letting the compass width slip between steps.

Answer Constructions — the steps above, checked with a protractor.

2. Which of the following pairs of angles are complementary? (i) $30^\circ + 60^\circ$ (ii) $45^\circ + 45^\circ$ (iii) $45^\circ + 135^\circ$ (iv) $30^\circ + 150^\circ$ (v) $75^\circ + 105^\circ$ (vi) $259^\circ + 180^\circ$

Two angles are complementary when they add to 90° , and supplementary when they add to 180° . So there is nothing to do but add each pair and compare the total with 90.

A way to keep the two words apart: C comes before S in the alphabet, and 90 comes before 180.

- (i) $30^\circ + 60^\circ$
 $30 + 60 = 90^\circ$
 they add to exactly 90°
 Complementary
- (ii) $45^\circ + 45^\circ$
 $45 + 45 = 90^\circ$
 they add to exactly 90°
 Complementary

- (iii) $45^\circ + 135^\circ$
 $45 + 135 = 180^\circ$
 they add to 180° , which makes them supplementary
 Supplementary, not complementary
- (iv) $30^\circ + 150^\circ$
 $30 + 150 = 180^\circ$
 they add to 180° , which makes them supplementary
 Supplementary, not complementary
- (v) $75^\circ + 105^\circ$
 $75 + 105 = 180^\circ$
 they add to 180° , which makes them supplementary
 Supplementary, not complementary
- (vi) $259^\circ + 180^\circ$
 $259 + 180 = 439^\circ$
 they add to neither 90° nor 180° — see the note
 Neither

Answer (i) complementary · (ii) complementary · (iii) supplementary · (iv) supplementary · (v) supplementary · (vi) neither

Please check — only (i) and (ii) are complementary. Parts (iii), (iv) and (v) are supplementary, and (vi) adds to 439° , which is neither — and 259° is a reflex angle besides.

3. Find the angle x in the following figures.

The figures are printed in the book, but every one of them yields to one of these facts. Find which applies:

- angles on a straight line add to 180°
- angles round a point add to 360°
- vertically opposite angles, made by two crossing lines, are equal
- the angles of a triangle add to 180°
- the angles of a quadrilateral add to 360°
- a right angle marked with a small square is 90° , even when no number is written

Method

1. decide which fact the figure is using
2. write the equation it gives, with x in it
3. solve for x
4. put x back into the figure and check the angles really do total what they should

Answer Worked from the printed figures by the method above.

■ EXERCISE 13.2

1. Classify the given triangles by their sides and by their angles.

Every triangle gets two names, one for its sides and one for its angles.

By sides

equilateral — all three sides equal (and then all three angles are 60°)

isosceles — exactly two sides equal, and the angles opposite them are equal too

scalene — all three sides different

By angles

acute-angled — all three angles under 90°

right-angled — one angle exactly 90°

obtuse-angled — one angle over 90°

A triangle can have at most one angle of 90° or more, because the three together only come to 180° .

So an equilateral triangle is always acute-angled, but an isosceles one may be acute, right or obtuse.

Answer Two names each, from the printed figures.

2. Find the unknown angle in the following triangles.

One fact settles all of them: the three angles of a triangle add to 180° .

Method

1. add the angles you are given
2. take that total from 180°
3. the answer is the missing angle

Two things to watch

if the triangle is marked isosceles, two of its angles are equal even though only one number may be printed

if it is equilateral, every angle is 60° and nothing needs calculating

Check

add all three angles back up — they must come to exactly 180°

An answer of 0° or less means the angles given were impossible for a triangle.

Answer 180° minus the sum of the angles given, for each figure.

3. Construct triangles from the given sides and angles, using ruler and compass.

Which construction to use depends on what is given.

Three sides (SSS)

1. draw the longest side as the base
 2. open the compass to the second side and draw an arc from one end
 3. open it to the third side and draw an arc from the other end
 4. where the arcs cross is the third corner — join it to both ends
- this only works if the two shorter sides together exceed the longest; otherwise the arcs never meet

Two sides and the angle between them (SAS)

1. draw one side
2. construct the given angle at one end of it
3. measure the second side along that new arm
4. join the two far ends

Two angles and the side between them (ASA)

1. draw the side
2. construct one angle at each end
3. extend both arms until they cross — that crossing is the third corner

Always finish by measuring

check every side and angle against what was asked for before calling it done.

Answer Constructions by SSS, SAS or ASA as the given data allows.

4. Construct a right-angled triangle DEF with $DE = 5$ cm, $DF = 12$ cm and $\angle D = 90^\circ$.

Two sides with the angle between them — so this is SAS, and the angle happens to be the easiest one to construct.

Step 1 draw $DE = 5$ cm with a ruler

Step 2 construct a right angle at D, by the arc method: equal arcs each side of D on the line, then two crossing arcs above

Step 3 along that perpendicular, measure $DF = 12$ cm from D

Step 4 join E to F

Step 5 Check

measure DE — it should be 5 cm

measure DF — 12 cm

measure the angle at D — 90°

the third side EF should measure 13 cm, because $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

That last check is the useful one: 5, 12, 13 is a Pythagorean triple, so a correct construction gives a whole number of centimetres to measure against.

Answer A right-angled triangle with EF measuring 13 cm.

5. Construct a scalene triangle PQR with $PQ = 4$ cm, $QR = 6$ cm and $\angle P = 40^\circ$.

Here the given angle is not between the two given sides, so the SAS method does not apply directly.

Step 1 draw $PQ = 4$ cm

Step 2 construct an angle of 40° at P, using a protractor, and draw a long arm from P

Step 3 open the compass to 6 cm, put the point at Q, and draw an arc cutting that arm

Step 4 where the arc cuts the arm is R — join Q to R

Step 5 Check

$PQ = 4$ cm, $QR = 6$ cm, $\angle P = 40^\circ$

all three sides should come out different, which is what makes it scalene

Because QR (6 cm) is longer than PQ (4 cm), the arc cuts the arm just once and there is only one possible triangle. Had the arc been shorter than PQ there could have been two answers, or none at all.

Answer One triangle only, since QR is longer than PQ.

6. Construct an obtuse-angled triangle LMN with $\angle L = 120^\circ$, $LM = 5$ cm and $LN = 8$ cm.

The angle is between the two given sides, so this is SAS.

Step 1 draw $LM = 5$ cm

Step 2 construct 120° at L — two 60° angles side by side, or a protractor

Step 3 measure $LN = 8$ cm along the new arm

Step 4 join M to N

Step 5 Check

$\angle L$ should measure 120° , which is over 90° — so the triangle is obtuse-angled as asked

the other two angles must together come to $180 - 120 = 60^\circ$, so both are acute

A triangle can only have one obtuse angle, and this calculation shows why.

Answer An obtuse-angled triangle; the other two angles total 60° .

7. Construct an acute-angled triangle ABC with $\angle A = 60^\circ$, $\angle B = 70^\circ$ and $AB = 6$ cm.

Two angles with the side between them — ASA.

Step 1 draw $AB = 6$ cm

Step 2 construct 60° at A (the compass method gives this exactly)

Step 3 construct 70° at B with a protractor

Step 4 extend both arms until they meet — that point is C

Step 5 Work out the third angle first as a check

$$\angle C = 180 - 60 - 70$$

$$= 50^\circ$$

measure the angle at C — it should be 50°

60° , 70° and 50° are all under 90° , so the triangle is acute-angled as asked ✓

Answer $\angle C = 50^\circ$; all three angles are acute.

8. Construct an equilateral triangle with each side 6 cm.

The simplest construction of all, because one compass width does the whole job.

Step 1 draw a base $AB = 6$ cm

Step 2 open the compass to exactly 6 cm — the length of the base

Step 3 with the point at A, draw an arc above the line

Step 4 without changing the width, put the point at B and draw a second arc crossing the first

Step 5 call that crossing C and join C to A and to B

Step 6 Check

all three sides should measure 6 cm

all three angles should measure 60° , since $180 \div 3 = 60$

The one thing that spoils this construction is changing the compass width between steps 3 and 4.

Answer All sides 6 cm, all angles 60° .

EXERCISE 13.3

1. Identify the following quadrilaterals.

Each quadrilateral is told apart by its sides, its angles and its diagonals.

square — 4 equal sides, 4 right angles; diagonals equal, perpendicular, and bisect each other

rectangle — opposite sides equal, 4 right angles; diagonals equal and bisect each other

rhombus — 4 equal sides but angles not right; diagonals perpendicular and bisect each other

parallelogram — opposite sides parallel and equal; diagonals bisect each other but are neither equal nor perpendicular

trapezium — exactly one pair of parallel sides

kite — two pairs of adjacent sides equal; diagonals perpendicular, but only one bisects the other

A useful order to work in: count the equal sides first, then look at the angles, then the diagonals.

Note that a square is a rectangle and a rhombus and a parallelogram — the names overlap, so give the most exact one.

Answer From the printed figures, using the properties above.

2. Find the sum of the interior angles of the polygons.

One formula covers every polygon:

$$\text{sum of interior angles} = (n - 2) \times 180^\circ$$

Where it comes from

draw all the diagonals from one corner; they cut the polygon into triangles

a polygon with n sides gives $(n - 2)$ triangles

each triangle contributes 180°

The common ones

$$\text{triangle, } n = 3: (3 - 2) \times 180 = 180^\circ$$

$$\text{quadrilateral, } n = 4: 2 \times 180 = 360^\circ$$

pentagon, $n = 5$: $3 \times 180 = 540^\circ$

hexagon, $n = 6$: $4 \times 180 = 720^\circ$

heptagon, $n = 7$: $5 \times 180 = 900^\circ$

octagon, $n = 8$: $6 \times 180 = 1,080^\circ$

This holds whether the polygon is regular or not — only the number of sides matters.

Answer $(n - 2) \times 180^\circ$ for each polygon shown.

3. Find the value of x .

Step 1 — count the sides of the polygon in the figure.

Step 2 — work out what all its angles should total: $(n - 2) \times 180^\circ$.

Step 3 — add up the angles that are given.

Step 4 — subtract that from the total; what is left is x .

Step 5 — put x back and check every angle adds to the right total.

If the figure marks an exterior angle rather than an interior one, use the 360° rule instead — the exterior angles of any polygon always total 360° , however many sides it has.

Answer Worked from each printed figure by the method above.

4. Identify concave and convex polygons.

The test is a single question: does any angle bend inward?

convex — every interior angle is less than 180° , and the shape bulges outward all round

concave — at least one interior angle is more than 180° , so the outline has a dent in it

Two quick ways to tell by eye

1. look for a corner that points into the shape rather than away from it

2. join any two corners with a straight line — if any such line passes outside the shape, it is concave

All regular polygons are convex, so a regular hexagon or a square can never be concave.

Answer Concave if any interior angle exceeds 180° ; otherwise convex.

5. Find the missing angles using the properties of quadrilaterals.

Which property to use depends on the quadrilateral.

any quadrilateral — the four angles add to 360°

parallelogram — opposite angles are equal, and neighbouring angles add to 180°

rectangle and square — every angle is 90°

rhombus — opposite angles equal, like a parallelogram

trapezium — the two angles on each non-parallel side add to 180°

kite — the two angles between the unequal sides are equal to each other

Method

1. name the quadrilateral

2. use its own property to fill in what you can

3. finish with the 360° rule

4. check all four angles total 360°

Answer From each figure, using the property of that quadrilateral.

6. Find the interior and exterior angles of a regular polygon with (i) $n = 5$ (ii) $n = 7$ (iii) $n = 9$ (iv) $n = 13$.

Two facts do all the work here.

the exterior angles of any polygon add to 360° , whatever its number of sides

an interior angle and its exterior angle sit on a straight line, so they add to 180°

In a regular polygon all the angles are equal, so each exterior angle is simply $360 \div n$.

Finding the exterior angle first is much quicker than working out $(n - 2) \times 180$ and then dividing — but each part below is checked both ways.

(i) $n = 5$

Exterior angle — the exterior angles of any polygon total 360° , and in a regular one they are all equal

exterior = $360 \div 5 = 72^\circ$

Interior angle — an interior angle and its exterior angle lie on a straight line

$$\text{interior} = 180 - 72^\circ = 108^\circ$$

Check by the other route

$$\text{sum of interior angles} = (n - 2) \times 180 = 3 \times 180 = 540^\circ$$

$$540 \div 5 = 108^\circ \checkmark \text{ — the same answer}$$

(ii) $n = 7$

Exterior angle — the exterior angles of any polygon total 360° , and in a regular one they are all equal

$$\text{exterior} = 360 \div 7 = 51.43^\circ$$

Interior angle — an interior angle and its exterior angle lie on a straight line

$$\text{interior} = 180 - 51.43^\circ = 128.57^\circ$$

Check by the other route

$$\text{sum of interior angles} = (n - 2) \times 180 = 5 \times 180 = 900^\circ$$

$$900 \div 7 = 128.57^\circ \checkmark \text{ — the same answer}$$

(iii) $n = 9$

Exterior angle — the exterior angles of any polygon total 360° , and in a regular one they are all equal

$$\text{exterior} = 360 \div 9 = 40^\circ$$

Interior angle — an interior angle and its exterior angle lie on a straight line

$$\text{interior} = 180 - 40^\circ = 140^\circ$$

Check by the other route

$$\text{sum of interior angles} = (n - 2) \times 180 = 7 \times 180 = 1260^\circ$$

$$1260 \div 9 = 140^\circ \checkmark \text{ — the same answer}$$

(iv) $n = 13$

Exterior angle — the exterior angles of any polygon total 360° , and in a regular one they are all equal

$$\text{exterior} = 360 \div 13 = 27.69^\circ$$

Interior angle — an interior angle and its exterior angle lie on a straight line

$$\text{interior} = 180 - 27.69^\circ = 152.31^\circ$$

Check by the other route

$$\text{sum of interior angles} = (n - 2) \times 180 = 11 \times 180 = 1980^\circ$$

$$1980 \div 13 = 152.31^\circ \checkmark \text{ — the same answer}$$

Answer (i) interior 108° , exterior 72°

(ii) interior 128.57° , exterior 51.43°

(iii) interior 140° , exterior 40°

(iv) interior 152.31° , exterior 27.69°

REVISION EXERCISE 13

1. Choose the correct option.

i. What is the sum of all exterior angles of any polygon?

- a) 360° b) 180° c) 540° d) Depends on the number of sides

The exterior angles of any polygon always total 360° .

Why — walking right round the outline, you turn through each exterior angle in turn and end up facing the way you started — one full turn, which is 360° .

Check on a square — each exterior angle is 90° , and $4 \times 90 = 360 \checkmark$

Check on a pentagon — each is 72° , and $5 \times 72 = 360 \checkmark$

Option (d) is the trap: the interior angles do depend on the number of sides, but the exterior ones never do.

Answer a) 360°

ii. Which of the following polygons is concave?

- a) Rectangle b) Square c) A polygon with at least one angle greater than 180° d) Regular hexagon

A polygon is concave when at least one interior angle is more than 180° — that is the definition.

- (a) a rectangle has four 90° angles $\times$
 (b) a square likewise $\times$
 (c) this is the definition $\checkmark$
 (d) a regular hexagon has angles of 120° , all under 180° $\times$

Every regular polygon is convex, so (a), (b) and (d) could never be concave.

Answer c) A polygon with at least one angle greater than 180°

iii. In a kite, the diagonals:

- a) are equal b) are perpendicular c) bisect each other d) are parallel

A kite has two pairs of equal adjacent sides.

- (a) equal diagonals belong to the rectangle and square $\times$
 (b) a kite's diagonals cross at right angles $\checkmark$
 (c) only one diagonal bisects the other in a kite — they do not bisect each other $\times$
 (d) diagonals cross, so they can never be parallel — true of every quadrilateral $\times$

Answer b) are perpendicular

iv. Which quadrilateral has all sides equal but angles not equal?

- a) Square b) Rectangle c) Parallelogram d) Rhombus

Two conditions must both hold: all four sides equal, and the angles not all equal.

- (a) a square has equal sides but its angles are all 90° — it fails the second $\times$
 (b) a rectangle has unequal sides — fails the first $\times$
 (c) a parallelogram has opposite sides equal, not all four $\times$
 (d) a rhombus has four equal sides with angles that are not right angles $\checkmark$

A square is a rhombus that happens to have right angles, which is why the question has to say 'angles not equal' to tell them apart.

Answer d) Rhombus

v. In a parallelogram, opposite angles are:

- a) equal b) supplementary c) complementary d) different

In a parallelogram opposite angles are equal and neighbouring angles are supplementary.

- (a) opposite angles equal $\checkmark$
 (b) supplementary describes the neighbouring pair, not the opposite one $\times$
 (c) complementary means 90° , which does not apply $\times$
 (d) 'different' is simply wrong $\times$

Check a parallelogram with angles 70° , 110° , 70° , 110° — the opposite pairs match, the neighbours add to 180° , and all four total 360° $\checkmark$

Answer a) equal

2. Fill in the blanks: (i) A polygon with 6 sides is called a _____. (ii) The formula for the sum of the interior angles of a polygon is _____. (iii) The interior angle of a regular pentagon is _____ degrees. (iv) The exterior angle of a regular polygon with n sides is _____. (v) A quadrilateral with two pairs of parallel sides is called a _____.

- (i) A polygon with 6 sides is called a _____.
 5 is a pentagon, 6 a hexagon, 7 a heptagon, 8 an octagon
 hexagon
 (ii) The formula for the sum of the interior angles of a polygon is _____.
 because the polygon splits into $(n - 2)$ triangles, each of 180°
 $(n - 2) \times 180^\circ$
 (iii) The interior angle of a regular pentagon is _____ degrees.
 exterior = $360 \div 5 = 72^\circ$, so interior = $180 - 72 = 108^\circ$
 108
 (iv) The exterior angle of a regular polygon with n sides is _____.
 since all n exterior angles are equal and total 360°

$$360^\circ \div n$$

- (v) A quadrilateral with two pairs of parallel sides is called a _____.
with only one pair it would be a trapezium
parallelogram

Answer (i) hexagon · (ii) $(n - 2) \times 180^\circ$ · (iii) 108 · (iv) $360^\circ \div n$ · (v) parallelogram

3. Construct an isosceles triangle with two sides of 5 cm and a base of 8 cm.

Three sides are given, so this is the SSS construction.

Step 1 draw the base $AB = 8$ cm

Step 2 open the compass to 5 cm

Step 3 draw an arc from A, then another from B at the same width

Step 4 where they cross is C — join C to A and to B

Step 5 Check it can be built at all

the two shorter sides must together exceed the longest

$$5 + 5 = 10, \text{ and } 10 > 8 \checkmark \text{ — so the arcs do meet}$$

had the base been 10 cm or more the arcs would never have crossed

Step 6 Check the finished triangle

CA and CB should both measure 5 cm

the angles at A and B should be equal, since they lie opposite the two equal sides

Answer An isosceles triangle; the angles at A and B are equal.

4. Construct a scalene triangle with sides 4 cm, 5 cm and 7 cm.

SSS again, with all three sides different.

Step 1 Check it is possible first

the two shorter sides must exceed the longest

$$4 + 5 = 9, \text{ and } 9 > 7 \checkmark$$

Step 2 draw the longest side, 7 cm, as the base

Step 3 set the compass to 4 cm and draw an arc from one end

Step 4 set it to 5 cm and draw an arc from the other end

Step 5 join the crossing point to both ends

Step 6 Check

measure all three sides — 4, 5 and 7 cm, all different, so it is scalene $\checkmark$

all three angles should come out different too

Answer A scalene triangle; $4 + 5 > 7$, so it can be built.

5. Construct a right-angled triangle with legs of 3 cm and 4 cm.

The right angle is between the two legs, so this is SAS.

Step 1 draw one leg, 4 cm

Step 2 construct a right angle at one end of it with compass and ruler

Step 3 measure the other leg, 3 cm, along the perpendicular

Step 4 join the two far ends to close the triangle

Step 5 Check with Pythagoras

$$\text{hypotenuse}^2 = 3^2 + 4^2$$

$$= 9 + 16$$

$$= 25$$

$$\text{hypotenuse} = \sqrt{25} = 5 \text{ cm}$$

measure the longest side — it should be exactly 5 cm

3, 4, 5 is the best known Pythagorean triple, which is why this construction checks so neatly.

Answer The hypotenuse measures 5 cm.

6. If the interior angle of a regular polygon is 144° , find the number of sides.

Go through the exterior angle — it is far quicker than working backwards from $(n - 2) \times 180$.

Step 1 Find the exterior angle

an interior angle and its exterior angle lie on a straight line

$$\text{exterior} = 180 - 144$$

$$= 36^\circ$$

Step 2 Use the 360° rule

all the exterior angles together make 360° , and in a regular polygon they are equal

$$n = 360 \div 36$$

$$= 10 \text{ sides}$$

Step 3 Check the long way round

$$\text{sum of interior angles} = (10 - 2) \times 180 = 8 \times 180 = 1,440^\circ$$

$$1,440 \div 10 = 144^\circ \checkmark \text{ — the interior angle given}$$

A 10-sided polygon is a decagon.

Answer 10 sides — a decagon

7. A quadrilateral has angles of 90° , 120° , 110° and x° . Find x .

Step 1 Recall the total

the angles of a quadrilateral add to 360°

$$\text{from the formula: } (4 - 2) \times 180 = 2 \times 180 = 360^\circ$$

Step 2 Add the angles given

$$90 + 120 = 210$$

$$210 + 110 = 320$$

Step 3 Subtract from 360

$$x = 360 - 320$$

$$x = 40^\circ$$

Step 4 Check

$$90 + 120 + 110 + 40 = 360^\circ \checkmark$$

All four angles are under 180° , so the quadrilateral is convex.

Answer $x = 40^\circ$



14

CHAPTER

Transformation

EXERCISE 14.1

1. Draw the lines of symmetry for the following shapes.

A line of symmetry is a fold line: the shape must land exactly on itself when folded along it.

The rule for regular shapes

a regular polygon has as many lines of symmetry as it has sides

equilateral triangle $\rightarrow 3$ · square $\rightarrow 4$ · regular pentagon $\rightarrow 5$ · regular hexagon $\rightarrow 6$

Shapes that are not regular

rectangle $\rightarrow 2$ (through the middle each way, but not along the diagonals)

isosceles triangle $\rightarrow 1$

parallelogram $\rightarrow 0$, which surprises most people

circle $\rightarrow$ infinitely many, since every diameter is one

How to test a line you have drawn

imagine folding along it. If every corner meets another corner and every edge meets another edge, it is a line of symmetry. If anything sticks out, it is not.

The rectangle's diagonal is the one to be careful with: it looks like a fold line, but folding along it leaves the corners sticking out.

Answer Regular polygons: as many lines as sides. Rectangle 2, parallelogram 0, circle infinitely many.

2. Find the order of rotational symmetry for the following shapes.

The order of rotational symmetry is how many times a shape looks the same during one complete turn.

Every shape matches itself after a full 360° turn, so the order is never less than 1. An order of exactly 1 means the shape has no rotational symmetry worth the name.

The angle of rotation is $360 \div \text{order}$ — the smallest turn that brings the shape back onto itself.

Equilateral Triangle

lines of symmetry: 3

order of rotational symmetry: 3

angle of rotation: $360 \div 3 = 120^\circ$

Square

lines of symmetry: 4

order of rotational symmetry: 4

angle of rotation: $360 \div 4 = 90^\circ$

Regular Pentagon

lines of symmetry: 5

order of rotational symmetry: 5

angle of rotation: $360 \div 5 = 72^\circ$

Regular Hexagon

lines of symmetry: 6

order of rotational symmetry: 6

angle of rotation: $360 \div 6 = 60^\circ$

Rectangle

lines of symmetry: 2

order of rotational symmetry: 2

angle of rotation: $360 \div 2 = 180^\circ$

Circle

lines of symmetry: infinitely many

order of rotational symmetry: infinite

a circle matches itself at any angle, so its order is infinite

How to count it

1. mark one corner

2. turn the shape slowly through a full circle

3. count how many times it looks exactly as it started, counting the finish

For a regular polygon the order always equals the number of sides, which is the same as its number of lines of symmetry.

Answer Regular polygons: order = number of sides. Rectangle 2, circle infinite.

3. Find the point of rotation of a square and of an octagon, and the angle of rotation for each.

The point of rotation — the centre the shape turns about — is the centre of the shape in both cases.

for a square, where the two diagonals cross

for a regular octagon, the centre of the circle that would pass through all eight corners

Square

4 sides, so order 4

angle = $360 \div 4 = 90^\circ$

it matches itself at 90° , 180° , 270° and 360°

Regular octagon

8 sides, so order 8

angle = $360 \div 8 = 45^\circ$

it matches itself at 45° , 90° , 135° and so on, eight times in a full turn

Check

$4 \times 90 = 360 \checkmark$ and $8 \times 45 = 360 \checkmark$

The more sides a regular polygon has, the smaller its angle of rotation — which is why a circle, with infinitely many, matches at any angle at all.

Answer Both rotate about their centre. Square: order 4, angle 90° . Octagon: order 8, angle 45° .

4. Create a 2-D shape with at least 2 lines of symmetry and rotational symmetry. Draw it and identify its lines of symmetry and order of rotational symmetry.

Any shape meeting both conditions is a right answer. Two worked examples:

A rectangle

lines of symmetry: 2 — one across the middle horizontally, one vertically

order of rotational symmetry: 2 — it matches at 180° and at 360°

angle of rotation: $360 \div 2 = 180^\circ$

note the diagonals are not lines of symmetry

A regular hexagon

lines of symmetry: 6 — three through opposite corners, three through the middles of opposite sides

order of rotational symmetry: 6

angle of rotation: $360 \div 6 = 60^\circ$

A shape that would not do

an isosceles triangle has only 1 line of symmetry, so it fails the first condition

a parallelogram has rotational symmetry of order 2 but no lines of symmetry, so it fails as well

— a useful reminder that the two kinds of symmetry are independent

Answer A rectangle (2 lines, order 2) or a regular hexagon (6 lines, order 6) both qualify.

■ EXERCISE 14.2

1. Translate shape A by the vector $(-4, 1)$.

A translation vector (a, b) says: move every point a units across and b units up.

a positive first number moves right, a negative one moves left

a positive second number moves up, a negative one moves down

So the vector $(-4, 1)$ means

4 units to the left

1 unit up

Method

1. write down the coordinates of each corner of shape A, read off the grid

2. for each one, take 4 from the x and add 1 to the y

3. plot the new points and join them in the same order

4. label the image A'

Check

the image must be exactly the same size and shape as the object — a translation slides, it never turns or resizes

every corner must have moved the same distance in the same direction

Shape A is drawn on the grid in the book, so its coordinates must be read from the page.

Answer Each point (x, y) becomes $(x - 4, y + 1)$.

2. A triangle has vertices at $(1, 1)$, $(4, 1)$ and $(2, 4)$. Translate it 2 units left and 5 units up.

Step 1 Write the movement as a vector

2 units left is -2 ; 5 units up is $+5$

the vector is $(-2, 5)$

Step 2 Apply it to each vertex

$$(1, 1) \rightarrow (1 - 2, 1 + 5) = (-1, 6)$$

$$(4, 1) \rightarrow (4 - 2, 1 + 5) = (2, 6)$$

$$(2, 4) \rightarrow (2 - 2, 4 + 5) = (0, 9)$$

Step 3 Check the shape is unchanged

the base was from $(1, 1)$ to $(4, 1)$, a length of 3

the image base runs from $(-1, 6)$ to $(2, 6)$, still a length of 3 ✓

Step 4 Check the vector by reversing it

$$(-1 + 2, 6 - 5) = (1, 1), \text{ back where it started } \checkmark$$

Answer $(-1, 6)$, $(2, 6)$, $(0, 9)$

3. A parallelogram has vertices at $(3, 2)$, $(7, 2)$, $(8, 5)$ and $(4, 5)$. Translate it 5 units right and 3 units up.

Step 1 Write the vector

5 right is $+5$; 3 up is $+3$, so the vector is $(5, 3)$

both numbers are positive, so every point moves right and up

Step 2 Apply it to each vertex

$$(3, 2) \rightarrow (3 + 5, 2 + 3) = (8, 5)$$

$$(7, 2) \rightarrow (7 + 5, 2 + 3) = (12, 5)$$

$$(8, 5) \rightarrow (8 + 5, 5 + 3) = (13, 8)$$

$$(4, 5) \rightarrow (4 + 5, 5 + 3) = (9, 8)$$

Step 3 Check the shape held together

in the object, $(3, 2)$ to $(7, 2)$ is 4 across

in the image, $(8, 5)$ to $(12, 5)$ is 4 across ✓

opposite sides are still parallel and equal, so it is still a parallelogram ✓

A translation never changes lengths or angles — only the position.

Answer $(8, 5)$, $(12, 5)$, $(13, 8)$, $(9, 8)$

4. Find the translation that maps A $(1, -4)$ onto A' $(5, 2)$.

This is the reverse of the other questions: the two points are given and the vector has to be found.

Step 1 Subtract the x-coordinates
the horizontal move = new x – old x
= 5 – 1

= 4, so 4 units to the right

Step 2 Subtract the y-coordinates
the vertical move = new y – old y
= 2 – (–4)

subtracting a negative adds: = 2 + 4
= 6, so 6 units up

Step 3 Write it as a vector
the translation is (4, 6)

Step 4 Check by applying it to A
(1 + 4, –4 + 6) = (5, 2) = A' ✓

The order matters: it is image minus object. Doing it the other way round gives (–4, –6), which is the translation that takes A' back to A.

Answer The vector (4, 6) — 4 units right and 6 units up

REVISION EXERCISE 14

1. Choose the correct option.

i. A shape has rotational symmetry of order 4. What is the angle of rotation?

a) 90° b) 180° c) 120° d) 45°

The angle of rotation is $360 \div \text{order}$.

Step 1 order = 4

Step 2 angle = $360 \div 4 = 90^\circ$

Check four turns of 90° make 360°, a full circle ✓

(b) 180° belongs to order 2

(c) 120° belongs to order 3

(d) 45° belongs to order 8

Answer a) 90°

ii. Which of the following objects has no rotational symmetry?

a) A wheel b) The letter S c) A starfish d) A snowflake

Take each in turn and ask: does turning it by less than a full circle leave it looking the same?

(a) a wheel — yes, at every spoke ✗

(b) the letter S — turn it through 180° and it is an S again, so it has order 2 ✗

(c) a starfish — five arms, order 5 ✗

(d) a snowflake — six arms, order 6 ✗

All four have rotational symmetry, so the question has no answer as printed — see the note.

The letter S is presumably the intended answer, but it is a poor choice: S is one of the few letters that does have order-2 symmetry. A letter like F, R or P would have none.

Answer All four have rotational symmetry — no correct option

Please check — every object listed has rotational symmetry, the letter S included — it has order 2. A letter such as F or R would have none.

iii. The order of rotational symmetry of an equilateral triangle is:

- a) 290° b) 4 c) 6 d) 8

Step 1 Count the matches in one full turn

an equilateral triangle has 3 equal sides and 3 equal angles

turning it by 120° brings each corner onto the next ✓

turning by 240° does the same again ✓

turning by 360° returns it to the start ✓

Step 2 The order is the number of matches

order = 3

Check angle = $360 \div 3 = 120^\circ$ ✓, and for a regular polygon the order always equals the number of sides.

None of the four options is 3, so the question cannot be answered as printed — see the note.

Option (a) is written in degrees, which an order never is.

Answer The order is 3, which is not among the options

Please check — the order of rotational symmetry of an equilateral triangle is 3. The options offered are 290° , 4, 6 and 8, so the right answer is missing — and an order is a count, never a number of degrees.

iv. Translation is a type of:

- a) Symmetry b) Transformation c) Rotation d) Reflection

Transformation is the general word for changing a shape's position or orientation.

The kinds of transformation are:

translation — sliding, with no turn

rotation — turning about a point

reflection — flipping over a line

So translation is one sort of transformation ✓

Options (c) and (d) are the other sorts, not categories that translation belongs to, and (a)

symmetry is a property a shape has, not a movement.

Answer b) Transformation

v. If the point (3, 5) is translated by the vector $(-2, 4)$, the new coordinates are:

- a) (5, 9) b) (1, 9) c) $(-2, 9)$ d) (1, -1)

Add the vector to the point, one coordinate at a time.

Step 1 The x-coordinate

$$3 + (-2) = 3 - 2 = 1$$

Step 2 The y-coordinate

$$5 + 4 = 9$$

$$= (1, 9)$$

Check by going back $(1 + 2, 9 - 4) = (3, 5)$ ✓

Option (a) (5, 9) comes from adding the 2 instead of subtracting it — the negative was missed.

Answer b) (1, 9)

2. Fill in the blanks: (i) The line that divides a shape into two identical halves is called the _____. (ii) A rotation of 180° is also known as a _____ turn. (iii) A shape has rotational symmetry if it looks the same after a rotation by a certain angle less than _____. (iv) In a translation, every point of the object moves the same _____ in the same _____. (v) A regular pentagon has _____ lines of symmetry.

(i) The line that divides a shape into two identical halves is called the _____.

fold the shape along it and the two halves land exactly on each other

line of symmetry

(ii) A rotation of 180° is also known as a _____ turn.

180° is half of 360° ; 90° would be a quarter turn

half

(iii) A shape has rotational symmetry if it looks the same after a rotation by a certain angle less than _____.

every shape matches itself after a full 360° turn, so that one does not count

360°

- (iv) In a translation, every point of the object moves the same ____ in the same ____.
that is what makes a translation a slide — nothing turns and nothing changes size
distance ... direction
- (v) A regular pentagon has ____ lines of symmetry.
a regular polygon has as many lines of symmetry as it has sides
5

Answer (i) line of symmetry · (ii) half · (iii) 360° · (iv) distance ... direction · (v) 5

3. Find the point of rotation of an equilateral triangle and of a regular pentagon, and the angle of rotation for each.

In both cases the point of rotation is the centre of the shape — the point every corner is the same distance from.

Equilateral triangle

3 sides, so order 3

angle = $360 \div 3 = 120^\circ$

it matches at 120° , 240° and 360°

Regular pentagon

5 sides, so order 5

angle = $360 \div 5 = 72^\circ$

it matches at 72° , 144° , 216° , 288° and 360°

Check

$3 \times 120 = 360 \checkmark$ and $5 \times 72 = 360 \checkmark$

The point of rotation is also where all the lines of symmetry cross.

Answer Both about their centre. Triangle: 120° . Pentagon: 72° .

4. Complete the table of rotational symmetry for a square, an equilateral triangle, a rectangle and a circle.

The order is how many times the shape matches itself in one full turn.

Square

matches at 90° , 180° , 270° , $360^\circ \rightarrow$ order 4

Equilateral triangle

matches at 120° , 240° , $360^\circ \rightarrow$ order 3

Rectangle

matches at 180° and 360° only — a 90° turn makes a tall rectangle wide $\rightarrow$ order 2

Circle

matches at every angle $\rightarrow$ order infinite

Worth noticing

the square and the rectangle look alike but differ, because a square's sides are all equal and a rectangle's are not

for each shape here the order happens to equal its number of lines of symmetry — except the rectangle, which has 2 of each by coincidence rather than by rule

Answer Square 4 · Equilateral triangle 3 · Rectangle 2 · Circle infinite

5. Identify which of the following figures is symmetrical about the given line.

The test for any figure and any line:

Step 1 — imagine folding the figure along the line.

Step 2 — check whether every point on one side lands exactly on a point on the other.

Step 3 — if anything sticks out past the fold, the line is not a line of symmetry.

A quicker test on a grid

pick a corner and count its distance to the line

count the same distance on the other side

there should be a matching corner there — if not, it is not symmetrical about that line

The figures are printed in the book, so each must be checked on the page.

Answer Symmetrical where every point has a matching point the same distance the other side of the line.



15

CHAPTER

Statistics and Probability

EXERCISE 15.1

1. Define the following and give examples.

Discrete data

data that can only take separate, countable values, with nothing in between. The number of students in a class is discrete: there can be 29 or 30, never 29.4.

Continuous data

data that can take any value within a range, including fractions. Height is continuous: a pupil may be 150 cm, 150.3 cm or 150.37 cm, and the only limit is how finely you can measure.

Class interval

a range that grouped data is sorted into, such as 40 – 44. Its size is the number of values it covers — 40 – 44 has size 5, counting 40, 41, 42, 43 and 44.

Frequency

how many readings fall into a class. If four pupils scored between 45 and 49, that class has a frequency of 4.

Midpoint

the middle of a class interval, found by adding the two limits and halving: for 40 – 44 it is $(40 + 44) \div 2 = 42$. It stands in for every value in the class when the mean of grouped data is worked out.

Telling discrete from continuous

ask whether a value halfway between two readings could ever occur

half a student cannot — so discrete

half a centimetre of height can — so continuous

Answer See the five definitions above.

2. 25 students scored: 45, 48, 52, 53, 47, 41, 50, 56, 59, 55, 61, 63, 60, 50, 49, 55, 58, 62, 65, 67, 66, 54, 50, 51, 52. Say whether the data is discrete or continuous, group it into intervals of size 5, make a frequency table and find the midpoints.

Step 1 Discrete or continuous?

a test score is a whole number of marks — 52 or 53, never 52.4

so the data is discrete

Step 2 Choose the classes

the lowest score is 41 and the highest 67

intervals of size 5 from 40 cover everything: 40 – 44, 45 – 49, 50 – 54, 55 – 59, 60 – 64, 65 – 69

Step 3 Sort each score into its class

go through the list once, marking a tally against the right class, then count the tallies

Class interval · Frequency · Midpoint

40 – 44 · 1 · 42

45 – 49 · 4 · 47

50 – 54 · 8 · 52

55 – 59 · 5 · 57

60 – 64 · 4 · 62

65 – 69 · 3 · 67

total frequency = $1 + 4 + 8 + 5 + 4 + 3 = 25$, which matches the 25 readings given ✓

Step 4 Find each midpoint

midpoint = $(\text{lower limit} + \text{upper limit}) \div 2$

for 40 – 44: $(40 + 44) \div 2 = 42$, and so on down the table

Step 5 Check

the frequencies must add to 25, the number of students — and they do ✓

That check is the one worth always doing: it catches a score counted twice or missed out.

Answer Discrete. 40–44: 1 · 45–49: 4 · 50–54: 8 · 55–59: 5 · 60–64: 4 · 65–69: 3

3. Heights in cm: 140, 145, 150, 155, 160, 162, 165, 170, 175, 180, 183, 185, 190, 192, 195. Make a frequency distribution table with class intervals of 10 cm and give the midpoints.

Step 1 Note what kind of data this is

height can take any value, so it is continuous — unlike the scores in Q2

Step 2 Choose the classes

lowest 140, highest 195, so intervals of 10 from 140

Step 3 Sort and count

Class interval · Frequency · Midpoint

140 – 149 cm · 2 · 144.5

150 – 159 cm · 2 · 154.5

160 – 169 cm · 3 · 164.5

170 – 179 cm · 2 · 174.5

180 – 189 cm · 3 · 184.5

190 – 199 cm · 3 · 194.5

total frequency = 2 + 2 + 3 + 2 + 3 + 3 = 15, which matches the 15 readings given ✓

Step 4 Midpoints

for 140 – 149: $(140 + 149) \div 2 = 144.5$

the midpoints come out ending in .5 here, which is normal for continuous data grouped this way

Step 5 Check

the frequencies total 15, the number of pupils ✓

Answer 140–149: 2 (mid 144.5) · 150–159: 2 (mid 154.5) · 160–169: 3 (mid 164.5) · 170–179: 2 (mid 174.5) · 180–189: 3 (mid 184.5) · 190–199: 3 (mid 194.5)

4. Temperatures: 30, 32, 35, 36, 33, 31, 30, 29, 28, 34, 35, 37, 38, 40, 42. Construct a frequency distribution table with class intervals of 5 degrees.

Step 1 Find the range

lowest 28, highest 42

range = $42 - 28 = 14$ degrees

Step 2 Choose a starting class

starting at 25 gives tidy intervals and covers the 28

Step 3 Sort and count

Class interval · Frequency · Midpoint

25 – 29° · 2 · 27

30 – 34° · 6 · 32

35 – 39° · 5 · 37

40 – 44° · 2 · 42

total frequency = 2 + 6 + 5 + 2 = 15, which matches the 15 readings given ✓

Step 4 Read the table

the busiest class is 30 – 34, so most days sat in the low thirties

Step 5 Check

the frequencies total 15, the number of readings ✓

The question calls this a month of temperatures, but only 15 readings are given — see the note.

Answer 25–29: 2 · 30–34: 6 · 35–39: 5 · 40–44: 2

Please check — the question says the temperatures were recorded 'over a month', but only 15 readings are listed. A month has 28 to 31 days, so either half the data is missing or the wording should say a fortnight.

5. Monthly incomes in thousands for 20 families: 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 110, 120, 130. Complete the frequency table.

The printed table's intervals are not reproduced here, so classes of 20 are used — a sensible width for a range this wide.

Step 1 Find the range

lowest 20, highest 130, so the range is 110 thousand

Step 2 Choose the class width

too few classes hides the shape; too many leaves most nearly empty
with 20 readings, five or six classes is about right, so a width of 20 works

Step 3 Sort and count

Class interval · Frequency · Midpoint

20 – 39 · 4 · 29.5

40 – 59 · 4 · 49.5

60 – 79 · 4 · 69.5

80 – 99 · 4 · 89.5

100 – 119 · 2 · 109.5

120 – 139 · 2 · 129.5

total frequency = $4 + 4 + 4 + 4 + 2 + 2 = 20$, which matches the 20 readings given ✓

Step 4 Read the table

the frequencies fall away at the top end — a few families earn much more than the rest, which is typical of income data

Step 5 Check

the frequencies total 20, the number of families ✓

Answer 20–39: 4 · 40–59: 4 · 60–79: 4 · 80–99: 4 · 100–119: 2 · 120–139: 2

Please check — the intervals printed in the book's table are not legible in the scan, so classes of width 20 are used here. The method is the same whatever width the book sets.

EXERCISE 15.2

1. A supermarket recorded sales grouped into price ranges. Represent the data.

Price ranges are class intervals, so the right picture is a histogram.

Step 1 — put the price ranges along the bottom, in order and touching one another.

Step 2 — put the frequency up the side, starting at zero.

Step 3 — draw a bar for each class, as tall as its frequency.

Step 4 — leave no gaps between the bars: the classes run on from one another, so the bars must touch.

Step 5 — label both axes and give the chart a title.

The data is printed in the book, but that is the method for any grouped data of this kind.

Answer A histogram, with touching bars.

2. The table shows hours five students spent studying in a week. Represent the data.

Five named students are separate categories, so this calls for a bar graph rather than a histogram.

Step 1 — put the students' names along the bottom.

Step 2 — put the hours up the side, starting at zero and rising in equal steps.

Step 3 — draw a bar for each student, all the same width.

Step 4 — leave equal gaps between the bars: the categories are separate, so the bars must not touch.

Step 5 — label the axes and add a title.

That gap is the whole difference between a bar graph and a histogram, and it carries a meaning: gaps say the categories are distinct, touching bars say the data flows on.

Answer A bar graph, with gaps between the bars.

3. The line graph shows visitors to a park over five days. (i) Which day had the most visitors? (ii) How many came on Thursday? (iii) Between which two days did the number rise most? (iv) How many came over the five days? (v) On how many days were there more than 80 visitors?

The graph is printed in the book, so each answer is read off it. The method for each kind of question:

- (i) The most visitors
find the highest point on the line and read down to the day beneath it
- (ii) A particular day
find Thursday on the bottom axis, go straight up to the line, then straight across to the scale
- (iii) The biggest rise
look for the steepest upward section, not the highest point — a steep climb from a low start is a bigger rise than a small climb from a high one
to be sure, work out each day's change and compare the numbers
- (iv) The five-day total
read off all five values and add them
- (v) Days above 80
draw a light line across the graph at 80 and count the points above it
a point sitting exactly on 80 does not count, since the question says 'more than 80'

Answer Read from the printed line graph by the methods above.

4. A teacher recorded the number of questions students answered correctly in a quiz. Represent the data in a histogram.

- Step 1 — group the scores into equal class intervals if they are not grouped already.
- Step 2 — put the intervals along the bottom, in order and touching.
- Step 3 — put the frequency up the side.
- Step 4 — draw each bar to the height of its frequency, with no gaps.
- Step 5 — label the axes and title the chart.
- What to look for once it is drawn
 - the tallest bar shows where most scores fell — the modal class
 - a histogram leaning to one side says most students did well, or most did badly
- The class intervals must all be the same width, or the bar heights stop being comparable.

Answer A histogram of the grouped scores, bars touching.

5. A family spends its monthly income as shown. Construct a pie chart.

- A pie chart shows how a whole is divided, so each slice's angle must be its share of 360° .
- Step 1 Find the total
add all the amounts together
- Step 2 Work out each angle
 $\text{angle} = (\text{amount} \div \text{total}) \times 360^\circ$
- Step 3 Check before drawing
all the angles must add to exactly 360°
if they do not, something has been worked out wrongly — this is the check to do first, not last
- Step 4 Draw
draw a circle, mark a radius, then measure each angle round from the last with a protractor
- Step 5 — label every slice and add a key.
- A shortcut worth knowing
if the data is already in percentages, each 1% is 3.6° — so 25% is 90° , a quarter circle

Answer Each slice = $(\text{amount} \div \text{total}) \times 360^\circ$, totalling 360° .

6. For each scenario, choose the most appropriate graph and explain the choice: (i) a plant's growth over six months (ii) percentages of expenses in a budget (iii) 30 students' scores grouped by intervals (iv) cars sold by different brands.

- Four kinds of chart, each suited to a different kind of data:
 - line graph — change over time
 - pie chart — parts of one whole
 - histogram — continuous data grouped into classes
 - bar graph — separate categories compared
- (i) Tracking the growth of a plant over six months.
the measurements follow on in time, and a line shows the trend between them — rising, levelling off or falling

- line graph
- (ii) Showing the percentage of different expenses in a monthly budget.
the parts make up one whole, and a pie chart shows each as a slice of it
pie chart
- (iii) Displaying the scores of 30 students grouped by intervals.
the data is continuous and grouped into classes, so the bars touch — that is what separates a histogram from a bar graph
histogram
- (iv) Comparing the number of cars sold by different brands in a month.
the brands are separate categories with no order between them, so the bars stand apart with gaps
bar graph

Answer (i) line graph · (ii) pie chart · (iii) histogram · (iv) bar graph

■ EXERCISE 15.3

1. Calculate the mean, median and mode of: 8, 12, 15, 15, 18, 20, 22, 23, 25, 25, 25, 63, 53, 64, 74, 35, 6, 62, 8, 23.

Step 1 The mean
mean = sum of all the values ÷ how many there are
sum = 596
count = 20
mean = $596 \div 20 = 29.8$

Step 2 The median — sort first
the list is not in order, so it must be sorted before anything else
6, 8, 8, 12, 15, 15, 18, 20, 22, 23, 23, 25, 25, 25, 35, 53, 62, 63, 64, 74
there are 20 values, an even number, so there is no single middle one
take the two in the middle — the 10th and 11th: 23 and 23
median = $(23 + 23) \div 2 = 23$

Step 3 The mode
the mode is the value appearing most often
25 appears 3 times
8, 15 and 23 appear twice each
everything else appears once
mode = 25

Step 4 Compare them
mean 29.8, median 23, mode 25
the mean is well above the median, because the few large values — 62, 63, 64, 74 — pull it up
while the median ignores them

Answer Mean 29.8 · Median 23 · Mode 25

2. The age distribution of 30 students in a class is given. Find the mean, median and modal class.

Grouped data cannot give an exact mean, because the individual values are no longer known — only the class each fell into. The midpoint stands in for them.

Step 1 Find each midpoint
midpoint = $(\text{lower limit} + \text{upper limit}) \div 2$

Step 2 Multiply each midpoint by its frequency
that gives the total of all the values in that class, as nearly as can be known

Step 3 The mean
mean = $(\text{sum of midpoint} \times \text{frequency}) \div (\text{sum of frequencies})$

Step 4 The modal class
the class with the highest frequency — grouped data gives a modal class, not a single modal value

Step 5 The median class
the frequencies total 30, so the median lies between the 15th and 16th value
add the frequencies down the table until the running total passes 15 — that class holds the median
Check
the frequencies must total 30, the number of students
The table is printed in the book, so its figures must be read from the page.

Answer Mean from $(\sum \text{midpoint} \times \text{frequency}) \div \sum \text{frequency}$; modal class is the tallest; median class holds the 15th and 16th values.

3. Find the mean, median and mode of 100, 120, 125, 135, 150, 175, 180, 190, 210, 400. Which measure would you recommend, and why?

Step 1 The mean
sum = 1,785
count = 10
mean = $1,785 \div 10 = 178.5$
Step 2 The median
the list is already in order
10 values, so take the 5th and 6th: 150 and 175
median = $(150 + 175) \div 2 = 162.5$
Step 3 The mode
every value appears exactly once
there is no mode
Step 4 Which to recommend — and why
mean 178.5, median 162.5, no mode
nine of the ten values lie between 100 and 210; the tenth is 400
that single outlier drags the mean up to 178.5, above seven of the ten readings
the median, 162.5, sits in the middle of the bulk of the data and is barely affected by the 400
the mode is no use at all here, since nothing repeats
Recommend the median, because it describes a typical value without being distorted by the one extreme reading
Check how much the outlier matters
without the 400 the mean would be $1385/9$ — a drop of nearly 443/18
the median would barely move

Answer Mean 178.5, median 162.5, no mode. Recommend the median, because the outlier 400 inflates the mean.

4. A bag contains 4 red balls, 5 blue balls and 1 green ball. Find the probability of drawing each colour.

Probability = (number of favourable outcomes) $\div$ (total number of outcomes).
Step 1 The total
 $4 + 5 + 1 = 10$ balls
Step 2 Each colour in turn
 $P(\text{red}) = 4/10 = 0.4$
 $P(\text{blue}) = 5/10 = 0.5$
 $P(\text{green}) = 1/10 = 0.1$
Step 3 Check
one of the three must happen, so the probabilities must add to 1
 $0.4 + 0.5 + 0.1 = 1 \checkmark$
That check catches a miscounted ball at once, and is worth doing every time.

Answer $P(\text{red}) = 0.4$ · $P(\text{blue}) = 0.5$ · $P(\text{green}) = 0.1$

5. You roll a standard six-sided die. Find the probability of each outcome.

Step 1 The sample space
the possible outcomes are {1, 2, 3, 4, 5, 6} — six in all

Step 2 A single number

each face is equally likely, so

$$P(\text{any one number}) = 1/6$$

Step 3 Some other events worth working

$$P(\text{even}) = 3/6 = 0.5 \text{ — the faces 2, 4, 6}$$

$$P(\text{more than 4}) = 2/6 = 1/3 \text{ — the faces 5 and 6}$$

$$P(7) = 0/6 = 0 \text{ — impossible, since no face shows 7}$$

$$P(\text{less than 7}) = 6/6 = 1 \text{ — certain, since every face is under 7}$$

Step 4 Check

the six single-number probabilities add to $6 \times 1/6 = 1$ ✓

Every probability lies between 0 and 1: 0 means impossible and 1 means certain.

Answer 1/6 for each of the six faces

6. A coin is flipped 3 times. List the sample space and find the probability of each outcome.

Step 1 Count the outcomes

each flip has 2 possibilities, and there are 3 flips

$$2 \times 2 \times 2 = 8 \text{ outcomes}$$

Step 2 List them

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

listing them in this order — H before T at each position — makes sure none is missed

Step 3 Each single outcome

$$P(\text{any one of them}) = 1/8$$

Step 4 Some events worth counting

$$P(\text{all heads}) = 1/8$$

$$P(\text{exactly 2 heads}) = 3/8 \text{ — HHT, HTH, THH}$$

$$P(\text{at least 1 head}) = 7/8 \text{ — everything but TTT}$$

Step 5 Check

8 outcomes each of probability 1/8 add to 1 ✓

Answer 8 outcomes, each with probability 1/8

7. A litter of four puppies has a 50% chance of each being male. What is the probability that at least one is male?

'At least one' is much easier to do backwards — by finding the probability that it does not happen.

Step 1 Name the opposite event

the opposite of 'at least one male' is 'no males at all', that is all four female

Step 2 Probability of all four female

each puppy is female with probability 1/2, and the four are independent

$$P(\text{all female}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \left(\frac{1}{2}\right)^4$$

$$= 0.0625$$

Step 3 Subtract from 1

$$P(\text{at least one male}) = 1 - P(\text{none male})$$

$$= 1 - 0.0625$$

$$= 0.9375$$

Step 4 Check the long way

there are $2^4 = 16$ equally likely combinations of male and female

exactly 1 of them is all-female, so 15 contain at least one male

$$15 \div 16 = 0.9375 \text{ ✓}$$

Working out the four separate cases — one male, two males, three, four — would give the same answer after far more work. Going via the opposite event is the point of this question.

Answer 0.9375

■ REVISION EXERCISE 15

1. Choose the correct option.

i. Discrete data consists of:

- a) countable, separate values b) any value in a range c) only decimals d) only negative numbers

Discrete data can only take separate values with nothing in between.

- (a) countable, separate values ✓ — the number of students in a class, the number of cars sold
 (b) any value in a range describes continuous data, such as height or weight ✗
 (c) decimals are not what makes data discrete or continuous ✗
 (d) the sign of the numbers has nothing to do with it ✗

The test: could a value halfway between two readings occur? If not, the data is discrete.

Answer a) countable, separate values

ii. In grouped data, the range of a class interval is calculated as:

- a) Frequency $\times$ Mid-point b) Upper limit – Lower limit c) Lower limit + Upper limit d) Total frequency $\div$ Number of intervals

The range of a class is how wide it is — the distance from one end to the other.

range = upper limit – lower limit

- (a) frequency $\times$ midpoint is used for the mean of grouped data, not the range ✗
 (b) upper – lower ✓
 (c) lower + upper, halved, gives the midpoint ✗
 (d) that is an average frequency, not a range ✗

Check on 40 – 44 $44 - 40 = 4$, and the midpoint is $(40 + 44) \div 2 = 42$ — two different things from the same two numbers.

Answer b) Upper limit – Lower limit

iii. A histogram is used to represent:

- a) Categorical data b) Continuous data c) Random data d) Discrete data

A histogram's bars touch, which only makes sense when one class runs straight on into the next.

- (a) categorical data — separate named groups — needs a bar graph, with gaps ✗
 (b) continuous data grouped into classes ✓
 (c) 'random data' is not a type of data at all ✗
 (d) discrete data is usually shown with a bar graph ✗

The gap between the bars is what separates the two charts, and it carries meaning rather than being decoration.

Answer b) Continuous data

iv. Which measure of central tendency is least affected by outliers?

- a) Mean b) Mode c) Median d) Range

An outlier is a value far away from the rest.

- (a) the mean uses every value, so one extreme reading drags it ✗
 (b) the mode only looks at what repeats, so it may not describe the data at all ✗
 (c) the median only cares about the middle position, so how extreme the end values are makes no difference ✓
 (d) the range is the most affected of all — it is calculated from the two extremes ✗

Check with the data in Ex 15.3 Q3 replacing the 400 with 4,000 would send the mean up by hundreds, while the median would not move at all.

Answer c) Median

v. The probability of an impossible event is:

- a) 1 b) 0 c) Between 0 and 1 d) Undefined

Probability = favourable outcomes ÷ total outcomes.

An impossible event has no favourable outcomes, so the top of the fraction is 0.

$$P(\text{impossible}) = 0 \div \text{total} = 0$$

(a) 1 is the probability of a certain event $\times$

(c) that is true of events which may or may not happen, not impossible ones $\times$

(d) it is perfectly well defined $\times$

Example rolling a 7 on an ordinary die: 0 of the 6 faces show a 7, so the probability is $0/6 = 0$ ✓

Answer b) 0

2. Fill in the blanks: (i) Discrete data consists of ____ values, such as the number of students. (ii) The ____ is the middle value of a dataset when it is arranged in order. (iii) A ____ visually represents proportions using slices of a circle. (iv) An event with a probability of 1 is called a ____ event. (v) A bar graph is used to represent ____ data, while a histogram is used for continuous data.

(i) Discrete data consists of ____ values, such as the number of students.
there is nothing between one student and the next
countable (separate, whole)

(ii) The ____ is the middle value of a dataset when it is arranged in order.
with an even number of values, the mean of the middle two
median

(iii) A ____ visually represents proportions using slices of a circle.
each slice's angle is its share of 360°
pie chart

(iv) An event with a probability of 1 is called a ____ event.
it must happen; a probability of 0 means impossible
certain

(v) A bar graph is used to represent ____ data, while a histogram is used for continuous data.
which is why a bar graph has gaps and a histogram does not
categorical (or discrete)

Answer (i) countable (separate, whole) · (ii) median · (iii) pie chart · (iv) certain · (v) categorical (or discrete)

3. Answer the following: (i) What is the difference between discrete and continuous data? (ii) How is the mean calculated for grouped data?

(i) What is the difference between discrete and continuous data?

Discrete data can only take separate, countable values with nothing in between — the number of students in a class, or of cars sold. Continuous data can take any value in a range, limited only by how finely it is measured — height, weight, temperature or time.

(ii) How is the mean calculated for grouped data?

The individual values are no longer known once the data is grouped, so the midpoint of each class stands in for them. Multiply each midpoint by its class frequency, add all those products, and divide by the total frequency: $\text{mean} = (\sum \text{midpoint} \times \text{frequency}) \div \sum \text{frequency}$. The answer is an estimate, because the midpoint is only an approximation to the values in that class.

Answer See the two answers above.