



Mathematics

Grade 6

WORKED SOLUTIONS

Worked solutions – the question, the working, then the answer

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Contents

Mathematics Grade 6 – every exercise worked through, with the question, the working and the answer.

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CHAPTER

Factors and Multiples

EXERCISE 1.1

1. Pick out the factors by finding the remainders in each case: (i) $144 \div 4$ (ii) $108 \div 14$ (iii) $89 \div 7$ (iv) $51 \div 3$ (v) $115 \div 8$ (vi) $45 \div 12$

Divide, then read the remainder: 0 means a factor, anything else means not.

- (i) $144 \div 4$

4 does not go into 1, so take the next digit too

4 goes into 14 3 times ($4 \times 3 = 12$)

$$14 - 12 = 2$$

bring down the 4 $\rightarrow 24$

4 goes into 24 6 times ($4 \times 6 = 24$)

$$24 - 24 = 0$$

$$144 \div 4 = 36 \text{ remainder } 0$$

Factor

- (ii) $108 \div 14$

14 does not go into 1, so take the next digit too

14 does not go into 10, so take the next digit too

14 goes into 108 7 times ($14 \times 7 = 98$)

$$108 - 98 = 10$$

$$108 \div 14 = 7 \text{ remainder } 10$$

Not a factor

- (iii) $89 \div 7$

7 goes into 8 1 times ($7 \times 1 = 7$)

$$8 - 7 = 1$$

bring down the 9 $\rightarrow 19$

7 goes into 19 2 times ($7 \times 2 = 14$)

$$19 - 14 = 5$$

$$89 \div 7 = 12 \text{ remainder } 5$$

Not a factor

- (iv) $51 \div 3$

3 goes into 5 1 times ($3 \times 1 = 3$)

$$5 - 3 = 2$$

bring down the 1 $\rightarrow 21$

3 goes into 21 7 times ($3 \times 7 = 21$)

$$21 - 21 = 0$$

$$51 \div 3 = 17 \text{ remainder } 0$$

Factor

- (v) $115 \div 8$

8 does not go into 1, so take the next digit too

8 goes into 11 1 times ($8 \times 1 = 8$)

$$11 - 8 = 3$$

bring down the 5 $\rightarrow 35$

8 goes into 35 4 times ($8 \times 4 = 32$)

$$35 - 32 = 3$$

$$115 \div 8 = 14 \text{ remainder } 3$$

Not a factor

- (vi) $45 \div 12$
 12 does not go into 4, so take the next digit too
 12 goes into 45 3 times ($12 \times 3 = 36$)
 $45 - 36 = 9$
 $45 \div 12 = 3$ remainder 9
 Not a factor

Answer (i) factor (ii) not a factor (iii) not a factor (iv) factor (v) not a factor (vi) not a factor

2. Read the statements and answer yes or no: (i) Is 8 a factor of 115? (ii) Is 3 a factor of 51? (iii) Is 12 a factor of 144? (iv) Is 6 a factor of 45? (v) Is 33 a factor of 333?

Divide; remainder 0 means yes.

- (i) $115 \div 8$
 8 does not go into 1, so take the next digit too
 8 goes into 11 1 times ($8 \times 1 = 8$)
 $11 - 8 = 3$
 bring down the 5 $\rightarrow 35$
 8 goes into 35 4 times ($8 \times 4 = 32$)
 $35 - 32 = 3$
 $115 \div 8 = 14$ remainder 3
 No

- (ii) $51 \div 3$
 3 goes into 5 1 times ($3 \times 1 = 3$)
 $5 - 3 = 2$
 bring down the 1 $\rightarrow 21$
 3 goes into 21 7 times ($3 \times 7 = 21$)
 $21 - 21 = 0$
 $51 \div 3 = 17$ remainder 0
 Yes

- (iii) $144 \div 12$
 12 does not go into 1, so take the next digit too
 12 goes into 14 1 times ($12 \times 1 = 12$)
 $14 - 12 = 2$
 bring down the 4 $\rightarrow 24$
 12 goes into 24 2 times ($12 \times 2 = 24$)
 $24 - 24 = 0$
 $144 \div 12 = 12$ remainder 0
 Yes

- (iv) $45 \div 6$
 6 does not go into 4, so take the next digit too
 6 goes into 45 7 times ($6 \times 7 = 42$)
 $45 - 42 = 3$
 $45 \div 6 = 7$ remainder 3
 No

- (v) $333 \div 33$
 33 does not go into 3, so take the next digit too
 33 goes into 33 1 times ($33 \times 1 = 33$)
 $33 - 33 = 0$
 bring down the 3 $\rightarrow 3$
 33 goes into 3 0 times ($33 \times 0 = 0$)
 $3 - 0 = 3$
 $333 \div 33 = 10$ remainder 3
 No

Answer (i) No (ii) Yes (iii) Yes (iv) No (v) No

3. Write all the factors of: (i) 30 (ii) 65 (iii) 78 (iv) 105 (v) 220

Find the factor pairs, then collect both halves of each.

(i) Factors of 30

$$1 \times 30 = 30$$

$$2 \times 15 = 30$$

$$3 \times 10 = 30$$

$$5 \times 6 = 30$$

Collecting both halves of every pair:

1, 2, 3, 5, 6, 10, 15, 30 (8 factors)

(ii) Factors of 65

$$1 \times 65 = 65$$

$$5 \times 13 = 65$$

Collecting both halves of every pair:

1, 5, 13, 65 (4 factors)

(iii) Factors of 78

$$1 \times 78 = 78$$

$$2 \times 39 = 78$$

$$3 \times 26 = 78$$

$$6 \times 13 = 78$$

Collecting both halves of every pair:

1, 2, 3, 6, 13, 26, 39, 78 (8 factors)

(iv) Factors of 105

$$1 \times 105 = 105$$

$$3 \times 35 = 105$$

$$5 \times 21 = 105$$

$$7 \times 15 = 105$$

Collecting both halves of every pair:

1, 3, 5, 7, 15, 21, 35, 105 (8 factors)

(v) Factors of 220

$$1 \times 220 = 220$$

$$2 \times 110 = 220$$

$$4 \times 55 = 220$$

$$5 \times 44 = 220$$

$$10 \times 22 = 220$$

$$11 \times 20 = 220$$

Collecting both halves of every pair:

1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, 220 (12 factors)

Answer (i) 30: 1, 2, 3, 5, 6, 10, 15, 30 (8 factors)

(ii) 65: 1, 5, 13, 65 (4 factors)

(iii) 78: 1, 2, 3, 6, 13, 26, 39, 78 (8 factors)

(iv) 105: 1, 3, 5, 7, 15, 21, 35, 105 (8 factors)

(v) 220: 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, 220 (12 factors)

4. Write all the factors of 36 and 40.

Step 1 Pair up the factors of 36

$$1 \times 36 = 36$$

$$2 \times 18 = 36$$

$$3 \times 12 = 36$$

$$4 \times 9 = 36$$

$$6 \times 6 = 36$$

1, 2, 3, 4, 6, 9, 12, 18, 36 (9 factors)

Step 2 Pair up the factors of 40

$$1 \times 40 = 40$$

$$2 \times 20 = 40$$

$$4 \times 10 = 40$$

$$5 \times 8 = 40$$

1, 2, 4, 5, 8, 10, 20, 40 (8 factors)

Step 3 Compare the two lists

Shared factors: 1, 2, 4

Largest shared factor = HCF = 4

36 has an odd number of factors because it is a perfect square — 6×6 pairs with itself, so one factor has no partner.

Answer 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

40: 1, 2, 4, 5, 8, 10, 20, 40

5. Which of the given numbers are factors of 42? (i) 3 (ii) 5 (iii) 8 (iv) 12 (v) 23

Factor pairs of 42:

$$1 \times 42 = 42$$

$$2 \times 21 = 42$$

$$3 \times 14 = 42$$

$$6 \times 7 = 42$$

Factors: 1, 2, 3, 6, 7, 14, 21, 42

(i) $42 \div 3 = 14$ remainder 0 → factor

(ii) $42 \div 5 = 8$ remainder 2 → not a factor

(iii) $42 \div 8 = 5$ remainder 2 → not a factor

(iv) $42 \div 12 = 3$ remainder 6 → not a factor

(v) $42 \div 23 = 1$ remainder 19 → not a factor

Answer (i) factor (ii) not a factor (iii) not a factor (iv) not a factor (v) not a factor

EXERCISE 1.2

1. Write the first five multiples of: (i) 27 (ii) 53 (iii) 64

A multiple is what you get when the number is multiplied by 1, 2, 3 and so on. The first multiple of a number is always the number itself, not double it.

(i) 27: $27 \times 1 = 27$, $27 \times 2 = 54$, $27 \times 3 = 81$, $27 \times 4 = 108$, $27 \times 5 = 135$

(ii) 53: $53 \times 1 = 53$, $53 \times 2 = 106$, $53 \times 3 = 159$, $53 \times 4 = 212$, $53 \times 5 = 265$

(iii) 64: $64 \times 1 = 64$, $64 \times 2 = 128$, $64 \times 3 = 192$, $64 \times 4 = 256$, $64 \times 5 = 320$

Answer (i) 27, 54, 81, 108, 135

(ii) 53, 106, 159, 212, 265

(iii) 64, 128, 192, 256, 320

2. List the multiples of 9 that are between 10 and 40.

Write out the multiples of 9 until you pass 40.

$$9 \times 1 = 9 \text{ — below 10, out}$$

$$9 \times 2 = 18 \text{ — keep}$$

$$9 \times 3 = 27 \text{ — keep}$$

$$9 \times 4 = 36 \text{ — keep}$$

$$9 \times 5 = 45 \text{ — above 40, out}$$

Keeping only those between 10 and 40: 18, 27, 36

Answer 18, 27, 36

3. List all numbers less than 50 that are multiples of both 2 and 3.

A number that is a multiple of both is a multiple of their LCM.

Step 1 Find the LCM

$$2 = 2, 3 = 3$$

$$\text{LCM}(2, 3) = 2 \times 3 = 6$$

Step 2 Count up in 6s until you pass 50

$$\begin{aligned}
 6 \times 1 &= 6 \\
 6 \times 2 &= 12 \\
 6 \times 3 &= 18 \\
 6 \times 4 &= 24 \\
 6 \times 5 &= 30 \\
 6 \times 6 &= 36 \\
 6 \times 7 &= 42 \\
 6 \times 8 &= 48 \\
 6, 12, 18, 24, 30, 36, 42, 48
 \end{aligned}$$

Answer 6, 12, 18, 24, 30, 36, 42, 48

4. List the first six multiples of 24, 48 and 72.

(i) Multiples of 24

$$\begin{aligned}
 24 \times 1 &= 24 \\
 24 \times 2 &= 48 \\
 24 \times 3 &= 72 \\
 24 \times 4 &= 96 \\
 24 \times 5 &= 120 \\
 24 \times 6 &= 144 \\
 24, 48, 72, 96, 120, 144
 \end{aligned}$$

(ii) Multiples of 48

$$\begin{aligned}
 48 \times 1 &= 48 \\
 48 \times 2 &= 96 \\
 48 \times 3 &= 144 \\
 48 \times 4 &= 192 \\
 48 \times 5 &= 240 \\
 48 \times 6 &= 288 \\
 48, 96, 144, 192, 240, 288
 \end{aligned}$$

(iii) Multiples of 72

$$\begin{aligned}
 72 \times 1 &= 72 \\
 72 \times 2 &= 144 \\
 72 \times 3 &= 216 \\
 72 \times 4 &= 288 \\
 72 \times 5 &= 360 \\
 72 \times 6 &= 432 \\
 72, 144, 216, 288, 360, 432
 \end{aligned}$$

48 and 72 are themselves multiples of 24, so their multiples all appear in 24's list too.

Answer (i) 24, 48, 72, 96, 120, 144

(ii) 48, 96, 144, 192, 240, 288

(iii) 72, 144, 216, 288, 360, 432

EXERCISE 1.3

1. Write all factors of: (i) 24 (ii) 32 (iii) 58 (iv) 95

Hunt the pairs, then collect both halves.

(i) Factors of 24

$$\begin{aligned}
 1 \times 24 &= 24 \\
 2 \times 12 &= 24 \\
 3 \times 8 &= 24 \\
 4 \times 6 &= 24 \\
 1, 2, 3, 4, 6, 8, 12, 24
 \end{aligned}$$

(ii) Factors of 32

$$1 \times 32 = 32$$

$$2 \times 16 = 32$$

$$4 \times 8 = 32$$

1, 2, 4, 8, 16, 32

(iii) Factors of 58

$$1 \times 58 = 58$$

$$2 \times 29 = 58$$

$$1, 2, 29, 58$$

(iv) Factors of 95

$$1 \times 95 = 95$$

$$5 \times 19 = 95$$

$$1, 5, 19, 95$$

Answer (i) 24: 1, 2, 3, 4, 6, 8, 12, 24

(ii) 32: 1, 2, 4, 8, 16, 32

(iii) 58: 1, 2, 29, 58

(iv) 95: 1, 5, 19, 95

2. Write the first five multiples of: (i) 4 (ii) 7 (iii) 11 (iv) 14

(i) Multiples of 4

$$4 \times 1 = 4$$

$$4 \times 2 = 8$$

$$4 \times 3 = 12$$

$$4 \times 4 = 16$$

$$4 \times 5 = 20$$

$$4, 8, 12, 16, 20$$

(ii) Multiples of 7

$$7 \times 1 = 7$$

$$7 \times 2 = 14$$

$$7 \times 3 = 21$$

$$7 \times 4 = 28$$

$$7 \times 5 = 35$$

$$7, 14, 21, 28, 35$$

(iii) Multiples of 11

$$11 \times 1 = 11$$

$$11 \times 2 = 22$$

$$11 \times 3 = 33$$

$$11 \times 4 = 44$$

$$11 \times 5 = 55$$

$$11, 22, 33, 44, 55$$

(iv) Multiples of 14

$$14 \times 1 = 14$$

$$14 \times 2 = 28$$

$$14 \times 3 = 42$$

$$14 \times 4 = 56$$

$$14 \times 5 = 70$$

$$14, 28, 42, 56, 70$$

Answer (i) 4, 8, 12, 16, 20

(ii) 7, 14, 21, 28, 35

(iii) 11, 22, 33, 44, 55

(iv) 14, 28, 42, 56, 70

3. Separate the odd and even numbers: 174, 247, 678, 4010, 9873, 43125, 612531, 843574

Only the last digit decides.

174 ends in 4 → even

247 ends in 7 → odd

678 ends in 8 → even
 4010 ends in 0 → even
 9873 ends in 3 → odd
 43125 ends in 5 → odd
 612531 ends in 1 → odd
 843574 ends in 4 → even
 Even: 174, 678, 4010, 843574
 Odd: 247, 9873, 43125, 612531

Answer Even: 174, 678, 4010, 843574
 Odd: 247, 9873, 43125, 612531

4. List all prime numbers between: (i) 6 and 30 (ii) 27 and 70 (iii) 33 and 49 (iv) 75 and 90

A prime has exactly two factors, 1 and itself. Test each number by dividing by 2, 3, 5, 7 ... up to its square root.

- (i) Between 6 and 30
 Test 7, 8, 9, 10, 11, 12, 13, 14...
 Primes found: 7, 11, 13, 17, 19, 23, 29
 (7 of them)
- (ii) Between 27 and 70
 Test 28, 29, 30, 31, 32, 33, 34, 35...
 Primes found: 29, 31, 37, 41, 43, 47, 53, 59, 61, 67
 (10 of them)
- (iii) Between 33 and 49
 Test 34, 35, 36, 37, 38, 39, 40, 41...
 Primes found: 37, 41, 43, 47
 (4 of them)
- (iv) Between 75 and 90
 Test 76, 77, 78, 79, 80, 81, 82, 83...
 Primes found: 79, 83, 89
 (3 of them)

Answer (i) 7, 11, 13, 17, 19, 23, 29
 (ii) 29, 31, 37, 41, 43, 47, 53, 59, 61, 67
 (iii) 37, 41, 43, 47
 (iv) 79, 83, 89

5. List all numbers less than 50 which are multiples of 6 and 8.

Step 1 Find the LCM of 6 and 8
 $6 = 2 \times 3$
 $8 = 2^3$
 Highest powers: 2^3 and 3
 $\text{LCM} = 8 \times 3 = 24$
 (not 48 — 6 and 8 share a factor 2)

Step 2 Count up in 24s below 50
 $24 \times 1 = 24$
 $24 \times 2 = 48$
 $24 \times 3 = 72$ — above 50, stop
 24, 48

Answer 24, 48

6. List all numbers less than 90 which are multiples of 6 and 9.

Step 1 Find the LCM of 6 and 9
 $6 = 2 \times 3$
 $9 = 3^2$
 Highest powers: 2 and 3^2

$$\text{LCM} = 2 \times 9 = 18$$

Step 2 Count up in 18s below 90

$$18 \times 1 = 18$$

$$18 \times 2 = 36$$

$$18 \times 3 = 54$$

$$18 \times 4 = 72$$

$$18 \times 5 = 90 \text{ — not below 90, stop}$$

18, 36, 54, 72

Answer 18, 36, 54, 72

7. Write all composite numbers less than 25.

A composite has more than two factors. 1 is neither prime nor composite — it has only one factor.

$$4 = 2 \times 2$$

$$6 = 2 \times 3$$

$$8 = 2 \times 4$$

$$9 = 3 \times 3$$

$$10 = 2 \times 5$$

$$12 = 2 \times 6$$

$$14 = 2 \times 7$$

$$15 = 3 \times 5$$

$$16 = 2 \times 8$$

$$18 = 2 \times 9$$

$$20 = 2 \times 10$$

$$21 = 3 \times 7$$

$$22 = 2 \times 11$$

$$24 = 2 \times 12$$

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24

That is 14 composites against 9 primes below 25.

Answer 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24

8. Write seven consecutive composite numbers just below 100.

We need seven numbers in a row with not one prime among them.

The primes near 100 are 83, 89 and 97. The gap between 89 and 97 is the one to use.

90, 91, 92, 93, 94, 95, 96 — every one composite, and seven of them.

Check: $90 = 2 \times 45$, $91 = 7 \times 13$, $92 = 4 \times 23$, $93 = 3 \times 31$, $94 = 2 \times 47$, $95 = 5 \times 19$, $96 = 2 \times 48$. No primes.

Answer 90, 91, 92, 93, 94, 95, 96

9. Write all prime numbers less than 80.

Work up from 2, crossing out every multiple of each prime as you go — the sieve method.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79.

There are 22 of them. 2 is the only even prime; every other even number has 2 as a third factor.

Answer 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79

EXERCISE 1.4

1. Fill the spaces to complete the factor trees for 42, 99 and 600. (the trees are printed in the book)

A factor tree splits a number into any two factors, then splits those again, until every branch ends in a prime.

$$42 \rightarrow 6 \times 7 \rightarrow (2 \times 3) \times 7, \text{ so } 42 = 2 \times 3 \times 7.$$

$$99 \rightarrow 9 \times 11 \rightarrow (3 \times 3) \times 11, \text{ so } 99 = 3^2 \times 11.$$

$$600 \rightarrow 10 \times 60 \rightarrow (2 \times 5) \times (6 \times 10) \rightarrow \dots \text{ giving } 600 = 2^3 \times 3 \times 5^2.$$

Whichever pair you split with first, the primes at the ends are always the same — that is the point of the exercise.

Answer $42 = 2 \times 3 \times 7$ · $99 = 3^2 \times 11$ · $600 = 2^3 \times 3 \times 5^2$

2. Find the prime factors by the tree method: (i) 18 (ii) 34 (iii) 385 (iv) 45 (v) 52 (vi) 64 (vii) 390 (viii) 2520 (ix) 1210 (x) 1260 (xi) 1024 (xii) 2520

Split each number into two factors and keep splitting until only primes remain.

(i) 18

$$\div 2 \rightarrow 9$$

$$\div 3 \rightarrow 3$$

$$18 = 2 \times 3^2$$

(ii) 34

$$\div 2 \rightarrow 17$$

$$34 = 2 \times 17$$

(iii) 385

$$\div 5 \rightarrow 77$$

$$\div 7 \rightarrow 11$$

$$385 = 5 \times 7 \times 11$$

(iv) 45

$$\div 3 \rightarrow 15$$

$$\div 3 \rightarrow 5$$

$$45 = 3^2 \times 5$$

(v) 52

$$\div 2 \rightarrow 26$$

$$\div 2 \rightarrow 13$$

$$52 = 2^2 \times 13$$

(vi) 64

$$\div 2 \rightarrow 32$$

$$\div 2 \rightarrow 16$$

$$\div 2 \rightarrow 8$$

$$\div 2 \rightarrow 4$$

$$\div 2 \rightarrow 2$$

$$64 = 2^6$$

(vii) 390

$$\div 2 \rightarrow 195$$

$$\div 3 \rightarrow 65$$

$$\div 5 \rightarrow 13$$

$$390 = 2 \times 3 \times 5 \times 13$$

(viii) 2520

$$\div 2 \rightarrow 1260$$

$$\div 2 \rightarrow 630$$

$$\div 2 \rightarrow 315$$

$$\div 3 \rightarrow 105$$

$$\div 3 \rightarrow 35$$

$$\div 5 \rightarrow 7$$

$$2520 = 2^3 \times 3^2 \times 5 \times 7$$

(ix) 1210

$$\div 2 \rightarrow 605$$

$$\begin{aligned}
 &\div 5 \rightarrow 121 \\
 &\div 11 \rightarrow 11 \\
 &1210 = 2 \times 5 \times 11^2 \\
 \text{(x)} \quad &1260 \\
 &\div 2 \rightarrow 630 \\
 &\div 2 \rightarrow 315 \\
 &\div 3 \rightarrow 105 \\
 &\div 3 \rightarrow 35 \\
 &\div 5 \rightarrow 7 \\
 &1260 = 2^2 \times 3^2 \times 5 \times 7 \\
 \text{(xi)} \quad &1024 \\
 &\div 2 \rightarrow 512 \\
 &\div 2 \rightarrow 256 \\
 &\div 2 \rightarrow 128 \\
 &\div 2 \rightarrow 64 \\
 &\div 2 \rightarrow 32 \\
 &\div 2 \rightarrow 16 \\
 &\div 2 \rightarrow 8 \\
 &\div 2 \rightarrow 4 \\
 &\div 2 \rightarrow 2 \\
 &1024 = 2^{10} \\
 \text{(xii)} \quad &2520 \\
 &\div 2 \rightarrow 1260 \\
 &\div 2 \rightarrow 630 \\
 &\div 2 \rightarrow 315 \\
 &\div 3 \rightarrow 105 \\
 &\div 3 \rightarrow 35 \\
 &\div 5 \rightarrow 7 \\
 &2520 = 2^3 \times 3^2 \times 5 \times 7
 \end{aligned}$$

(viii) and (xii) are the same number printed twice — 2,520 appears in both positions.

Answer (i) $18 = 2 \times 3^2$

(ii) $34 = 2 \times 17$

(iii) $385 = 5 \times 7 \times 11$

(iv) $45 = 3^2 \times 5$

(v) $52 = 2^2 \times 13$

(vi) $64 = 2^6$

(vii) $390 = 2 \times 3 \times 5 \times 13$

(viii) $2520 = 2^3 \times 3^2 \times 5 \times 7$

(ix) $1210 = 2 \times 5 \times 11^2$

(x) $1260 = 2^2 \times 3^2 \times 5 \times 7$

(xi) $1024 = 2^{10}$

(xii) $2520 = 2^3 \times 3^2 \times 5 \times 7$

3. Find the prime factors by the division method: (i) 46 (ii) 150 (iii) 198 (iv) 210 (v) 420 (vi) 268 (vii) 570 (viii) 1050 (ix) 8436 (x) 2500 (xi) 9999 (xii) 1225

Divide by the smallest prime that goes in, then divide the quotient the same way, until you reach 1.

(i) 46

$$2 \mid 46 \rightarrow 23$$

$$23 \mid 23 \rightarrow 1$$

$$46 = 2 \times 23$$

(ii) 150

$$2 \mid 150 \rightarrow 75$$

$$3 \mid 75 \rightarrow 25$$

- $5 \mid 25 \rightarrow 5$
 $5 \mid 5 \rightarrow 1$
 $150 = 2 \times 3 \times 5^2$
- (iii) 198
- $2 \mid 198 \rightarrow 99$
 $3 \mid 99 \rightarrow 33$
 $3 \mid 33 \rightarrow 11$
 $11 \mid 11 \rightarrow 1$
 $198 = 2 \times 3^2 \times 11$
- (iv) 210
- $2 \mid 210 \rightarrow 105$
 $3 \mid 105 \rightarrow 35$
 $5 \mid 35 \rightarrow 7$
 $7 \mid 7 \rightarrow 1$
 $210 = 2 \times 3 \times 5 \times 7$
- (v) 420
- $2 \mid 420 \rightarrow 210$
 $2 \mid 210 \rightarrow 105$
 $3 \mid 105 \rightarrow 35$
 $5 \mid 35 \rightarrow 7$
 $7 \mid 7 \rightarrow 1$
 $420 = 2^2 \times 3 \times 5 \times 7$
- (vi) 268
- $2 \mid 268 \rightarrow 134$
 $2 \mid 134 \rightarrow 67$
 $67 \mid 67 \rightarrow 1$
 $268 = 2^2 \times 67$
- (vii) 570
- $2 \mid 570 \rightarrow 285$
 $3 \mid 285 \rightarrow 95$
 $5 \mid 95 \rightarrow 19$
 $19 \mid 19 \rightarrow 1$
 $570 = 2 \times 3 \times 5 \times 19$
- (viii) 1050
- $2 \mid 1050 \rightarrow 525$
 $3 \mid 525 \rightarrow 175$
 $5 \mid 175 \rightarrow 35$
 $5 \mid 35 \rightarrow 7$
 $7 \mid 7 \rightarrow 1$
 $1050 = 2 \times 3 \times 5^2 \times 7$
- (ix) 8436
- $2 \mid 8436 \rightarrow 4218$
 $2 \mid 4218 \rightarrow 2109$
 $3 \mid 2109 \rightarrow 703$
 $19 \mid 703 \rightarrow 37$
 $37 \mid 37 \rightarrow 1$
 $8436 = 2^2 \times 3 \times 19 \times 37$
- (x) 2500
- $2 \mid 2500 \rightarrow 1250$
 $2 \mid 1250 \rightarrow 625$
 $5 \mid 625 \rightarrow 125$
 $5 \mid 125 \rightarrow 25$
 $5 \mid 25 \rightarrow 5$

$$\begin{aligned}
 &5 \mid 5 \rightarrow 1 \\
 &2500 = 2^2 \times 5^4 \\
 \text{(xi)} \quad &9999 \\
 &3 \mid 9999 \rightarrow 3333 \\
 &3 \mid 3333 \rightarrow 1111 \\
 &11 \mid 1111 \rightarrow 101 \\
 &101 \mid 101 \rightarrow 1 \\
 &9999 = 3^2 \times 11 \times 101 \\
 \text{(xii)} \quad &1225 \\
 &5 \mid 1225 \rightarrow 245 \\
 &5 \mid 245 \rightarrow 49 \\
 &7 \mid 49 \rightarrow 7 \\
 &7 \mid 7 \rightarrow 1 \\
 &1225 = 5^2 \times 7^2 \\
 \text{(xii)} \quad &1,225 = 5^2 \times 7^2 \text{ — every power is even, so 1,225 is a perfect square } (35^2).
 \end{aligned}$$

Answer (i) $46 = 2 \times 23$

(ii) $150 = 2 \times 3 \times 5^2$

(iii) $198 = 2 \times 3^2 \times 11$

(iv) $210 = 2 \times 3 \times 5 \times 7$

(v) $420 = 2^2 \times 3 \times 5 \times 7$

(vi) $268 = 2^2 \times 67$

(vii) $570 = 2 \times 3 \times 5 \times 19$

(viii) $1050 = 2 \times 3 \times 5^2 \times 7$

(ix) $8436 = 2^2 \times 3 \times 19 \times 37$

(x) $2500 = 2^2 \times 5^4$

(xi) $9999 = 3^2 \times 11 \times 101$

(xii) $1225 = 5^2 \times 7^2$

4. Write in index notation: (i) $6 \times 6 \times 6 \times 7$ (ii) $7 \times 7 \times 7 \times 5 \times 5$ (iii) $2 \times 2 \times 8 \times 8 \times 8 \times 11$ (iv) $11 \times 11 \times 13 \times 13$ (v) $7 \times 2 \times 1 \times 3$ (vi) $a \times a \times b \times c$

Index notation counts how many times each factor is repeated and writes that count as a small raised number.

Count each factor separately — only identical factors combine.

(i) $6 \times 6 \times 6 \times 7 = 6^3 \times 7$

(ii) $7 \times 7 \times 7 \times 5 \times 5 = 7^3 \times 5^2$

(iii) $2 \times 2 \times 8 \times 8 \times 8 \times 11 = 2^2 \times 8^3 \times 11$

(iv) $11 \times 11 \times 13 \times 13 = 11^2 \times 13^2$

(v) $7 \times 2 \times 1 \times 3 = 7 \times 2 \times 3$ (the 1 may be dropped)

(vi) $a \times a \times b \times c = a^2bc$

Answer (i) $6^3 \times 7$ (ii) $7^3 \times 5^2$ (iii) $2^2 \times 8^3 \times 11$ (iv) $11^2 \times 13^2$ (v) $7 \times 2 \times 3$ (the 1 may be dropped) (vi) a^2bc

5. Write these in expanded form: (i) $2^4 \times 3^2$ (ii) $3^5 \times 5^4 \times 23^2$ (iii) $5^6 \times 7^5 \times 2^2$ (iv) $2^7 \times 3^4 \times 13^3 \times 17^2$

The index tells you how many times to write the factor out.

(i) $2^4 \times 3^2$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 144$

(ii) $3^5 \times 5^4 \times 23^2$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 23 \times 23$

$= 80,341,875$

(iii) $5^6 \times 7^5 \times 2^2$

$= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 7 \times 7 \times 2 \times 2$

$= 1,050,437,500$

(iv) $2^7 \times 3^4 \times 13^3 \times 17^2$

$$= 2 \text{ seven times} \times 3 \text{ four times} \times 13 \text{ three times} \times 17 \text{ twice}$$

$$= 6,582,985,344$$

Answer (i) $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

(ii) $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 23 \times 23 = 80,341,875$

(iii) $5 \times 5 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7 \times 7 \times 2 \times 2 = 1,050,437,500$

(iv) $2 \text{ seven times} \times 3 \text{ four times} \times 13 \text{ three times} \times 17 \text{ twice} = 6,582,985,344$

EXERCISE 1.5

1. Find the HCF by complete prime factorisation: (i) 84, 132, 156 (ii) 198, 360 (iii) 27, 36, 45 (iv) 48, 56, 72 (v) 104, 576 (vi) 360, 540, 720 (vii) 96, 406

The method has three steps, and they are the same every time.

Step 1 — break every number into its prime factors.

Step 2 — find the primes that appear in all of them.

Step 3 — take each shared prime to the lowest power it reaches, and multiply.

- (i) 84, 132, 156

Step 1 — prime factors:

$$84 = 2^2 \times 3 \times 7$$

$$132 = 2^2 \times 3 \times 11$$

$$156 = 2^2 \times 3 \times 13$$

Step 2 — primes in all of them: 2, 3

Step 3 — each at its lowest power: $2^2 \times 3$

$$\text{HCF} = 4 \times 3 = 12$$

Check: $84 \div 12 = 7$, $132 \div 12 = 11$, $156 \div 12 = 13$ — all whole ✓

- (ii) 198, 360

Step 1 — prime factors:

$$198 = 2 \times 3^2 \times 11$$

$$360 = 2^3 \times 3^2 \times 5$$

Step 2 — primes in all of them: 2, 3

Step 3 — each at its lowest power: 2×3^2

$$\text{HCF} = 2 \times 9 = 18$$

Check: $198 \div 18 = 11$, $360 \div 18 = 20$ — all whole ✓

- (iii) 27, 36, 45

Step 1 — prime factors:

$$27 = 3^3$$

$$36 = 2^2 \times 3^2$$

$$45 = 3^2 \times 5$$

Step 2 — primes in all of them: 3

Step 3 — each at its lowest power: 3^2

$$\text{HCF} = 9$$

Check: $27 \div 9 = 3$, $36 \div 9 = 4$, $45 \div 9 = 5$ — all whole ✓

- (iv) 48, 56, 72

Step 1 — prime factors:

$$48 = 2^4 \times 3$$

$$56 = 2^3 \times 7$$

$$72 = 2^3 \times 3^2$$

Step 2 — primes in all of them: 2

Step 3 — each at its lowest power: 2^3

$$\text{HCF} = 8$$

Check: $48 \div 8 = 6$, $56 \div 8 = 7$, $72 \div 8 = 9$ — all whole ✓

- (v) 104, 576

Step 1 — prime factors:

$$104 = 2^3 \times 13$$

$$576 = 2^6 \times 3^2$$

Step 2 — primes in all of them: 2

Step 3 — each at its lowest power: 2^3

$$\text{HCF} = 8$$

Check: $104 \div 8 = 13$, $576 \div 8 = 72$ — all whole ✓

(vi) 360, 540, 720

Step 1 — prime factors:

$$360 = 2^3 \times 3^2 \times 5$$

$$540 = 2^2 \times 3^3 \times 5$$

$$720 = 2^4 \times 3^2 \times 5$$

Step 2 — primes in all of them: 2, 3, 5

Step 3 — each at its lowest power: $2^2 \times 3^2 \times 5$

$$\text{HCF} = 4 \times 9 \times 5 = 180$$

Check: $360 \div 180 = 2$, $540 \div 180 = 3$, $720 \div 180 = 4$ — all whole ✓

(vii) 96, 406

Step 1 — prime factors:

$$96 = 2^5 \times 3$$

$$406 = 2 \times 7 \times 29$$

Step 2 — primes in all of them: 2

Step 3 — each at its lowest power: 2

$$\text{HCF} = 2$$

Check: $96 \div 2 = 48$, $406 \div 2 = 203$ — all whole ✓

Answer (i) 12 (ii) 18 (iii) 9 (iv) 8 (v) 8 (vi) 180 (vii) 2

2. Find the HCF by long division: (i) 120, 168 (ii) 84, 144 (iii) 325, 775, 990 (iv) 632, 790, 869 (v) 430, 516, 817 (vi) 291, 582, 776 (vii) 399, 633, 999

Divide the larger by the smaller; then divide the divisor by the remainder; keep going until the remainder is 0. The last non-zero remainder is the HCF.

For three numbers, find the HCF of two, then take the HCF of that with the third.

(i) 120, 168

$$168 \div 120 = 1 \text{ remainder } 48$$

$$120 \div 48 = 2 \text{ remainder } 24$$

$$48 \div 24 = 2 \text{ remainder } 0$$

$$\text{HCF} = 24$$

(ii) 84, 144

$$144 \div 84 = 1 \text{ remainder } 60$$

$$84 \div 60 = 1 \text{ remainder } 24$$

$$60 \div 24 = 2 \text{ remainder } 12$$

$$24 \div 12 = 2 \text{ remainder } 0$$

$$\text{HCF} = 12$$

(iii) 325, 775, 990

$$775 \div 325 = 2 \text{ remainder } 125$$

$$325 \div 125 = 2 \text{ remainder } 75$$

$$125 \div 75 = 1 \text{ remainder } 50$$

$$75 \div 50 = 1 \text{ remainder } 25$$

$$50 \div 25 = 2 \text{ remainder } 0$$

$$990 \div 25 = 39 \text{ remainder } 15$$

$$25 \div 15 = 1 \text{ remainder } 10$$

$$15 \div 10 = 1 \text{ remainder } 5$$

$$10 \div 5 = 2 \text{ remainder } 0$$

$$\text{HCF} = 5$$

(iv) 632, 790, 869

$$790 \div 632 = 1 \text{ remainder } 158$$

$$\begin{aligned}
 632 \div 158 &= 4 \text{ remainder } 0 \\
 869 \div 158 &= 5 \text{ remainder } 79 \\
 158 \div 79 &= 2 \text{ remainder } 0 \\
 \text{HCF} &= 79 \\
 \text{(v)} \quad 430, 516, 817 \\
 516 \div 430 &= 1 \text{ remainder } 86 \\
 430 \div 86 &= 5 \text{ remainder } 0 \\
 817 \div 86 &= 9 \text{ remainder } 43 \\
 86 \div 43 &= 2 \text{ remainder } 0 \\
 \text{HCF} &= 43 \\
 \text{(vi)} \quad 291, 582, 776 \\
 582 \div 291 &= 2 \text{ remainder } 0 \\
 776 \div 291 &= 2 \text{ remainder } 194 \\
 291 \div 194 &= 1 \text{ remainder } 97 \\
 194 \div 97 &= 2 \text{ remainder } 0 \\
 \text{HCF} &= 97 \\
 \text{(vii)} \quad 399, 633, 999 \\
 633 \div 399 &= 1 \text{ remainder } 234 \\
 399 \div 234 &= 1 \text{ remainder } 165 \\
 234 \div 165 &= 1 \text{ remainder } 69 \\
 165 \div 69 &= 2 \text{ remainder } 27 \\
 69 \div 27 &= 2 \text{ remainder } 15 \\
 27 \div 15 &= 1 \text{ remainder } 12 \\
 15 \div 12 &= 1 \text{ remainder } 3 \\
 12 \div 3 &= 4 \text{ remainder } 0 \\
 999 \div 3 &= 333 \text{ remainder } 0 \\
 \text{HCF} &= 3
 \end{aligned}$$

Answer (i) 24 (ii) 12 (iii) 5 (iv) 79 (v) 43 (vi) 97 (vii) 3

3. What is the greatest possible length that can measure exactly 7 m, 3 m 85 cm and 12 m 95 cm?

'Measures exactly' = divides with no remainder. 'Greatest' = the HCF.

Step 1 Put every length in the same unit

$$7 \text{ m} = 700 \text{ cm}$$

$$3 \text{ m } 85 \text{ cm} = 300 + 85 = 385 \text{ cm}$$

$$12 \text{ m } 95 \text{ cm} = 1,200 + 95 = 1,295 \text{ cm}$$

Step 2 Factorise each

$$700 = 2^2 \times 5^2 \times 7$$

$$385 = 5 \times 7 \times 11$$

$$1,295 = 5 \times 7 \times 37$$

Step 3 Take the shared primes at their lowest powers

Shared: 5 and 7

$$\text{HCF} = 5 \times 7 = 35 \text{ cm}$$

Step 4 Check

$$700 \div 35 = 20 \checkmark$$

$$385 \div 35 = 11 \checkmark$$

$$1,295 \div 35 = 37 \checkmark$$

Answer 35 cm

4. What is the largest number which divides 64, 136 and 238 to leave the same remainder in each case?

If the same remainder is left each time, the differences divide exactly — the remainder cancels when you subtract.

Step 1 Find the differences

$$136 - 64 = 72$$

$$238 - 136 = 102$$

$$238 - 64 = 174$$

Step 2 Take their HCF

$$\text{HCF}(72, 102, 174) = 6$$

Step 3 Check the remainders match

$$64 \div 6 \text{ leaves } 4$$

$$136 \div 6 \text{ leaves } 4$$

$$238 \div 6 \text{ leaves } 4$$

The same remainder each time ✓

Answer 6

5. Find the greatest number which divides 24, 28 and 64 exactly.

'Divides exactly' with nothing left over, and 'greatest' — so this is simply the HCF.

$$24 = 2^3 \times 3, 28 = 2^2 \times 7, 64 = 2^6.$$

The only prime in all three is 2, and the lowest power is 2^2 , so $\text{HCF} = 4$.

Answer 4

6. Two ropes are 275 cm and 385 cm long. What is the greatest length of pieces into which both can be cut without remainder?

Each piece must divide both lengths exactly, and we want the longest — so the HCF.

Step 1 Factorise

$$275 = 5^2 \times 11$$

$$385 = 5 \times 7 \times 11$$

Step 2 Take the shared primes

Shared: 5 and 11

$$\text{HCF} = 5 \times 11 = 55 \text{ cm}$$

Step 3 Check

$$275 \div 55 = 5 \text{ pieces}$$

$$385 \div 55 = 7 \text{ pieces}$$

12 pieces in all, none wasted ✓

Answer 55 cm

EXERCISE 1.6

1. Find the LCM by prime factorisation: (i) 40, 56, 60 (ii) 207, 138 (iii) 72, 96, 120 (iv) 120, 150, 135 (v) 102, 170, 136 (vi) 50, 100, 200 (vii) 660, 420, 240 (viii) 112, 140, 168 (ix) 144, 180, 384

The LCM method mirrors the HCF method, with two words changed:

Step 1 — break every number into its prime factors.

Step 2 — list every prime that appears anywhere, not just the shared ones.

Step 3 — take each to the highest power it reaches, and multiply.

(For the HCF it was the shared primes at their lowest powers. That is the only difference, and mixing the two up is the commonest error in this unit.)

- (i) 40, 56, 60

Step 1 — prime factors:

$$40 = 2^3 \times 5$$

$$56 = 2^3 \times 7$$

$$60 = 2^2 \times 3 \times 5$$

Step 2 — every prime appearing anywhere: 2, 3, 5, 7

Step 3 — each at its highest power: $2^3 \times 3 \times 5 \times 7$

$$\text{LCM} = 8 \times 3 \times 5 \times 7$$

$$= 8 \times 3 = 24$$

$$= 24 \times 5 = 120$$

$$= 120 \times 7 = 840$$

$$\text{LCM} = 840$$

Check: $840 \div 40 = 21$, $840 \div 56 = 15$, $840 \div 60 = 14$ — all whole ✓

(ii) 207, 138

Step 1 — prime factors:

$$207 = 3^2 \times 23$$

$$138 = 2 \times 3 \times 23$$

Step 2 — every prime appearing anywhere: 2, 3, 23

Step 3 — each at its highest power: $2 \times 3^2 \times 23$

$$\text{LCM} = 2 \times 9 \times 23$$

$$= 2 \times 9 = 18$$

$$= 18 \times 23 = 414$$

$$\text{LCM} = 414$$

Check: $414 \div 207 = 2$, $414 \div 138 = 3$ — all whole ✓

(iii) 72, 96, 120

Step 1 — prime factors:

$$72 = 2^3 \times 3^2$$

$$96 = 2^5 \times 3$$

$$120 = 2^3 \times 3 \times 5$$

Step 2 — every prime appearing anywhere: 2, 3, 5

Step 3 — each at its highest power: $2^5 \times 3^2 \times 5$

$$\text{LCM} = 32 \times 9 \times 5$$

$$= 32 \times 9 = 288$$

$$= 288 \times 5 = 1,440$$

$$\text{LCM} = 1,440$$

Check: $1,440 \div 72 = 20$, $1,440 \div 96 = 15$, $1,440 \div 120 = 12$ — all whole ✓

(iv) 120, 150, 135

Step 1 — prime factors:

$$120 = 2^3 \times 3 \times 5$$

$$150 = 2 \times 3 \times 5^2$$

$$135 = 3^3 \times 5$$

Step 2 — every prime appearing anywhere: 2, 3, 5

Step 3 — each at its highest power: $2^3 \times 3^3 \times 5^2$

$$\text{LCM} = 8 \times 27 \times 25$$

$$= 8 \times 27 = 216$$

$$= 216 \times 25 = 5,400$$

$$\text{LCM} = 5,400$$

Check: $5,400 \div 120 = 45$, $5,400 \div 150 = 36$, $5,400 \div 135 = 40$ — all whole ✓

(v) 102, 170, 136

Step 1 — prime factors:

$$102 = 2 \times 3 \times 17$$

$$170 = 2 \times 5 \times 17$$

$$136 = 2^3 \times 17$$

Step 2 — every prime appearing anywhere: 2, 3, 5, 17

Step 3 — each at its highest power: $2^3 \times 3 \times 5 \times 17$

$$\text{LCM} = 8 \times 3 \times 5 \times 17$$

$$= 8 \times 3 = 24$$

$$= 24 \times 5 = 120$$

$$= 120 \times 17 = 2,040$$

$$\text{LCM} = 2,040$$

Check: $2,040 \div 102 = 20$, $2,040 \div 170 = 12$, $2,040 \div 136 = 15$ — all whole ✓

(vi) 50, 100, 200

Step 1 — prime factors:

$$50 = 2 \times 5^2$$

$$100 = 2^2 \times 5^2$$

$$200 = 2^3 \times 5^2$$

Step 2 — every prime appearing anywhere: 2, 5

Step 3 — each at its highest power: $2^3 \times 5^2$

$$\text{LCM} = 8 \times 25$$

$$= 8 \times 25 = 200$$

$$\text{LCM} = 200$$

Check: $200 \div 50 = 4$, $200 \div 100 = 2$, $200 \div 200 = 1$ — all whole ✓

(vii) 660, 420, 240

Step 1 — prime factors:

$$660 = 2^2 \times 3 \times 5 \times 11$$

$$420 = 2^2 \times 3 \times 5 \times 7$$

$$240 = 2^4 \times 3 \times 5$$

Step 2 — every prime appearing anywhere: 2, 3, 5, 7, 11

Step 3 — each at its highest power: $2^4 \times 3 \times 5 \times 7 \times 11$

$$\text{LCM} = 16 \times 3 \times 5 \times 7 \times 11$$

$$= 16 \times 3 = 48$$

$$= 48 \times 5 = 240$$

$$= 240 \times 7 = 1,680$$

$$= 1,680 \times 11 = 18,480$$

$$\text{LCM} = 18,480$$

Check: $18,480 \div 660 = 28$, $18,480 \div 420 = 44$, $18,480 \div 240 = 77$ — all whole ✓

(viii) 112, 140, 168

Step 1 — prime factors:

$$112 = 2^4 \times 7$$

$$140 = 2^2 \times 5 \times 7$$

$$168 = 2^3 \times 3 \times 7$$

Step 2 — every prime appearing anywhere: 2, 3, 5, 7

Step 3 — each at its highest power: $2^4 \times 3 \times 5 \times 7$

$$\text{LCM} = 16 \times 3 \times 5 \times 7$$

$$= 16 \times 3 = 48$$

$$= 48 \times 5 = 240$$

$$= 240 \times 7 = 1,680$$

$$\text{LCM} = 1,680$$

Check: $1,680 \div 112 = 15$, $1,680 \div 140 = 12$, $1,680 \div 168 = 10$ — all whole ✓

(ix) 144, 180, 384

Step 1 — prime factors:

$$144 = 2^4 \times 3^2$$

$$180 = 2^2 \times 3^2 \times 5$$

$$384 = 2^7 \times 3$$

Step 2 — every prime appearing anywhere: 2, 3, 5

Step 3 — each at its highest power: $2^7 \times 3^2 \times 5$

$$\text{LCM} = 128 \times 9 \times 5$$

$$= 128 \times 9 = 1,152$$

$$= 1,152 \times 5 = 5,760$$

$$\text{LCM} = 5,760$$

Check: $5,760 \div 144 = 40$, $5,760 \div 180 = 32$, $5,760 \div 384 = 15$ — all whole ✓

Answer (i) 840 (ii) 414 (iii) 1,440 (iv) 5,400 (v) 2,040 (vi) 200 (vii) 18,480 (viii) 1,680 (ix) 5,760

2. Find the LCM by the division method: (i) 45, 120 (ii) 36, 24, 40 (iii) 84, 90 (iv) 70, 110, 150 (v) 36, 60, 120 (vi) 30, 150, 300 (vii) 192, 288 (viii) 225, 375, 525

Write the numbers in a row. Divide the row by a prime that goes into at least one of them, carrying anything it does not divide down unchanged. Stop at all 1s, then multiply every divisor used.

(i) 45, 120

$$\div 2 \rightarrow 45, 60$$

$$\div 2 \rightarrow 45, 30$$

$$\div 2 \rightarrow 45, 15$$

$$\div 3 \rightarrow 15, 5$$

$$\div 3 \rightarrow 5, 5$$

$$\div 5 \rightarrow 1, 1$$

Multiply the divisors: LCM = 360

(ii) 36, 24, 40

$$\div 2 \rightarrow 18, 12, 20$$

$$\div 2 \rightarrow 9, 6, 10$$

$$\div 2 \rightarrow 9, 3, 5$$

$$\div 3 \rightarrow 3, 1, 5$$

$$\div 3 \rightarrow 1, 1, 5$$

$$\div 5 \rightarrow 1, 1, 1$$

Multiply the divisors: LCM = 360

(iii) 84, 90

$$\div 2 \rightarrow 42, 45$$

$$\div 2 \rightarrow 21, 45$$

$$\div 3 \rightarrow 7, 15$$

$$\div 3 \rightarrow 7, 5$$

$$\div 5 \rightarrow 7, 1$$

$$\div 7 \rightarrow 1, 1$$

Multiply the divisors: LCM = 1,260

(iv) 70, 110, 150

$$\div 2 \rightarrow 35, 55, 75$$

$$\div 3 \rightarrow 35, 55, 25$$

$$\div 5 \rightarrow 7, 11, 5$$

$$\div 5 \rightarrow 7, 11, 1$$

$$\div 7 \rightarrow 1, 11, 1$$

$$\div 11 \rightarrow 1, 1, 1$$

Multiply the divisors: LCM = 11,550

(v) 36, 60, 120

$$\div 2 \rightarrow 18, 30, 60$$

$$\div 2 \rightarrow 9, 15, 30$$

$$\div 2 \rightarrow 9, 15, 15$$

$$\div 3 \rightarrow 3, 5, 5$$

$$\div 3 \rightarrow 1, 5, 5$$

$$\div 5 \rightarrow 1, 1, 1$$

Multiply the divisors: LCM = 360

(vi) 30, 150, 300

$$\div 2 \rightarrow 15, 75, 150$$

$$\div 2 \rightarrow 15, 75, 75$$

$$\div 3 \rightarrow 5, 25, 25$$

$$\div 5 \rightarrow 1, 5, 5$$

$$\div 5 \rightarrow 1, 1, 1$$

Multiply the divisors: LCM = 300

(vii) 192, 288

$$\div 2 \rightarrow 96, 144$$

$$\div 2 \rightarrow 48, 72$$

$$\div 2 \rightarrow 24, 36$$

$$\div 2 \rightarrow 12, 18$$

$$\div 2 \rightarrow 6, 9$$

$$\div 2 \rightarrow 3, 9$$

$$\div 3 \rightarrow 1, 3$$

$$\div 3 \rightarrow 1, 1$$

Multiply the divisors: LCM = 576

(viii) 225, 375, 525

$$\div 3 \rightarrow 75, 125, 175$$

$$\div 3 \rightarrow 25, 125, 175$$

$$\div 5 \rightarrow 5, 25, 35$$

$$\div 5 \rightarrow 1, 5, 7$$

$$\div 5 \rightarrow 1, 1, 7$$

$$\div 7 \rightarrow 1, 1, 1$$

Multiply the divisors: LCM = 7,875

Check for two numbers: LCM $\times$ HCF = the product. For (i): $360 \times 15 = 5,400 = 45 \times 120$ ✓

Answer (i) 360 (ii) 360 (iii) 1,260 (iv) 11,550 (v) 360 (vi) 300 (vii) 576 (viii) 7,875

3. A park hosts events every 75, 150 and 225 days. If all are held today, how many days until all three fall together again?

They coincide on a day that is a multiple of all three cycles — the first such day is the LCM.

Step 1 Factorise each cycle

$$75 = 3 \times 5^2$$

$$150 = 2 \times 3 \times 5^2$$

$$225 = 3^2 \times 5^2$$

Step 2 Take every prime at its highest power

2^1 (from 150), 3^2 (from 225), 5^2 (from all three)

$$\text{LCM} = 2 \times 3^2 \times 5^2$$

$$= 2 \times 9 = 18$$

$$= 18 \times 25$$

$$= 450 \text{ days}$$

Step 3 Check

$$450 \div 75 = 6 \checkmark$$

$$450 \div 150 = 3 \checkmark$$

$$450 \div 225 = 2 \checkmark$$

Answer 450 days

4. Three trains depart at intervals of 144, 192 and 216 minutes. If they depart together now, when will they next depart together?

$$144 = 2^4 \times 3^2, 192 = 2^6 \times 3, 216 = 2^3 \times 3^3$$

Highest powers: 2^6 and $3^3 \rightarrow \text{LCM} = 1,728$ minutes.

In hours that is $1728 \div 60 = 28.8$ hours, or about 1.20 days.

Answer After 1,728 minutes (28.8 hours)

5. A stadium holds events every 70, 140 and 210 days. If all are held today, when will all three coincide again?

'All together again' means the first day that is a multiple of all three cycles — the LCM.

Step 1 — prime factors:

$$70 = 2 \times 5 \times 7$$

$$140 = 2^2 \times 5 \times 7$$

$$210 = 2 \times 3 \times 5 \times 7$$

Step 2 — every prime appearing anywhere: 2, 3, 5, 7

Step 3 — each at its highest power: $2^2 \times 3 \times 5 \times 7$

$$\text{LCM} = 4 \times 3 \times 5 \times 7$$

$$= 4 \times 3 = 12$$

$$= 12 \times 5 = 60$$

$$= 60 \times 7 = 420$$

$$\text{LCM} = 420$$

Check: $420 \div 70 = 6$, $420 \div 140 = 3$, $420 \div 210 = 2$ — all whole ✓

So they next coincide after 420 days.

Answer After 420 days

6. Three friends run on cycles of 225, 375 and 525 days. If they all run together today, how many days before they all run together again?

They run together again on the first day that is a multiple of all three cycles — the LCM.

Step 1 — prime factors:

$$225 = 3^2 \times 5^2$$

$$375 = 3 \times 5^3$$

$$525 = 3 \times 5^2 \times 7$$

Step 2 — every prime appearing anywhere: 3, 5, 7

Step 3 — each at its highest power: $3^2 \times 5^3 \times 7$

$$\text{LCM} = 9 \times 125 \times 7$$

$$= 9 \times 125 = 1,125$$

$$= 1,125 \times 7 = 7,875$$

$$\text{LCM} = 7,875$$

Check: $7,875 \div 225 = 35$, $7,875 \div 375 = 21$, $7,875 \div 525 = 15$ — all whole ✓

So they next run together after 7,875 days.

Answer After 7,875 days

EXERCISE 1.7

1. The LCM of two numbers is 180 and their HCF is 12. If one number is 36, find the other.

The rule for any two numbers: $\text{HCF} \times \text{LCM} = \text{product of the numbers}$.

Step 1 Write the rule with what you know

$$12 \times 180 = 36 \times \text{other}$$

Step 2 Work out the left side

$$12 \times 180 = 2,160$$

Step 3 Divide to find the other number

$$\text{other} = 2,160 \div 36$$

$$= 60$$

Step 4 Check

$$\text{HCF}(36, 60) = 12 \checkmark$$

$$\text{LCM}(36, 60) = 180 \checkmark$$

Answer 60

2. The product of two numbers is 360 and their HCF is 12. What is their LCM?

$\text{HCF} \times \text{LCM} = \text{product of the numbers}$.

$$12 \times \text{LCM} = 360.$$

$$\text{LCM} = 360 \div 12 = 30.$$

Answer 30

3. The product of two numbers is 540 and their HCF is 15. Find their LCM.

For any two numbers, $\text{HCF} \times \text{LCM} = \text{the product of the numbers}$.

Put in what is known:

$$15 \times \text{LCM} = 540$$

Divide both sides by 15:

$$\text{LCM} = 540 \div 15$$

$$\text{LCM} = 36$$

Check: $15 \times 36 = 540 \checkmark$

Answer 36

4. The product of the HCF and LCM of two numbers is 1,050. Find the product of the numbers.

Step 1 Write the rule

$$\text{HCF} \times \text{LCM} = \text{product of the two numbers}$$

Step 2 Substitute what is given

$$\text{HCF} \times \text{LCM} = 1,050$$

Step 3 Read off the answer

$$\text{product of the numbers} = 1,050$$

The individual numbers are not needed, and could be 1,050 and 1, or 30 and 35, or other pairs — the product is the same.

Answer 1,050

5. The LCM of three numbers is 630 and their HCF is 3. If two of them are 18 and 21, find the third.

The HCF $\times$ LCM rule holds only for two numbers, so it cannot be used here. Work from the prime factors instead.

$$630 = 2 \times 3^2 \times 5 \times 7, 18 = 2 \times 3^2, 21 = 3 \times 7.$$

Between them 18 and 21 supply 2, 3^2 and 7. The LCM also needs a 5, which neither of them has — so the third number must contribute the 5.

It must also keep the HCF of all three at 3, so it needs a factor 3 but no factor 2.

The smallest such number is $3 \times 5 = 15$.

$$\text{Check: LCM}(18, 21, 15) = 630 \checkmark \text{ and HCF}(18, 21, 15) = 3 \checkmark$$

Answer 15

6. The product of two numbers is 1,260 and their LCM is 84. Find their HCF.

$$\text{HCF} \times \text{LCM} = \text{product.}$$

$$\text{HCF} \times 84 = 1,260.$$

$$\text{HCF} = 1,260 \div 84 = 15.$$

Answer 15

7. The HCF and LCM of two numbers are 825 and 25 respectively. If one number is 275, find the other.

There is a problem with the question as printed: it gives the HCF as 825 and the LCM as 25. That cannot happen — the HCF divides both numbers and the LCM is a multiple of both, so the HCF can never be the larger of the two.

Taking them the right way round, $\text{HCF} = 25$ and $\text{LCM} = 825$:

$$25 \times 825 = 275 \times \text{the other number.}$$

$$20,625 = 275 \times \text{other.}$$

$$\text{Other} = 20,625 \div 275 = 75.$$

$$\text{Check: HCF}(275, 75) = 25 \checkmark \text{ and LCM} = 825 \checkmark$$

Answer 75 (reading HCF 25 and LCM 825)

Please check — the question swaps its HCF and LCM. An HCF cannot be larger than an LCM, so as printed it has no solution. Read with HCF 25 and LCM 825 it gives 75.

8. Calculate the squares of: (i) 8 (ii) 12 (iii) 19 (iv) 42 (v) 57 (vi) 63 (vii) 76 (viii) 84 (ix) 29 (x) 65 (xi) 38 (xii) 99

To square a number, multiply it by itself. Split the second number into its tens and units, multiply by each, then add.

$$(i) \quad 8^2 = 8 \times 8$$

$$8 \times 8 = 64$$

$$8^2 = 64$$

$$(ii) \quad 12^2 = 12 \times 12$$

$$12 \times 2 = 24$$

$$12 \times 10 = 120 \text{ (the 1 is 10)}$$

$$24 + 120 = 144$$

$$12^2 = 144$$

$$(iii) \quad 19^2 = 19 \times 19$$

$$19 \times 9 = 171$$

$$19 \times 10 = 190 \text{ (the 1 is 10)}$$

$$\begin{aligned}
 &171 + 190 = 361 \\
 &19^2 = 361 \\
 \text{(iv) } &42^2 = 42 \times 42 \\
 &42 \times 2 = 84 \\
 &42 \times 40 = 1,680 \text{ (the 4 is 40)} \\
 &84 + 1,680 = 1,764 \\
 &42^2 = 1,764 \\
 \text{(v) } &57^2 = 57 \times 57 \\
 &57 \times 7 = 399 \\
 &57 \times 50 = 2,850 \text{ (the 5 is 50)} \\
 &399 + 2,850 = 3,249 \\
 &57^2 = 3,249 \\
 \text{(vi) } &63^2 = 63 \times 63 \\
 &63 \times 3 = 189 \\
 &63 \times 60 = 3,780 \text{ (the 6 is 60)} \\
 &189 + 3,780 = 3,969 \\
 &63^2 = 3,969 \\
 \text{(vii) } &76^2 = 76 \times 76 \\
 &76 \times 6 = 456 \\
 &76 \times 70 = 5,320 \text{ (the 7 is 70)} \\
 &456 + 5,320 = 5,776 \\
 &76^2 = 5,776 \\
 \text{(viii) } &84^2 = 84 \times 84 \\
 &84 \times 4 = 336 \\
 &84 \times 80 = 6,720 \text{ (the 8 is 80)} \\
 &336 + 6,720 = 7,056 \\
 &84^2 = 7,056 \\
 \text{(ix) } &29^2 = 29 \times 29 \\
 &29 \times 9 = 261 \\
 &29 \times 20 = 580 \text{ (the 2 is 20)} \\
 &261 + 580 = 841 \\
 &29^2 = 841 \\
 \text{(x) } &65^2 = 65 \times 65 \\
 &65 \times 5 = 325 \\
 &65 \times 60 = 3,900 \text{ (the 6 is 60)} \\
 &325 + 3,900 = 4,225 \\
 &65^2 = 4,225 \\
 \text{(xi) } &38^2 = 38 \times 38 \\
 &38 \times 8 = 304 \\
 &38 \times 30 = 1,140 \text{ (the 3 is 30)} \\
 &304 + 1,140 = 1,444 \\
 &38^2 = 1,444 \\
 \text{(xii) } &99^2 = 99 \times 99 \\
 &99 \times 9 = 891 \\
 &99 \times 90 = 8,910 \text{ (the 9 is 90)} \\
 &891 + 8,910 = 9,801 \\
 &99^2 = 9,801
 \end{aligned}$$

Answer (i) $8^2 = 64$ (ii) $12^2 = 144$ (iii) $19^2 = 361$ (iv) $42^2 = 1,764$ (v) $57^2 = 3,249$ (vi) $63^2 = 3,969$
 (vii) $76^2 = 5,776$ (viii) $84^2 = 7,056$ (ix) $29^2 = 841$ (x) $65^2 = 4,225$ (xi) $38^2 = 1,444$ (xii) $99^2 = 9,801$

9. A square field has a side of 12 metres. What is its area?

Area of a square = side $\times$ side.

$$= 12 \times 12 = 144 \text{ square metres.}$$

This is why $n \times n$ is called 'n squared' — it is literally the area of a square of side n .

Answer 144 m²

REVIEW EXERCISE

- i. What is the smallest factor of every number?

- a) 1 b) The number itself c) 0 d) 2

Every number divides exactly by 1, so 1 is a factor of everything.

No number smaller than 1 can be a factor: 0 cannot divide anything.

The number itself is the largest factor, not the smallest.

Answer a) 1

- ii. Which of the following is a factor of 48?

- a) 7 b) 6 c) 13 d) 9

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

$$48 \div 6 = 8 \text{ exactly } \checkmark$$

$$48 \div 7 = 6 \text{ r } 6, 48 \div 13 = 3 \text{ r } 9, 48 \div 9 = 5 \text{ r } 3 \text{ — none exact.}$$

Answer b) 6

- iii. Which of these is a multiple of 9?

- a) 17 b) 18 c) 20 d) 22

Test each option by dividing by 9.

$$17 \div 9 = 1 \text{ remainder } 8 \text{ } \times$$

$$18 \div 9 = 2 \text{ remainder } 0 \text{ } \checkmark$$

$$20 \div 9 = 2 \text{ remainder } 2 \text{ } \times$$

$$25 \div 9 = 2 \text{ remainder } 7 \text{ } \times$$

$$\text{Only 18 divides exactly: } 18 = 9 \times 2 \text{ } \checkmark$$

A quick test: a number is a multiple of 9 when its digits add to a multiple of 9. For 18, $1 + 8 = 9 \checkmark$; for 17, $1 + 7 = 8 \times$.

Answer b) 18

- iv. Which pair has the highest HCF?

- a) 24 and 36 b) 15 and 25 c) 9 and 12 d) 10 and 20

$$\text{HCF}(24, 36) = 12$$

$$\text{HCF}(15, 25) = 5$$

$$\text{HCF}(9, 12) = 3$$

$$\text{HCF}(10, 20) = 10$$

The largest is 12, from 24 and 36.

Answer a) 24 and 36

- v. The HCF of two numbers is 1. What can be concluded?

- a) They are divisible by 2 b) They are prime numbers c) They are co-prime (relatively prime) d) They have the same factors

An HCF of 1 means they share no factor except 1 — that is exactly what co-prime means.

They need not be prime themselves: 8 and 9 are both composite, yet $\text{HCF}(8, 9) = 1$.

So 'they are prime numbers' is the tempting but wrong answer.

Answer c) They are co-prime (relatively prime)

vi. Which statement about HCF and LCM is true?

- a) $\text{HCF} \times \text{LCM} = \text{sum of the two numbers}$ b) $\text{HCF} \times \text{LCM} = \text{product of the two numbers}$ c) $\text{HCF} \times \text{LCM} = \text{difference of the two numbers}$ d) $\text{HCF} \times \text{LCM} = \text{half the product of the two numbers}$

Test it on a case: 12 and 18. $\text{HCF} = 6$, $\text{LCM} = 36$.

$$6 \times 36 = 216, \text{ and } 12 \times 18 = 216 \checkmark$$

This holds for any two numbers — though not for three or more.

Answer b) $\text{HCF} \times \text{LCM} = \text{product of the two numbers}$

vii. If two numbers are co-prime, their HCF is:

- a) The smaller number b) The larger number c) 1 d) 0

Co-prime means sharing no common factor except 1.

So the highest common factor is 1 itself.

It cannot be 0 — 0 is not a factor of anything.

Answer c) 1

viii. Which statement is true?

- a) Every even number is a perfect square b) Every odd number is a perfect square c) Perfect squares can be both even and odd d) Perfect squares are always odd

$4 = 2^2$ is an even perfect square; $9 = 3^2$ is an odd one. So both kinds exist $\checkmark$

6 is even but not a square, and 7 is odd but not a square, so the first two statements fail.

An even number squared is even and an odd number squared is odd — that is the pattern behind it.

Answer c) Perfect squares can be both even and odd

ix. What is the square of 13?

- a) 156 b) 169 c) 144 d) 162

$$13^2 = 13 \times 13.$$

$$13 \times 13 = 130 + 39 = 169.$$

144 is 12^2 and is there to catch a slip of one.

Answer b) 169

2. Find the greatest number that can exactly divide 120, 160 and 240.

'Exactly divide' and 'greatest' means the HCF.

$$120 = 2^3 \times 3 \times 5, 160 = 2^5 \times 5, 240 = 2^4 \times 3 \times 5.$$

Common primes at their lowest powers: 2^3 and 5 $\rightarrow \text{HCF} = 40$.

Answer 40

3. Find the smallest number that is exactly divisible by 9, 18 and 24.

'Divisible by all' and 'smallest' means the LCM.

$$9 = 3^2, 18 = 2 \times 3^2, 24 = 2^3 \times 3.$$

Highest powers: 2^3 and $3^2 \rightarrow \text{LCM} = 72$.

$$\text{Check: } 72 \div 9 = 8, \div 18 = 4, \div 24 = 3 \checkmark$$

Answer 72



2

CHAPTER

Integers

EXERCISE 2.1

1. Arrange the integers in descending order: (i) 9, -2, 3, 0, -5, -7, 7, -1 (ii) -11, 17, -2, 2, -6, -15, 0, 1 (iii) 12, -21, -18, 14, -5, -1, 1, 10 (iv) -6, 4, -18, 21, 29, -8, -16, 19

Descending = largest first. With negatives, the one nearest zero is the largest.

(i)

Positives: 9, 7, 3

Zero: 0

Negatives, nearest zero first: -1, -2, -5, -7

$9 > 7 > 3 > 0 > -1 > -2 > -5 > -7$

(ii)

Positives: 17, 2, 1

Zero: 0

Negatives, nearest zero first: -2, -6, -11, -15

$17 > 2 > 1 > 0 > -2 > -6 > -11 > -15$

(iii)

Positives: 14, 12, 10, 1

Zero: none

Negatives, nearest zero first: -1, -5, -18, -21

$14 > 12 > 10 > 1 > -1 > -5 > -18 > -21$

(iv)

Positives: 29, 21, 19, 4

Zero: none

Negatives, nearest zero first: -6, -8, -16, -18

$29 > 21 > 19 > 4 > -6 > -8 > -16 > -18$

Answer (i) $9 > 7 > 3 > 0 > -1 > -2 > -5 > -7$

(ii) $17 > 2 > 1 > 0 > -2 > -6 > -11 > -15$

(iii) $14 > 12 > 10 > 1 > -1 > -5 > -18 > -21$

(iv) $29 > 21 > 19 > 4 > -6 > -8 > -16 > -18$

2. Arrange the following integers in ascending order: (i) -3, 12, 7, 0, -8, 6 (ii) 0, -9, -15, 15, 9, -6, -18, 29 (iii) -25, 0, -1, 8, -6, -13, 24, 6 (iv) -706, 409, 170, 109, -75, -555

Ascending = smallest first. Negatives come first, and among them the one furthest from zero is smallest.

(i)

Negatives, furthest from zero first: -8, -3

Zero: 0

Positives: 6, 7, 12

$-8 < -3 < 0 < 6 < 7 < 12$

(ii)

Negatives, furthest from zero first: -18, -15, -9, -6

Zero: 0

Positives: 9, 15, 29

$-18 < -15 < -9 < -6 < 0 < 9 < 15 < 29$

(iii)

Negatives, furthest from zero first: -25, -13, -6, -1

Zero: 0

Positives: 6, 8, 24
 $-25 < -13 < -6 < -1 < 0 < 6 < 8 < 24$
 (iv)
 Negatives, furthest from zero first: -706, -555, -75
 Zero: none
 Positives: 109, 170, 409
 $-706 < -555 < -75 < 109 < 170 < 409$

Answer (i) $-8 < -3 < 0 < 6 < 7 < 12$
 (ii) $-18 < -15 < -9 < -6 < 0 < 9 < 15 < 29$
 (iii) $-25 < -13 < -6 < -1 < 0 < 6 < 8 < 24$
 (iv) $-706 < -555 < -75 < 109 < 170 < 409$

3. Write the next three integers in each sequence: (i) -9, -6, -3 (ii) 7, 4, 1 (iii) 2, 7, 12

Find the step first by subtracting one term from the next.
 (i) $-6 - (-9) = 3$, so the sequence goes up in 3s: 0, 3, 6.
 (ii) $4 - 7 = -3$, so it goes down in 3s: -2, -5, -8 — straight through zero into the negatives.
 (iii) $7 - 2 = 5$, so up in 5s: 17, 22, 27.

Answer (i) 0, 3, 6 (ii) -2, -5, -8 (iii) 17, 22, 27

4. Give two possible integers for the placeholder: (i) $__ < 0$ (ii) $__ > -2$ (iii) $__ < -3$

Any integer satisfying the condition will do — there are infinitely many in each case.
 (i) Less than 0 means any negative integer: -1 and -2.
 (ii) Greater than -2 means -1, 0, 1, 2 and upward: -1 and 0. Note -2 itself does not qualify.
 (iii) Less than -3 means further from zero than -3: -4 and -5.

Answer (i) -1, -2 (ii) -1, 0 (iii) -4, -5 (any others satisfying the condition are equally correct)

5. Which is greater, -111 or -1111?

Both are negative, so the one nearer zero is greater.
 Distance from zero:
 $|-111| = 111$
 $|-1111| = 1,111$
 $1,111 - 111 = 1,000$ further away
 $-111 > -1111$
 The reverse of what happens with positives: extra digits make a negative number smaller, not larger.

Answer -111 is greater

6. Using a number line, find: (i) $-2 - 7$ (ii) $-4 - 5$ (iii) $-1 + 6$ (iv) $-9 - 3$ (v) $-10 + 8$ (vi) $-4 + 1$

Start at the first number. Subtracting moves left; adding moves right.
 (i) Start at -2 and move 7 left: -9.
 (iii) Start at -1 and move 6 right: you pass zero and finish at +5.
 (v) Start at -10 and move 8 right: still short of zero, at -2.
 (i) $-2 - 7 = -9$
 (ii) $-4 - 5 = -9$
 (iii) $-1 + 6 = 5$
 (iv) $-9 - 3 = -12$
 (v) $-10 + 8 = -2$
 (vi) $-4 + 1 = -3$

Answer (i) -9 (ii) -9 (iii) 5 (iv) -12 (v) -2 (vi) -3

7. Compare and put the right sign $<$, $>$ or $=$: (i) -17 and 0 (ii) $(-6 - 10)$ and $(-10 + 6)$ (iii) 45 and -45 (iv) $(-5 + 5)$ and 0 (v) -12 and -9 (vi) $(+8 - 3)$ and $(-3 + 8)$

Work out each side first, then compare.

(ii) $-6 - 10 = -16$ and $-10 + 6 = -4$. -16 is further from zero, so it is the smaller.

(iv) $-5 + 5 = 0$, so the two sides are equal.

(vi) $+8 - 3 = 5$ and $-3 + 8 = 5$ — the same, because addition can be done in any order.

(i) -17 and $0 \rightarrow -17 < 0$

(ii) $(-6 - 10)$ and $(-10 + 6) \rightarrow -16 < -4$

(iii) 45 and $-45 \rightarrow 45 > -45$

(iv) $(-5 + 5)$ and $0 \rightarrow 0 = 0$

(v) -12 and $-9 \rightarrow -12 < -9$

(vi) $(+8 - 3)$ and $(-3 + 8) \rightarrow 5 = 5$

Answer (i) $<$ (ii) $<$ (iii) $>$ (iv) $=$ (v) $<$ (vi) $=$

8. Compare using a number line: (i) -2 and -6 (ii) 3 and -4 (iii) 0 and -5 (iv) -3 and -1 (v) 4 and 2 (vi) -7 and -2

Whichever lies further right on the line is greater. Every positive is right of every negative; zero sits between.

(i) -2 and -6

-2 is 2 from zero (left)

-6 is 6 from zero (left)

$-2 > -6$

(ii) 3 and -4

3 is 3 from zero (right)

-4 is 4 from zero (left)

$3 > -4$

(iii) 0 and -5

0 is 0 from zero

-5 is 5 from zero (left)

$0 > -5$

(iv) -3 and -1

-3 is 3 from zero (left)

-1 is 1 from zero (left)

$-3 < -1$

(v) 4 and 2

4 is 4 from zero (right)

2 is 2 from zero (right)

$4 > 2$

(vi) -7 and -2

-7 is 7 from zero (left)

-2 is 2 from zero (left)

$-7 < -2$

Answer (i) $-2 > -6$ (ii) $3 > -4$ (iii) $0 > -5$ (iv) $-3 < -1$ (v) $4 > 2$ (vi) $-7 < -2$

9. Add: (i) -57 and -476 (ii) 274 and -342 (iii) $+145$ and $+264$ (iv) -814 and $+415$

When the signs are the same, add the digits and keep the sign. When they differ, subtract the smaller from the larger and take the sign of the larger.

(i) Both negative: $57 + 476 = 533$, so the answer is -533 .

(ii) Different signs: $342 - 274 = 68$, and 342 is the larger and negative, so -68 .

(iii) Both positive: $145 + 264 = 409$.

(iv) Different signs: $814 - 415 = 399$, and 814 is negative, so -399 .

Answer (i) -533 (ii) -68 (iii) 409 (iv) -399

10. Solve using a number line: (i) $3 + 5$ (ii) $(-6) + 5$ (iii) $(-5) + (-3)$ (iv) $(-3) + 5$

Begin at the first number and move by the second — right if it is positive, left if negative.

(ii) From -6 move 5 right, reaching -1 : still negative, because 5 was not enough to clear the 6.

(iii) From -5 move 3 further left, to -8 .

(iv) From -3 move 5 right, crossing zero to $+2$.

(i) $3 + (5) = 8$

(ii) $-6 + (5) = -1$

(iii) $-5 + (-3) = -8$

(iv) $-3 + (5) = 2$

Answer (i) 8 (ii) -1 (iii) -8 (iv) 2

11. Find the absolute value of: (i) -12 (ii) 30 (iii) 0 (iv) -8 (v) 15 (vi) -25

The absolute value is the distance from zero, so it is never negative — the sign is simply dropped.

$|0| = 0$, the only absolute value that is zero.

(i) $|-12| = 12$

(ii) $|30| = 30$

(iii) $|0| = 0$

(iv) $|-8| = 8$

(v) $|15| = 15$

(vi) $|-25| = 25$

Answer (i) 12 (ii) 30 (iii) 0 (iv) 8 (v) 15 (vi) 25

Please check — the book letters the six parts i, ii, iii, vi, v, vi — 'vi' twice and no 'iv'. They are lettered i to vi above.

12. Find the value of $24 + |-14|$.

Deal with the absolute value first: $|-14| = 14$.

Then $24 + 14 = 38$.

Answering 10 is the error — that would be $24 + (-14)$, ignoring the absolute value bars.

Answer 38

13. Subtract: (i) $-7 - (-4)$ (ii) $+8 - (-18)$ (iii) $-16 - (-46)$ (iv) $+41 - (-76)$ (v) $-63 - (+87)$

Subtracting a negative is the same as adding: two minus signs together make a plus.

(i) $-7 - (-4) = -7 + 4 = -3$.

(ii) $8 - (-18) = 8 + 18 = 26$.

(iii) $-16 - (-46) = -16 + 46 = 30$ — a negative start giving a positive answer, because 46 is larger than 16.

(v) $-63 - (+87)$ has only one minus to subtract, so it really does take away: $-63 - 87 = -150$.

(i) $-7 - (-4) = -7 + (4) = -3$

(ii) $8 - (-18) = 8 + (18) = 26$

(iii) $-16 - (-46) = -16 + (46) = 30$

(iv) $41 - (-76) = 41 + (76) = 117$

(v) $-63 - (87) = -63 + (-87) = -150$

Answer (i) -3 (ii) 26 (iii) 30 (iv) 117 (v) -150

14. A car travelled 48 km north of California, then 215 km south from there. How far was it finally from California?

Take north as positive and south as negative.

Step 1 Write each leg as a signed number

$$48 \text{ km north} = +48$$

$$215 \text{ km south} = -215$$

Step 2 Add them

$$\text{Position} = +48 + (-215)$$

$$= 48 - 215$$

$$= -167$$

Step 3 Read the sign

The minus means south of California.

Final position = 167 km south

Answer 167 km south of California

15. Mount Everest is at 29,028 feet and the Dead Sea at -485 feet. What is the difference in elevation?

Difference = higher - lower.

Step 1 Write the subtraction

$$29,028 - (-485)$$

Step 2 Two minuses make a plus

$$= 29,028 + 485$$

Step 3 Add

$$= 29,513 \text{ feet}$$

The Dead Sea is below sea level, so the full drop is more than Everest's own height ✓

Answer 29,513 feet

16. What is the balance resulting from a credit of Rs 516 and a debit of Rs 497?

Credit is money in (positive); debit is money out (negative).

$$\text{Credit} = +516$$

$$\text{Debit} = -497$$

$$\text{Balance} = 516 + (-497)$$

$$= 516 - 497$$

$$= +19$$

Positive, so Rs 19 is left rather than owed.

Answer Rs 19 in credit

17. An elevator is on the twentieth floor. It goes down 11 floors, then up 5. What floor is it on now?

Down is negative, up is positive.

Step 1 Write the journey

$$20 - 11 + 5$$

Step 2 Work left to right

$$20 - 11 = 9$$

$$9 + 5 = 14$$

The elevator is on the 14th floor.

Answer The 14th floor

18. Steve's temperature was 102 °F. Two hours later it was 3° lower. What was it then?

'Lower' means subtract.

$$\text{Starting temperature} = 102 \text{ °F}$$

$$\text{Change} = -3 \text{ °F}$$

$$102 - 3 = 99 \text{ °F}$$

Normal is 98.6 °F, so his fever had fallen but not gone.

Answer 99 °F

REVIEW EXERCISE

- i. On the number line, which is greater: -15 or -3 ?

a) -15 b) 0 c) -3 d) 3

Both are negative, so the one nearer zero is greater.

$|-3| = 3$ — three steps from zero

$|-15| = 15$ — fifteen steps from zero

$3 < 15$, so -3 lies nearer zero

$-3 > -15$

Answer c) -3

- ii. Arrange in decreasing order: $-25, +23, +3, -12, 0$

a) $-12, +3, 0, -25, +23$ b) $-25, -12, 0, +3, +23$ c) $+23, +3, 0, -12, -25$ d) $-25, +23, +3, -12, 0$

Decreasing = largest first.

Positives: $+23, +3$

Zero: 0

Negatives, nearest zero first: $-12, -25$

$+23 > +3 > 0 > -12 > -25$

Option (b) is the same list in increasing order — the trap.

Answer c) $+23, +3, 0, -12, -25$

- iii. $-33, -22, -11, _, _$ — the next two integers are:

a) $+11, +22$ b) $+11, 0$ c) $-1, 0$ d) $0, +11$

Find the step: $-22 - (-33) = 11$, so the sequence rises by 11 each time.

$-11 + 11 = 0$.

$0 + 11 = +11$.

So $0, +11$. Jumping straight to $+11, +22$ skips the zero.

Answer d) $0, +11$

- iv. 0 lies to the right of -25 . Which statement is true?

a) $-25 > 0$ b) -25 (incomplete in the book) c) $-(-25) < 0$ d) $-25 = 0$

0 is to the right of -25 , so $-25 < 0$.

(a) $-25 > 0 \rightarrow$ false, since $-25 < 0$

(c) $-(-25) = +25$, and $25 > 0$, so $-(-25) < 0 \rightarrow$ false

(d) $-25 = 0 \rightarrow$ false

None of the readable options is true — see the note.

Answer c) $-(-25) < 0$

Please check — option (b) is printed as just ' -25 ' with no relation sign, so it cannot be judged. Of the three readable options none is true, which means the intended answer is in the missing text of (b).

2. State whether each is true or false: (i) $-12 + 15 = 15 + (-12)$ (ii) $3 + (-5)$ is an integer (iii) $5 + (-5)$ is a positive integer (iv) $(-39) + 14 = 14 - 39$ (v) $(-32) \times 0 = 0$ (vi) Negative of (-5) does not exist

(i) $-12 + 15 = 15 + (-12)$

both sides come to 3; addition can be done in any order (commutative law)

True

(ii) $3 + (-5)$ is an integer

$3 + (-5) = -2$, which is an integer — adding two integers always gives an integer

True

(iii) $5 + (-5)$ is a positive integer

$5 + (-5) = 0$, and zero is neither positive nor negative

False

(iv) $(-39) + 14 = 14 - 39$

both sides come to -25 ; adding a negative is the same as subtracting

True

(v) $(-32) \times 0 = 0$

anything multiplied by zero is zero, whatever its sign

True

(vi) Negative of (-5) does not exist

the negative of -5 is $+5$; every integer has a negative

False

Answer (i) True (ii) True (iii) False (iv) True (v) True (vi) False

3. Arrange in ascending order: (i) $|-11|$, $|-13|$, -6 , $|8|$, $|-5|$ (ii) 18 , $|-9|$, $|14|$, $|-7|$, -27

Work out every absolute value first — the bars make a number positive, so $|-13|$ is 13 .

(i) $|-11|$, $|-13|$, -6 , $|8|$, $|-5|$

$$|-11| = 11$$

$$|-13| = 13$$

$$-6 = -6$$

$$|8| = 8$$

$$|-5| = 5$$

$$-6 < 5 < 8 < 11 < 13$$

(ii) 18 , $|-9|$, $|14|$, $|-7|$, -27

$$18 = 18$$

$$|-9| = 9$$

$$|14| = 14$$

$$|-7| = 7$$

$$-27 = -27$$

$$-27 < 7 < 9 < 14 < 18$$

Answer (i) $-6 < 5 < 8 < 11 < 13$

(ii) $-27 < 7 < 9 < 14 < 18$

4. Arrange in descending order: (i) $|-30|$, -52 , $|-66|$, $|74|$, 39 (ii) $|-2|$, $|-18|$, 85 , $|-76|$, -45

Take the absolute values first, then order from largest down.

(i) $|-30|$, -52 , $|-66|$, $|74|$, 39

$$|-30| = 30$$

$$-52 = -52$$

$$|-66| = 66$$

$$|74| = 74$$

$$39 = 39$$

$$74 > 66 > 39 > 30 > -52$$

(ii) $|-2|$, $|-18|$, 85 , $|-76|$, -45

$$|-2| = 2$$

$$|-18| = 18$$

$$85 = 85$$

$$|-76| = 76$$

$$-45 = -45$$

$$85 > 76 > 18 > 2 > -45$$

Answer (i) $74 > 66 > 39 > 30 > -52$

(ii) $85 > 76 > 18 > 2 > -45$

5. Simplify: (i) $-5 + 7$ (ii) $3 + (-8)$ (iii) $6 + (-4)$ (iv) $10 + (-15)$ (v) $-12 + 9$ (vi) $-9 + (-8)$

Same signs: add and keep the sign. Different signs: subtract the smaller from the larger and take the larger one's sign.

$$(i) -5 + (7) = 2$$

$$(ii) 3 + (-8) = -5$$

$$(iii) 6 + (-4) = 2$$

$$(iv) 10 + (-15) = -5$$

$$(v) -12 + (9) = -3$$

$$(vi) -9 + (-8) = -17$$

Answer (i) 2 (ii) -5 (iii) 2 (iv) -5 (v) -3 (vi) -17

6. Simplify: (i) $8 - (-3)$ (ii) $-6 - 4$ (iii) $-2 - (-5)$ (iv) $7 - 10$ (v) $-9 - (-2)$ (vi) $-15 - (+8)$

Change every subtraction into an addition first: $a - b$ becomes $a + (-b)$. Two minuses together make a plus.

$$(i) 8 - (-3) = 8 + (3) = 11$$

$$(ii) -6 - (4) = -6 + (-4) = -10$$

$$(iii) -2 - (-5) = -2 + (5) = 3$$

$$(iv) 7 - (10) = 7 + (-10) = -3$$

$$(v) -9 - (-2) = -9 + (2) = -7$$

$$(vi) -15 - (8) = -15 + (-8) = -23$$

Answer (i) 11 (ii) -10 (iii) 3 (iv) -3 (v) -7 (vi) -23



3

CHAPTER

Laws of Integers

EXERCISE 3.1

1. Solve: (i) -8×-4 (ii) $(-2) \times (-2) \times (-2) \times (-2)$ (iii) $[(+3) \times (+4)] \times (-5)$ (iv) $-1044 \div 12$ (v) $-1886 \div (-23)$ (vi) $(+294) \div (-6)$ (vii) $[(-200) \times (-5)] \times 4$ (viii) $(+49) \div (+7)$

The sign rule for multiplying and dividing: two like signs give a plus, two unlike signs give a minus.

(i) Both negative, so the answer is positive: $8 \times 4 = 32$.

(ii) Four negatives multiplied — count them. An even number of minus signs gives a positive, so $2 \times 2 \times 2 \times 2 = 16$.

(iii) Positive $\times$ positive = positive, then $\times$ negative turns it negative: $12 \times -5 = -60$.

(v) Negative $\div$ negative = positive: $1886 \div 23 = 82$.

(vii) Two negatives give +1000, then $\times 4 = 4000$.

(i) $-8 \times -4 = 32$

(ii) $(-2) \times (-2) \times (-2) \times (-2) = 16$

(iii) $[(+3) \times (+4)] \times (-5) = -60$

(iv) $-1044 \div 12 = -87$

(v) $-1886 \div (-23) = 82$

(vi) $(+294) \div (-6) = -49$

(vii) $[(-200) \times (-5)] \times 4 = 4000$

(viii) $(+49) \div (+7) = 7$

Answer (i) 32 (ii) 16 (iii) -60 (iv) -87 (v) 82 (vi) -49 (vii) 4000 (viii) 7

2. Prove each and name the property involved: (i) $8 \times (2 + 7) = (8 \times 2) + (8 \times 7)$ (ii) $6 + (-3) = (-3) + 6$ (iii) $-4 \times (3 + 2) = (-4 \times 3) - (-4 \times 2)$ (iv) $-3 \times (7 + 4) = (-3 \times 7) + (-3 \times 4)$ (v) $[(-15) \times 4] \times 3$ (vi) $[(+15) - (-7)] - (+30)$ (vii) $[(+200) - (+100)] - (+50)$ (viii) $(+36) + [(+40) + (+10)]$ (ix) $[(-12) + (-14)] + (-50)$

To prove an identity, work out each side on its own and only then compare them.

(i) LHS = 72, RHS = 72 — equal ✓. Distributive property of multiplication over addition: the 8 is shared out over both numbers inside the bracket.

(ii) LHS = 3, RHS = 3 — equal ✓. Commutative property of addition: the order of the two numbers may be swapped without changing the sum.

(iii) LHS = -20, RHS = -4 — not equal ✗. Distributive property — but misprinted: the sign between the two products should be a plus, not a minus.

(iv) LHS = -33, RHS = -33 — equal ✓. Distributive property of multiplication over addition: the -3 multiplies each term inside the bracket.

(v) $[(-15) \times 4] \times 3 = -180$, and regrouped it is also -180 ✓ — Associative property of multiplication: the brackets may be moved: $(a \times b) \times c = a \times (b \times c)$.

(vi) $[(+15) - (-7)] - (+30) = -8$ — Order of operations — not associative: subtraction is not associative, so the brackets must be worked as written.

(vii) $[(+200) - (+100)] - (+50) = 50$ — Order of operations — not associative: again subtraction, so work left to right as bracketed.

(viii) $(+36) + [(+40) + (+10)] = 86$, and regrouped it is also 86 ✓ — Associative property of addition: $(a + b) + c = a + (b + c)$, so the grouping makes no difference.

(ix) $[(-12) + (-14)] + (-50) = -76$, and regrouped it is also -76 ✓ — Associative property of addition: regrouping leaves the sum unchanged.

Answer (i) Distributive ✓ (ii) Commutative ✓ (iii) false as printed (iv) Distributive ✓ (v) Associative, = -180 (vi) = -8 (vii) = 50 (viii) Associative, = 86 (ix) Associative, = -76

Please check — part (iii) cannot be proved, because it is not true. $-4 \times (3 + 2) = -20$, but $(-4 \times 3) - (-4 \times 2) = -4$. The sign between the two products should be a plus; with a plus it comes to -20 and the distributive property holds.

REVIEW EXERCISE

- i. The product of any integer and one is the integer itself. This multiplication property is called:

a) Commutative b) Identity c) Associative d) Distributive

An identity element leaves the other number unchanged.

For multiplication that element is 1, since $a \times 1 = a$.

So this is the multiplicative identity property.

(For addition the identity is 0, since $a + 0 = a$.)

Answer b) Identity

- ii. The sum of two integers is always a/an:

a) Integer b) Non-integer c) Fraction d) Decimal

Add any two integers and the result is another integer: $5 + 3 = 8$, $-5 + 3 = -2$, $-5 + (-3) = -8$.

There is no way to add two whole numbers and get a fraction.

This is the closure property — integers are closed under addition.

Answer a) Integer

- iii. The value of $5 \times (-3) - (-3)$ is:

a) -12 b) 18 c) -18 d) 12

Multiplication comes before subtraction.

$5 \times (-3) = -15$.

Then $-15 - (-3)$. Subtracting a negative adds: $-15 + 3 = -12$.

Answering -18 is the trap — that comes from subtracting 3 instead of adding it.

Answer a) -12

- iv. Multiplication over addition and subtraction is called:

a) Commutative b) Associative c) Distributive d) Identity

The distributive property shares a multiplier out over everything inside a bracket: $a \times (b + c) = ab + ac$.

Commutative is about changing the order; associative is about moving the brackets.

Only the distributive property involves two different operations at once, which is what the question describes.

Answer c) Distributive

- v. The product of two negative integers is always:

a) Positive b) Negative c) Zero d) Undefined

Two like signs give a plus.

$(-4) \times (-3) = +12$, $(-1) \times (-1) = +1$.

Positive — always, with no exception among the integers.

Answer a) Positive

- vi. The sum of any integer and zero is the integer itself. This addition property is called:

a) Associative b) Distributive c) Identity d) Commutative

$a + 0 = a$, so 0 leaves the number unchanged under addition.

That makes 0 the additive identity.

Compare question (i): 1 is the multiplicative identity, 0 the additive one. Mixing the two up is the commonest slip in this unit.

Answer c) Identity

2. State whether each is true or false: (i) $2 \times (3 + 5) = 2 \times 3 + 2 \times 5$ (ii) $3 \times (5 + (-2)) = 3 \times 5 + (-2)$ (iii) $0 \times (2 + 3) = 0 + 3$ (iv) $27 + (-27) = 54$

Work out both sides of each statement separately, then compare.

(i) LHS = 16, RHS = 16 → True — the distributive property, correctly applied.

(ii) LHS = 9, RHS = 13 → False — the 3 must multiply both terms; the right side multiplies only the 5. It should be $3 \times 5 + 3 \times (-2) = 9$.

(iii) LHS = 0, RHS = 3 → False — anything times zero is zero, so the left side is 0, not 3.

(iv) LHS = 0, RHS = 54 → False — adding a number to its own negative always gives 0, not double it.

Answer (i) True (ii) False (iii) False (iv) False

3. Match the columns: Identity element in addition · Commutative property · Distributive property · Closure property — with — $a + b = b + a$ · $a \times (b + c) = ab + ac$ · Zero · If a and b are integers then $(a + b)$ is also an integer

Take each property and recall what it states.

The identity element in addition is the number that changes nothing when added — Zero.

The commutative property is about order — $a + b = b + a$.

The distributive property shares a multiplier over a bracket — $a \times (b + c) = ab + ac$.

The closure property says the operation keeps you inside the set — if a and b are integers then $(a + b)$ is also an integer.

Answer Identity element in addition → Zero

Commutative property → $a + b = b + a$

Distributive property → $a \times (b + c) = ab + ac$

Closure property → If a and b are integers then $(a + b)$ is also an integer

■ SOLVE THE FOLLOWINGS

1. $(-4) + 7$

This is adding integers of unlike signs.

the signs are unlike, so subtract and keep the sign of the bigger number
bigger number without its sign: 7

$$7 - 4 = 3$$

7 is positive, so the answer is positive

$$= 3$$

Answer 3

2. $(3 + (-2)) + 5 = 3 + ((-2) + 5)$

This is the associative property of addition.

Left side — bracket first:

$$3 + (-2) = 3 - 2 = 1$$

$$1 + 5 = 6$$

Right side — bracket first:

$$(-2) + 5 = 5 - 2 = 3$$

$$3 + 3 = 6$$

Both sides come to 6, so the grouping made no difference

$$= 6$$

Answer 6

3. $8 \div (-2)$

This is the sign rule for division.

divide the numbers first: $8 \div 2 = 4$

one number is positive and one negative — unlike signs

unlike signs give a negative quotient

$$= -4$$

Answer -4

4. $4 + 0$

This is the additive identity.
adding zero changes nothing
 $4 + 0 = 4$
0 is called the additive identity for exactly this reason
 $= 4$

Answer 4

5. $12 + \underline{\hspace{1cm}} = 0$

This is the additive inverse.
the blank must cancel the 12 exactly
 $\underline{\hspace{1cm}} = 0 - 12$
 $\underline{\hspace{1cm}} = -12$
check: $12 + (-12) = 12 - 12 = 0 \checkmark$
 $= -12$

Answer -12

6. $(-4) \times (-3)$

This is the sign rule for multiplication.
multiply the numbers first: $4 \times 3 = 12$
both numbers are negative — like signs
like signs give a positive product
 $= 12$

Answer 12

7. $12 \div 4$

This is the sign rule for division.
divide the numbers: $12 \div 4 = 3$
both are positive — like signs
like signs give a positive quotient
 $= 3$

Answer 3

8. $(-5) \times 8$

This is the sign rule for multiplication.
multiply the numbers first: $5 \times 8 = 40$
one is negative and one positive — unlike signs
unlike signs give a negative product
 $= -40$

Answer -40

9. $(2 \times (-4)) \times 5 = 2 \times ((-4) \times 5)$

This is the associative property of multiplication.
Left side — bracket first:
 $2 \times (-4) = -8$
 $(-8) \times 5 = -40$
Right side — bracket first:
 $(-4) \times 5 = -20$
 $2 \times (-20) = -40$
Both sides come to -40, so the grouping made no difference
 $= -40$

Answer -40

10. 4×1

This is the multiplicative identity.
 multiplying by one changes nothing
 $4 \times 1 = 4$
 1 is called the multiplicative identity for exactly this reason
 $= 4$

Answer 4

11. $-3 \times \underline{\hspace{1cm}} = 1$

This is the multiplicative inverse.
 the blank must turn -3 into 1
 $\underline{\hspace{1cm}} = 1 \div (-3)$
 $\underline{\hspace{1cm}} = -1/3$
 check: $(-3) \times (-1/3) = 3/3 = 1 \checkmark$
 No integer works here, which is why the integers are not closed under division
 $= -1/3$

Answer $-1/3$

12. $64.85 + [3.2 \times \{3.2 \times 4.4 \div (3.3 - 1.1)\}]$

Work from the innermost bracket outward — BODMAS.
 Innermost: $3.3 - 1.1 = 2.2$.
 Then inside the braces, left to right: $3.2 \times 4.4 = 14.08$, and $14.08 \div 2.2 = 6.4$.
 Then the square bracket: $3.2 \times 6.4 = 20.48$.
 Finally: $64.85 + 20.48 = 85.33$.

Answer 85.33

13. $5.4318 \div [2.067 + \{3.12 \div (9.261 \div 3.32 + 1.34)\}]$

Innermost bracket first: $9.261 \div 3.32 = 2.7895$ (to 4 d.p.), then $+ 1.34 = 4.1295$.
 Braces: $3.12 \div 4.1295 = 0.7556$.
 Square bracket: $2.067 + 0.7556 = 2.8226$.
 Finally $5.4318 \div 2.8226 = 1.9244$ to 4 decimal places.
 Note this does not come out exactly, unlike every other question here — see the note below.

Answer ≈ 1.9244

Please check — this does not give a terminating decimal. Since the rest of the exercise is exact, one of the figures has probably been mistyped in the book.

14. $0.8 \times [4.7 \times \{0.6328 \div (0.1 + 0.01 + 0.001)\}]$

Innermost: $0.1 + 0.01 + 0.001 = 0.111$.
 Braces: $0.6328 \div 0.111 = 5.7009$ (to 4 d.p.).
 Square bracket: $4.7 \times 5.7009 = 26.7942$.
 Finally $0.8 \times 26.7942 = 21.4354$ to 4 decimal places.

Answer ≈ 21.4354

Please check — this also fails to terminate — 0.6328 does not divide by 0.111 exactly.

15. $200.14 - [1.3754 \div \{0.074 \div 8.25 - (5.231 + 1.07)\}]$

Innermost: $5.231 + 1.07 = 6.301$.
 Then $0.074 \div 8.25 = 0.00897$, and $0.00897 - 6.301 = -6.2920$ — a negative, which is the point of meeting it in this unit.
 Braces: $1.3754 \div (-6.2920) = -0.2186$.
 Finally $200.14 - (-0.2186) = 200.14 + 0.2186 = 200.3586$.
 Subtracting a negative adds — the rule from Unit 2, met again here with decimals.

Answer ≈ 200.3586

16. Mehmood had Rs 3,500 for meat shopping. He paid for 2.25 kg beef at Rs 950 per kg, 2.50 kg chicken at Rs 300 per kg and 0.75 kg mutton at Rs 1,200 per kg. What amount is left with him?

Work out each item, then total.

Beef: $2.25 \times 950 = \text{Rs } 2,137.50$

Chicken: $2.50 \times 300 = \text{Rs } 750.00$

Mutton: $0.75 \times 1,200 = \text{Rs } 900.00$

Total = Rs 3,787.50

But he had only Rs 3,500, and 3,787.50 is more than that.

$3,500 - 3,787.50 = -287.50$.

So nothing is left: he is Rs 287.50 short and cannot buy all three items.

Answer Nothing is left — the bill is Rs 3,787.50, so he is Rs 287.50 short

Please check — the question asks what is left, but the shopping costs more than the money given. Either the Rs 3,500 or one of the prices needs correcting — worth checking with your academics.

17. A man walks 3 km in 1 hour. How far can he travel in 18 seconds?

Step 1 Put both into the same units

3 km = 3,000 m

1 hour = $60 \times 60 = 3,600$ seconds

Step 2 Distance in one second

$3,000 \div 3,600 = 5/6$ m

Step 3 Distance in 18 seconds

$5/6 \times 18 = 90 \div 6$

= 15 m

Step 4 Check

18 s = $1/200$ of an hour

$3,000 \div 200 = 15$ m ✓

Answer 15 metres

18. A book has 240 pages. A student reads 10 pages on the first day, 14 on the second and 16 on the third. What fraction of the book has been read, and what fraction is left?

Step 1 Add the pages read

$10 + 14 = 24$

$24 + 16 = 40$ pages

Step 2 Fraction read

40 out of 240 = $40/240$

= $1/6$

Step 3 Pages left

$240 - 40 = 200$ pages

Step 4 Fraction left

200 out of 240 = $200/240$

= $5/6$

Step 5 Check

$1/6 + 5/6 = 1$ — the whole book ✓

Answer Read $1/6$, left $5/6$

19. If 7 sheets of paper weigh 28 grams, how many sheets would weigh 2.5 kg?

Step 1 Weight of one sheet

$$28 \div 7 = 4 \text{ grams}$$

Step 2 Put the target in the same unit

$$2.5 \text{ kg} = 2.5 \times 1,000$$

$$= 2,500 \text{ grams}$$

Step 3 Divide

$$2,500 \div 4 = 625 \text{ sheets}$$

Step 4 Check

$$625 \times 4 = 2,500 \text{ g} = 2.5 \text{ kg} \checkmark$$

Answer 625 sheets



4

CHAPTER

Ratio, Rate and Percentage

EXERCISE 4.1

1. Find the ratio of: (i) 48 and 64 (ii) 6.4 and 8 (iii) 36 kg and 80 kg (iv) Rs 27 and Rs 63 (v) 4 dozen and 3 dozen (vi) 60 cm and 3 m (vii) Rs 2 and Rs 80 (viii) 0.5 m and 75 cm

Two rules before simplifying. First, both quantities must be in the same unit — (vi) and (viii) mix metres with centimetres and must be converted first. Second, clear any decimals by multiplying both parts by 10.

Then divide both parts by their HCF.

(i) 48 and 64 → $48 : 64 = 3 : 4$

(ii) 6.4 and 8 → $64 : 80 = 4 : 5$ (multiply both by 10 to clear the decimal)

(iii) 36 kg and 80 kg → $36 : 80 = 9 : 20$

(iv) Rs 27 and Rs 63 → $27 : 63 = 3 : 7$

(v) 4 dozen and 3 dozen → $4 : 3 = 4 : 3$ (the dozens cancel — no need to work out 48 and 36)

(vi) 60 cm and 3 m → $60 : 300 = 1 : 5$ (3 m = 300 cm, so both are in centimetres)

(vii) Rs 2 and Rs 80 → $2 : 80 = 1 : 40$

(viii) 0.5 m and 75 cm → $50 : 75 = 2 : 3$ (0.5 m = 50 cm)

Answer (i) 3 : 4 (ii) 4 : 5 (iii) 9 : 20 (iv) 3 : 7 (v) 4 : 3 (vi) 1 : 5 (vii) 1 : 40 (viii) 2 : 3

2. Write each of the following in ratio form: (i) $(a-b)/(p+q)$ (ii) $34/37$ (iii) $53/59$ (iv) $x/(x+1)$ (v) x/y (vi) $101/103$ (vii) $3/5$ (viii) $7/12$

A fraction and a ratio say the same thing: the top becomes the first term and the bottom the second.

None of these simplifies further — 34 and 37 share no factor, nor do 53 and 59 or 101 and 103 (all four of those are prime).

Where the terms are algebraic, keep the brackets so the whole expression stays together.

(i) $(a-b)/(p+q) = (a-b) : (p+q)$

(ii) $34/37 = 34 : 37$

(iii) $53/59 = 53 : 59$

(iv) $x/(x+1) = x : (x+1)$

(v) $x/y = x : y$

(vi) $101/103 = 101 : 103$

(vii) $3/5 = 3 : 5$

(viii) $7/12 = 7 : 12$

Answer (i) $(a-b) : (p+q)$ (ii) 34 : 37 (iii) 53 : 59 (iv) $x : (x+1)$ (v) $x : y$ (vi) 101 : 103 (vii) 3 : 5 (viii) 7 : 12

3. An alloy has 13.5 grams of copper and 4.5 grams of zinc. What is the ratio of copper to zinc by weight?

Clear the decimals first by multiplying both by 10: 135 and 45.

$HCF(135, 45) = 45$.

$135 \div 45 = 3$ and $45 \div 45 = 1$, so the ratio is 3 : 1.

Check: there is three times as much copper as zinc, and 13.5 is indeed three times 4.5 ✓

Answer 3 : 1

4. Simplify the continued ratio 2 : 4 : 8.

Divide every term by the HCF of all three.

Step 1 Find the HCF

Factors of 2: 1, 2

Factors of 4: 1, 2, 4

Factors of 8: 1, 2, 4, 8

Highest shared = 2

Step 2 Divide each term

$$2 \div 2 = 1$$

$$4 \div 2 = 2$$

$$8 \div 2 = 4$$

$$1 : 2 : 4$$

Answer 1 : 2 : 4

5. Simplify the continued ratio 48 : 64 : 96.

Step 1 Find the HCF of all three terms

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Factors of 64: 1, 2, 4, 8, 16, 32, 64

Factors of 96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

Highest shared factor = 16

Step 2 Divide every term by it

$$48 \div 16 = 3$$

$$64 \div 16 = 4$$

$$96 \div 16 = 6$$

$$3 : 4 : 6$$

Answer 3 : 4 : 6

6. Write the continued ratio 3 : 6 : 12 in its simplest form.

Step 1 Find the HCF of all three terms

Factors of 3: 1, 3

Factors of 6: 1, 2, 3, 6

Factors of 12: 1, 2, 3, 4, 6, 12

Highest shared factor = 3

Step 2 Divide every term by it

$$3 \div 3 = 1$$

$$6 \div 3 = 2$$

$$12 \div 3 = 4$$

$$1 : 2 : 4$$

Answer 1 : 2 : 4

7. Write the continued ratio 180 : 240 : 300 in its simplest form.

Step 1 Find the HCF of all three terms

Factors of 180: 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180

Factors of 240: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 40, 48, 60, 80, 120, 240

Factors of 300: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300

Highest shared factor = 60

Step 2 Divide every term by it

$$180 \div 60 = 3$$

$$240 \div 60 = 4$$

$$300 \div 60 = 5$$

$$3 : 4 : 5$$

Answer 3 : 4 : 5

8. The continued ratio of three numbers is 5 : 10 : 15. Express it in its simplest form.

Step 1 Find the HCF of all three terms

Factors of 5: 1, 5

Factors of 10: 1, 2, 5, 10

Factors of 15: 1, 3, 5, 15

Highest shared factor = 5

Step 2 Divide every term by it

$$5 \div 5 = 1$$

$$10 \div 5 = 2$$

$$15 \div 5 = 3$$

$$1 : 2 : 3$$

Answer 1 : 2 : 3

9. Write the continued ratio 3 : 6 : 12 in its simplest form.

Divide every term by the HCF of all three.

Step 1 Find the HCF

factors of 3: 1, 3

factors of 6: 1, 2, 3, 6

factors of 12: 1, 2, 3, 4, 6, 12

the largest shared factor is 3, so HCF = 3

Step 2 Divide each term by 3

$$3 \div 3 = 1$$

$$6 \div 3 = 2$$

$$12 \div 3 = 4$$

$$1 : 2 : 4$$

Step 3 Check

1, 2 and 4 share no factor but 1, so it cannot be reduced further ✓

This is the same ratio as question 6.

Answer 1 : 2 : 4

Please check — this question repeats question 6 word for word. One of the two should probably be a different ratio.

10. If $a : b = 2 : \underline{\quad}$ and $b : c = 4 : 5$, find the continued ratio $a : b : c$.

The question cannot be answered as printed — the second term of $a : b$ is missing.

The method, though, is this: make the shared term (b) the same in both ratios, then join them.

If $a : b$ were $2 : 4$, then b already matches the 4 in $b : c = 4 : 5$, and $a : b : c = 2 : 4 : 5 = 2 : 4 : 5$.

If instead $a : b$ were $2 : 3$, you would scale $2 : 3$ by 4 and $4 : 5$ by 3 to make b equal 12 in both, giving $8 : 12 : 15$.

Answer Cannot be answered as printed. Reading $a : b = 2 : 4$ gives $2 : 4 : 5$

Please check — the first ratio is printed as ' $a : b = 2 :$ ' with nothing after the colon. The missing number needs restoring from the original manuscript.

11. The continued ratio $x : y : z$ is $3 : 4 : 5$. If $z = 20$, find x and y.

z is 5 parts and equals 20, so one part = $20 \div 5 = 4$.

$$x = 3 \text{ parts} = 3 \times 4 = 12.$$

$$y = 4 \text{ parts} = 4 \times 4 = 16.$$

$$\text{Check: } 12 : 16 : 20 = 3 : 4 : 5 \checkmark$$

Answer $x = 12$, $y = 16$

12. In school X, 275 out of 300 students received an A grade; in school Y, 120 out of 150 did. Which school has the better record?

The totals differ, so turn each into a percentage first.

Step 1 School X

$$275 \text{ out of } 300 = 275/300$$

$$= 11/12$$

$$= 275 \div 300 = 0.9167$$

$$= 91.67\%$$

Step 2 School Y

$$120 \text{ out of } 150 = 120/150$$

$$= 4/5$$

$$= 120 \div 150 = 0.8$$

$$= 80\%$$

Step 3 Compare

$$91.67\% > 80\%$$

So school X has the better record.

Answer School X, with about 91.67% against school Y's 80%

13. Distribute Rs 98,000 among Ali, Yasir and Shahid so that Ali : Yasir = 4 : 5 and Yasir : Shahid = 5 : 6.

Join the two ratios through the shared name, Yasir. He is 5 in both, so no scaling is needed.

$$\text{Ali : Yasir : Shahid} = 4 : 5 : 6.$$

$$\text{Total parts} = 4 + 5 + 6 = 15.$$

One part = $98,000 \div 15 = \text{Rs } 6,533.33$ — which is not a whole number of rupees.

Shares: Ali Rs 26,133.33, Yasir Rs 32,666.67, Shahid Rs 39,200.

Only Shahid's share comes out whole. See the note.

Answer Ali Rs 26,133.33, Yasir Rs 32,666.67, Shahid Rs 39,200

Please check — Rs 98,000 does not divide into 15 equal parts, so two of the three shares are recurring decimals. The method is right; the total appears to be mistyped. Rs 90,000 would give whole shares of Rs 24,000, Rs 30,000 and Rs 36,000.

14. Distribute Rs 48,000 among Sara, Zoya and Rabaal so that Sara : Zoya = 2 : 4 and Zoya : Rabaal = 3 : 5.

Here Zoya is 4 in the first ratio but 3 in the second, so they must be made to agree before joining.

The LCM of 4 and 3 is 12. Scale the first by 3: 6 : 12. Scale the second by 4: 12 : 20.

$$\text{So Sara : Zoya : Rabaal} = 6 : 12 : 20 = 3 : 6 : 10.$$

$$\text{Total parts} = 3 + 6 + 10 = 19.$$

One part = $48,000 \div 19 = \text{Rs } 2,526.32$ — again not whole.

Shares: Sara Rs 7,578.95, Zoya Rs 15,157.89, Rabaal Rs 25,263.16.

Answer Sara Rs 7,578.95, Zoya Rs 15,157.89, Rabaal Rs 25,263.16

Please check — Rs 48,000 does not divide into 19 equal parts. The joining of the two ratios is the part being tested and is shown in full; the total looks mistyped.

EXERCISE 4.2

1. A factory produces 1,800 widgets in 6 hours. How many in 15 hours at the same rate?

Find the rate for one hour first.

$$1,800 \div 6 = 300 \text{ widgets per hour.}$$

$$\text{In 15 hours: } 300 \times 15 = 4,500 \text{ widgets.}$$

Answer 4,500 widgets

2. Noor walked 2 km 500 m in 50 minutes. What was his rate of walking in kilometres per hour?

Step 1 Convert the distance to kilometres

$$500 \text{ m} = 500 \div 1,000 = 0.5 \text{ km}$$

$$2 \text{ km } 500 \text{ m} = 2 + 0.5 = 2.5 \text{ km}$$

Step 2 Convert the time to hours

$$50 \text{ min} = 50 \div 60 \text{ h}$$

$$= 5/6 \text{ h}$$

Step 3 Divide distance by time

$$\text{Rate} = 2.5 \div 5/6$$

$$= 2.5 \times 6/5$$

$$= 15 \div 5$$

$$= 3 \text{ km/h}$$

Step 4 Check
 $3 \times 50/60 = 2.5 \text{ km } \checkmark$

Answer 3 km/h

3. Water flows from a tap at 12 litres per minute. How much flows out in 25 minutes?

Step 1 Read the rate as an amount for one minute
 1 minute $\rightarrow$ 12 litres

Step 2 Multiply by the number of minutes

Amount = rate $\times$ time

$$= 12 \times 25$$

$$= 300 \text{ litres}$$

Step 3 Check

$$300 \div 25 = 12 \text{ litres a minute } \checkmark$$

Answer 300 litres

4. Abida worked 35 hours last week and earned Rs 39,995. (i) How much does she earn per hour? (ii) How much for 10 hours?

(i) Hourly rate = total $\div$ hours = $39,995 \div 35$.

= Rs 1,142.71 per hour — which does not come out to a whole number of rupees.

(ii) For 10 hours: $1,142.71 \times 10 = \text{Rs } 11,427.14$.

Or, more simply, 10 hours is $10/35 = 2/7$ of the week's work, so $2/7$ of 39,995 = Rs 11,427.14 — the same figure, reached without the recurring decimal.

Answer (i) Rs 1,142.71 per hour (ii) Rs 11,427.14

Please check — Rs 39,995 over 35 hours gives a recurring hourly rate, which is unusual for a wage. Rs 39,900 would give exactly Rs 1,140 an hour.

5. A jogger runs at 8 km/h. After 1.5 hours, how much farther to complete a 20 km marathon?

Distance covered = speed $\times$ time = $8 \times 1.5 = 12 \text{ km}$.

Remaining = $20 - 12 = 8 \text{ km}$.

The question asks how much farther, not how far he has run — so 12 km is the wrong answer.

Answer 8 km

6. If a person earns Rs 500 in 8 hours, what is the hourly wage?

A wage 'per hour' means the money for one hour, so divide the total by the hours.

Step 1 Set up the division

Hourly wage = total $\div$ hours

$$= 500 \div 8$$

Step 2 Divide

8 goes into 50 six times ($8 \times 6 = 48$), remainder 2

bring down the 0 $\rightarrow$ 20; 8 goes into 20 twice ($8 \times 2 = 16$), remainder 4

add a decimal point and a 0 $\rightarrow$ 40; $8 \times 5 = 40$, remainder 0

= Rs 62.50 per hour

Step 3 Check

$$62.50 \times 8 = 500 \checkmark$$

Answer Rs 62.50 per hour

7. A car travels 400 km at a constant 80 km/h. How long does the journey take?

Time = distance $\div$ speed.

$$= 400 \div 80 = 5 \text{ hours.}$$

$$\text{Check: } 80 \times 5 = 400 \text{ km } \checkmark$$

Answer 5 hours

8. A machine produces 150 items in 3 hours. What is its production rate per hour?

'Per hour' means the number made in one hour, so divide the items by the hours.

Step 1 Set up the division

Rate = items $\div$ hours

$$= 150 \div 3$$

Step 2 Divide

3 goes into 15 five times, so $150 \div 3 = 50$

= 50 items per hour

Step 3 Check

$$50 \times 3 = 150 \text{ items } \checkmark$$

Answer 50 items per hour

9. Akbar drove at an average rate of 7 km per hour. How far did he drive in 6 hours?

Distance = rate $\times$ time.

$$= 7 \times 6 = 42 \text{ km.}$$

Worth noticing: 7 km/h is walking pace. A car averaging 7 km/h would be a very unusual journey — see the note.

Answer 42 km

Please check — 7 km per hour is walking pace rather than a car's. The figure may be meant as 70 km/h, which would give 420 km. The method is the same either way.

10. If a car travels 240 km in 4 hours, what is its speed in km/h?

Speed is the distance covered in one hour, so divide the distance by the time.

Step 1 Set up the division

Speed = distance $\div$ time

$$= 240 \div 4$$

Step 2 Divide

4 goes into 24 six times, so $240 \div 4 = 60$

= 60 km/h

Step 3 Check

$$60 \times 4 = 240 \text{ km } \checkmark$$

Answer 60 km/h

EXERCISE 4.4

1. Write the percentage of the coloured part of each figure. (the figures are printed in the book)

Count the coloured parts and the total parts.

Percentage = (coloured $\div$ total) $\times$ 100.

If a figure is divided into 100 small squares the count is already the percentage; otherwise scale it up.

Answer Count the shaded parts out of the total and multiply by 100.

Please check — the figures are printed in the book, so the counts have to be read from the page.

2. Write these percentages as fractions in simplest form: (i) 25% (ii) 20% (iii) 84% (iv) 34% (v) 93%

Per cent means 'out of a hundred', so write the number over 100 and then simplify.

$$(i) 25\% = 25/100 = 1/4$$

$$(ii) 20\% = 20/100 = 1/5$$

$$(iii) 84\% = 84/100 = 21/25$$

$$(iv) 34\% = 34/100 = 17/50$$

$$(v) 93\% = 93/100 = 93/100$$

(v) 93/100 will not simplify — 93 is 3×31 and 100 is $2^2 \times 5^2$, so they share no factor.

Answer (i) $1/4$ (ii) $1/5$ (iii) $21/25$ (iv) $17/50$ (v) $93/100$

3. Solve: (i) 8% of 100 (ii) 15% of 60 litres (iii) 35% of 160 (iv) 80% of 900 (v) 8.9% of 900

'Of' means multiply, and a percentage is the number over 100.

(i) $8/100 \times 100 = 8$ — any percentage of 100 is just the percentage itself.

(ii) $15/100 \times 60 = 9$ litres.

(iii) $35/100 \times 160 = 56$.

(iv) $80/100 \times 900 = 720$.

(v) $8.9/100 \times 900 = 80.1$ — the decimal in the percentage causes no difficulty; treat it as $8.9 \div 100$.

Answer (i) 8 (ii) 9 (iii) 56 (iv) 720 (v) 80.1

4. There are 400 students in a school and 30% are left-handed. How many are right-handed?

If 30% are left-handed, the rest are right-handed: $100\% - 30\% = 70\%$.

70% of 400 = $70/100 \times 400 = 280$.

Check: 30% of 400 = 120 left-handed, and $120 + 280 = 400$ ✓

Answering 120 is the trap — that is the left-handed count, not what was asked.

Answer 280 students

5. 20% of a school's students were absent one day. If 1,200 were present, find the total number of students.

Step 1 Find what percentage 1,200 is

Absent = 20%

Present = $100\% - 20\% = 80\%$

So $80\% = 1,200$ students

Step 2 Find 1%

$1\% = 1,200 \div 80$

= 15 students

Step 3 Scale up to 100%

$100\% = 15 \times 100$

= 1,500 students

Step 4 Check

20% of 1,500 = 300 absent

$1,500 - 300 = 1,200$ present ✓

Answer 1,500 students

REVIEW EXERCISE

- i. If 5 books cost Rs 725, how much will 9 books cost?

a) Rs 6,525 b) Rs 3,625 c) Rs 1,305 d) Rs 145

Find the cost of one book first: $725 \div 5 = \text{Rs } 145$.

Then 9 books: $145 \times 9 = \text{Rs } 1,305$.

Rs 145 is the cost of one book — the trap for anyone who stops a step early.

Answer c) Rs 1,305

- ii. At a zoo there are 4 lions and 2 tigers. What is the simplified ratio of lions to tigers?

a) 4 : 6 b) 2 : 4 c) 2 : 1 d) 1 : 2

Lions to tigers is 4 : 2 — in that order, because the question names lions first.

HCF(4, 2) = 2, so divide both: 2 : 1.

1 : 2 is the same ratio the wrong way round, which would be tigers to lions.

Answer c) 2 : 1

iii. Change 125 g : 1 kg into a common unit and simplify:

- a) 1 : 8 b) 25 : 200 c) 8 : 1 d) 5 : 40

Convert first: 1 kg = 1,000 g, so the ratio is 125 : 1,000.

HCF(125, 1000) = 125.

$125 \div 125 : 1,000 \div 125 = 1 : 8$.

25 : 200 and 5 : 40 are both equal to 1 : 8 but are not in simplest form; 8 : 1 is the reverse.

Answer a) 1 : 8

Please check — the book letters two of these options (c). They are lettered a to d above.

iv. Ratio 1 = 3 : 5 : 8, Ratio 2 = 12 : 20 : 32, Ratio 3 = 9 : 25 : 64. Which statement is true?

- a) Ratio 1 = Ratio 3 b) Ratio 1 = Ratio 2 = Ratio 3 c) Ratio 3 = Ratio 2 d) Ratio 1 = Ratio 2

Simplify each and compare.

Ratio 2: HCF(12, 20, 32) = 4, so it reduces to 3 : 5 : 8 — the same as Ratio 1 ✓

Ratio 3 is 9 : 25 : 64, which is each of 3, 5, 8 squared. Squaring does not preserve a ratio, so it is not equal to the others.

Ratio 1 = Ratio 2.

Answer d) Ratio 1 = Ratio 2

vi. To make a chocolate milkshake, mix 1 cup of whole milk and 2 cups of chocolate ice cream. Which statement is correct?

- a) Ice cream : Milk = 2 : 1 b) Milk : Ice cream = 1 : 2 c) Ice cream : Milk = 1 : 2 d) Milkshake : Ice cream = 2 : 1

There is 1 cup of milk and 2 of ice cream.

Ice cream : Milk = 2 : 1 ✓ — and Milk : Ice cream = 1 : 2 ✓ as well.

Both (a) and (b) are correct statements of the same fact. Option (c) reverses it wrongly, and (d) is wrong because the whole milkshake is 3 cups, so Milkshake : Ice cream = 3 : 2.

The question as printed has two right answers.

Answer a) Ice cream : Milk = 2 : 1

Please check — options (a) and (b) are both true — they state the same ratio from each side. Only one should be offered.



5

CHAPTER

Sets

EXERCISE 5.1

1. Write the following sets in tabular form: (i) M = the names of the Presidents of Pakistan (ii) A = multiples of 3 (iii) D = planets in the solar system (iv) F = days of the week (v) Q = names of three countries beginning with B

Tabular form means listing the members inside braces, separated by commas.

Where the members never run out, write the first few and then three dots — that is how an infinite set is shown.

(i) M = the names of the Presidents of Pakistan

{Iskander Mirza, Ayub Khan, Yahya Khan, Zulfikar Ali Bhutto, Fazal Ilahi Chaudhry, Zia-ul-Haq, Ghulam Ishaq Khan, Farooq Leghari, Rafiq Tarar, Pervez Musharraf, Asif Ali Zardari, Mamnoon Hussain, Arif Alvi, ...} (this is a finite set, but a long one, so the names are listed and the list closed)

(ii) A = multiples of 3

{3, 6, 9, 12, 15, ...} (the three dots are essential — the multiples never stop, so the set is infinite and cannot be closed with a brace after a last member)

(iii) D = planets in the solar system

{Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune} (eight members, all listable)

(iv) F = days of the week

{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} (seven members)

(v) Q = names of three countries beginning with B

{Bangladesh, Brazil, Belgium} (any three will do — Bahrain, Bhutan and Belarus would be equally correct)

Answer (i) {Iskander Mirza, Ayub Khan, Yahya Khan, Zulfikar Ali Bhutto, Fazal Ilahi Chaudhry, Zia-ul-Haq, Ghulam Ishaq Khan, Farooq Leghari, Rafiq Tarar, Pervez Musharraf, Asif Ali Zardari, Mamnoon Hussain, Arif Alvi, ...}

(ii) {3, 6, 9, 12, 15, ...}

(iii) {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}

(iv) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

(v) {Bangladesh, Brazil, Belgium}

2. Write the following sets in descriptive form: (i) $P = \{a, b, c, \dots, z\}$ (ii) $X = \{\text{violet, indigo, blue, green, yellow, orange, red}\}$ (iii) $R = \{1, 3, 5, \dots, 9\}$ (iv) $A = \{4, 8, 12, 16, \dots\}$ (v) $C = \{3, 6, 9, \dots\}$ (vi) $M = \{2\}$ (vii) $E = \{21, 28, 35, 42, 49\}$ (viii) $N = \{6, 12, 18\}$

Descriptive form means saying in words what the members have in common, instead of listing them.

Look for the rule: are they consecutive? multiples of something? members of a known group?

(iii) and (v) both end in dots, but they mean different things — (iii) stops at 9, so it is finite, while (v) goes on for ever.

(i) $P = \{a, b, c, \dots, z\} \rightarrow P$ is the set of letters of the English alphabet

(ii) $X = \{\text{violet, indigo, blue, green, yellow, orange, red}\} \rightarrow X$ is the set of colours of the rainbow

(iii) $R = \{1, 3, 5, \dots, 9\} \rightarrow R$ is the set of odd numbers from 1 to 9

(iv) $A = \{4, 8, 12, 16, \dots\} \rightarrow A$ is the set of multiples of 4

(v) $C = \{3, 6, 9, \dots\} \rightarrow C$ is the set of multiples of 3

(vi) $M = \{2\} \rightarrow M$ is the set of even prime numbers — 2 is the only one

(vii) $E = \{21, 28, 35, 42, 49\} \rightarrow E$ is the set of multiples of 7 from 21 to 49

(viii) $N = \{6, 12, 18\} \rightarrow N$ is the set of the first three multiples of 6

Answer (i) P is the set of letters of the English alphabet
 (ii) X is the set of colours of the rainbow
 (iii) R is the set of odd numbers from 1 to 9
 (iv) A is the set of multiples of 4
 (v) C is the set of multiples of 3
 (vi) M is the set of even prime numbers — 2 is the only one
 (vii) E is the set of multiples of 7 from 21 to 49
 (viii) N is the set of the first three multiples of 6

■ EXERCISE 5.2

1. Separate the empty and singleton sets: (i) odd numbers between 31 and 33 (ii) even numbers less than 0 (iii) monkeys having five legs (iv) months starting with 'B' (v) capital cities of Pakistan (vi) days of a week starting with 'M' (vii) natural numbers less than 1 (viii) even numbers between 100 and 150

An empty set has no members at all; a singleton has exactly one. A set with two or more is neither.

Test each by trying to list its members.

- (i) The set of odd numbers between 31 and 33 — the only whole number strictly between 31 and 33 is 32, which is even, so no odd number qualifies. Empty.
 (ii) The set of even numbers less than 0 — -2, -4, -6 and so on without end — negative numbers can be even too, so this is neither empty nor a singleton but infinite. Neither.
 (iii) The set of monkeys having five legs — no such monkey exists. Empty.
 (iv) The set of months of a year starting with 'B' — no month begins with B. Empty.
 (v) The set of capital cities of Pakistan — Pakistan has exactly one capital. Singleton.
 (vi) The set of days of a week starting with 'M' — only Monday begins with M. Singleton.
 (vii) The set of natural numbers less than 1 — the natural numbers start at 1, so none is below it. Empty.
 (viii) The set of even numbers between 100 and 150 — 102, 104, ... 148 — twenty-four members, so neither empty nor a singleton. Neither.

Answer Empty: (i), (iii), (iv), (vii) · Singleton: (v), (vi) · Neither: (ii) infinite, (viii) 24 members

2. Which pairs of sets are equal or equivalent? Write them in symbolic form. (i) $M = \{2, 4, 6, 9\}$, $N = \{2, 4, 9, 6\}$ (ii) $F = \{a, b, c, d, e\}$, $G = \{a, c, d, e, b\}$ (iii) $X = \{1, II, III, V\}$, $Y = \{III, II, I, V\}$ (iv) $P = \{a, b, c, d, e\}$, $Q = \{1, 2, 3, 4, 5\}$ (v) suns on Earth and even numbers between 31 and 33 (vi) natural numbers less than 1 and vowels starting with 'k' (vii) months starting with 'O' and even numbers less than 0

Two sets are equal when they have exactly the same members, written $=$. They are equivalent when they merely have the same number of members, written $\leftrightarrow$.

Equal sets are always equivalent; equivalent sets need not be equal.

(i) $M = \{2, 4, 6, 9\}$ and $N = \{2, 4, 9, 6\}$ — the same four members, only written in a different order — and order never matters in a set. $M = N$

(ii) $F = \{a, b, c, d, e\}$ and $G = \{a, c, d, e, b\}$ — again the same five members reordered. $F = G$

(iii) $X = \{1, II, III, V\}$ and $Y = \{III, II, I, V\}$ — both have four members, so they are equivalent ($X \leftrightarrow Y$). Whether they are equal depends on whether X's '1' is meant to be the Roman I — see the note

(iv) $P = \{a, b, c, d, e\}$ and $Q = \{1, 2, 3, 4, 5\}$ — different members, but five in each, so $P \leftrightarrow Q$ — equivalent but not equal

(v) The set of suns on the Earth and the set of even numbers between 31 and 33 — the first has one member (the Sun); the second has none, since 32 is the only number between them and it is even — so the second is empty. One member against none, so they are neither equal nor equivalent

(vi) The set of natural numbers less than 1 and the set of vowels starting with 'k' — both are empty, and all empty sets are equal. $=$

(vii) The set of months starting with 'O' and the set of even numbers less than 0 — the first is {October}, a singleton; the second is infinite. Neither equal nor equivalent

Answer (i) $M = N$ · (ii) $F = G$ · (iii) $X \leftrightarrow Y$ · (iv) $P \leftrightarrow Q$ · (v) neither · (vi) equal (both empty) · (vii) neither

Please check — parts (viii), (ix) and (x) in the book repeat (i), (ii) and (iii) word for word, so there are really seven pairs, not ten. In (iii), X is printed as $\{1, II, III, V\}$ — the Arabic 1 among Roman numerals, probably meant to be I.

3. Separate the finite and infinite sets: (i) stars in the universe (ii) fingers in a hand (iii) planets in our solar system (iv) all prime numbers (v) all even numbers (vi) neutral integers

A set is finite if its members can be counted to an end, and infinite if the counting never stops.

(i) Set of stars in the universe — infinite, they cannot be counted to an end.

(ii) Set of fingers in a hand — finite, five.

(iii) Set of planets in our solar system — finite, eight.

(iv) Set of all prime numbers — infinite, the primes never run out — a fact proved by Euclid.

(v) Set of all even numbers — infinite, they continue in both directions without end.

(vi) Set of neutral integers — finite, zero is the only integer that is neither positive nor negative, so this is a singleton — $\{0\}$, with one member.

Answer Finite: (ii), (iii), (vi) · Infinite: (i), (iv), (v)

EXERCISE 5.3

1. Draw a Venn diagram for: (i) names of 6 boys starting with M and names of 6 girls starting with A (ii) students who live in apartments and those who live in houses (iii) animals that eat plants and animals that eat meat and plants (iv) four presidents of Pakistan and four presidents of India (v) five cities in Sindh and five cities in KPK

The shape of the diagram depends on whether the two sets share any members.

(i), (iv) and (v) are disjoint — no name, president or city can be in both — so draw two separate circles that do not touch.

(ii) is disjoint too, since a student lives in one or the other.

(iii) overlaps: animals that eat meat and plants also eat plants, so the second set sits partly inside the first. Draw two circles that cross, with the omnivores in the overlap.

Label every circle and put the members inside.

Answer Disjoint circles for (i), (ii), (iv) and (v); overlapping circles for (iii), with the omnivores in the intersection.

2. Use $\subset$ or $\subseteq$ in the blanks: (i) $\{3,5,7\}$ ____ $\{1,2,3,\dots,10\}$ (ii) $\{2,4,6\}$ ____ $\{2,4,6\}$ (iii) $\{0,1,2\}$ ____ $\{0,1,2,\dots\}$

$\subseteq$ means 'is a subset of', and allows the two sets to be equal. $\subset$ means 'is a proper subset' — a subset that is definitely smaller.

So the question is whether the two sets could be the same.

(i) $\{3, 5, 7\} \subset \{1, 2, 3, \dots, 10\}$ — every member of the first is in the second, and the second has others besides, so it is a proper subset

(ii) $\{2, 4, 6\} \subseteq \{2, 4, 6\}$ — the two sets are identical. A set is a subset of itself, but not a proper one, so $\subseteq$ is correct and $\subset$ is not

(iii) $\{0, 1, 2\} \subset \{0, 1, 2, \dots\}$ — the second set continues without end, so it has members the first does not — a proper subset again

Answer (i) $\subset$ (ii) $\subseteq$ (iii) $\subset$

3. Find the total number of subsets of: (i) $\{2,3\}$ (ii) $\{-1,-2\}$ (iii) $\{7,9,2,3\}$ (iv) $\{-1,-2,-3,-4\}$

A set with n members has 2^n subsets, because each member is either in a given subset or out of it — two choices, made n times.

Counting them first tells you when your list is complete.

(i) 2 members $\rightarrow 2^2 = 4$ subsets

(ii) 2 members $\rightarrow 2^2 = 4$ subsets

(iii) 4 members $\rightarrow 2^4 = 16$ subsets

(iv) 4 members $\rightarrow 2^4 = 16$ subsets

Answer (i) 4 (ii) 4 (iii) 16 (iv) 16

4. Find all possible subsets of: (i) $\{1\}$ (ii) $\{0\}$ (iii) $\{0,1\}$ (iv) $\{+, -, \times\}$ (v) $\{-1, -5, 7, 9\}$

List them by size: the empty set first, then the singletons, then the pairs, and so on up to the whole set.

Working in that order is what stops one being missed.

Remember both ends: the empty set and the set itself are both subsets.

(i) 1 members $\rightarrow 2$ subsets: $\{\}, \{1\}$

(ii) 1 members $\rightarrow 2$ subsets: $\{\}, \{0\}$

(iii) 2 members $\rightarrow 4$ subsets: $\{\}, \{0\}, \{1\}, \{0, 1\}$

(iv) 3 members $\rightarrow 8$ subsets: $\{\}, \{+\}, \{-\}, \{\times\}, \{+, -\}, \{+, \times\}, \{-, \times\}, \{+, -, \times\}$

(v) 4 members $\rightarrow 16$ subsets: $\{\}, \{-1\}, \{-5\}, \{7\}, \{9\}, \{-1, -5\}, \{-1, 7\}, \{-1, 9\}, \{-5, 7\}, \{-5, 9\}, \{7, 9\}, \{-1, -5, 7\}, \{-1, -5, 9\}, \{-1, 7, 9\}, \{-5, 7, 9\}, \{-1, -5, 7, 9\}$

Answer (i) $\{\}, \{1\}$

(ii) $\{\}, \{0\}$

(iii) $\{\}, \{0\}, \{1\}, \{0, 1\}$

(iv) $\{\}, \{+\}, \{-\}, \{\times\}, \{+, -\}, \{+, \times\}, \{-, \times\}, \{+, -, \times\}$

(v) $\{\}, \{-1\}, \{-5\}, \{7\}, \{9\}, \{-1, -5\}, \{-1, 7\}, \{-1, 9\}, \{-5, 7\}, \{-5, 9\}, \{7, 9\}, \{-1, -5, 7\}, \{-1, -5, 9\}, \{-1, 7, 9\}, \{-5, 7, 9\}, \{-1, -5, 7, 9\}$

5. Draw the Venn diagram for: (i) $U = \{1,2,3,4,12\}$, $A = \{4,5,6,7,9,12\}$, $B = \{6,9,12\}$ (ii) $U = \{1,3,5,\dots,21\}$, $A = \{1,3,5,7,9\}$, $B = \{13,15\}$ (iii) $U = \{2,4,6,\dots,301\}$, $A = \{6,8,10,12,18,22,26\}$

Draw the universal set U as a rectangle, and every other set as a circle inside it.

(i) cannot be drawn as printed: A contains 5, 6, 7 and 9, none of which is in U . A set can never have a member that the universal set lacks — see the note.

(ii) works: $A = \{1,3,5,7,9\}$ and $B = \{13,15\}$ are both inside U , and they share no member, so draw two separate circles inside the rectangle.

(iii) $A = \{6,8,10,12,18,22,26\}$ sits inside U as one circle. Note U is described as even numbers but ends at 301, which is odd.

Answer (i) cannot be drawn as printed · (ii) two disjoint circles inside U · (iii) one circle inside U

Please check — in (i) the set A contains 5, 6, 7 and 9, which are not members of $U = \{1,2,3,4,12\}$. Every set must lie inside the universal set, so this diagram is impossible as printed. In (iii) the universal set lists even numbers but ends at 301, an odd number — it should end at 300.



6

CHAPTER

Introduction to Algebra

EXERCISE 6.1

1. Complete the number sequences: (i) 10, 20, 30, ... (ii) 300, 290, 280, 270, ... (iii) 1000, 950, 900, 850, 800, ...
(iv) 3, 3, -3, 3, -3, ... (v) 17, -12, -7, -2, 3, ... (vi) 5000, 1000, 200, ...

Find the rule first by comparing one term with the next — look for a constant difference or a constant ratio. Then apply it, one term at a time.

- (i) 10, 20, 30, ...

$20 - 10 = 10$, so the rule is $+ 10$ each time

$$30 + 10 = 40$$

$$40 + 10 = 50$$

$$50 + 10 = 60$$

- (ii) 300, 290, 280, 270, ...

$290 - 300 = -10$, so the rule is $- 10$ each time

$$270 - 10 = 260$$

$$260 - 10 = 250$$

$$250 - 10 = 240$$

- (iii) 1,000, 950, 900, 850, 800, ...

$950 - 1,000 = -50$, so the rule is $- 50$ each time

$$800 - 50 = 750$$

$$750 - 50 = 700$$

$$700 - 50 = 650$$

- (iv) 3, 3, -3, 3, -3, ...

This one alternates rather than steps: after the opening 3 the terms run 3, -3, 3, -3 the next three keep alternating: 3, -3, 3

- (v) -17, -12, -7, -2, 3 (see the note)

$-12 - -17 = 5$, so the rule is $+ 5$ each time

$$3 + 5 = 8$$

$$8 + 5 = 13$$

$$13 + 5 = 18$$

- (vi) 5000, 1000, 200, ...

$1000 \div 5000 = 1/5$ — a constant ratio, not a difference

$$200 \times 1/5 = 40$$

$$40 \times 1/5 = 8$$

$$8 \times 1/5 = 1.6$$

Answer (i) 40, 50, 60 · (ii) 260, 250, 240 · (iii) 750, 700, 650 · (iv) 3, -3, 3 · (v) 8, 13, 18 · (vi) 40, 8, 1.6

Please check — part (v) is printed as 17, -12, -7, -2, 3. The jump from 17 to -12 is -29 while every later step is +5, so the first term is almost certainly -17. It is read that way above.

2. There are 50 chairs in room 1, 100 in room 2 and 150 in room 3. Find the number of chairs in room 6.

The rooms go up by 50 chairs each time: $100 - 50 = 50$.

So room n holds $50 \times n$ chairs.

Room 6: $50 \times 6 = 300$ chairs.

Check by continuing the list: room 4 has 200, room 5 has 250, room 6 has 300 ✓

Answer 300 chairs

3. Naveed walked 50 steps on the first day, 90 on the second and 130 on the third. Using the same pattern, how many steps on the seventh day?

The difference is $90 - 50 = 40$ steps a day.

Starting at 50 on day 1, day n gives $50 + 40 \times (n - 1)$.

Day 7: $50 + 40 \times 6 = 50 + 240 = 290$ steps.

Check by listing: 50, 90, 130, 170, 210, 250, 290 ✓

Note it is $(n - 1)$, not n — day 1 has had no increase yet. Using 40×7 would give 330, the commonest error here.

Answer 290 steps

EXERCISE 6.2

1. The general terms of sequences are given. Find the first three terms of each: (i) $2n + 5$ (ii) $n^3 - 2$ (iii) $2/(2n - 1)$ (iv) $3n/2$

The general term is a formula in n , where n is the position — so the first term is found by putting $n = 1$, the second by $n = 2$, and so on.

Substitute carefully and work out the powers before the additions.

(i) $2n + 5$: put $n = 1, 2, 3 \rightarrow 7, 9, 11$

(ii) $n^3 - 2$: put $n = 1, 2, 3 \rightarrow -1, 6, 25$

(iii) $2/(2n - 1)$: put $n = 1, 2, 3 \rightarrow 2, 0.666667, 0.4$

(iv) $3n/2$: put $n = 1, 2, 3 \rightarrow 1.5, 3, 4.5$

Answer (i) 7, 9, 11

(ii) -1, 6, 25

(iii) 2, 0.666667, 0.4

(iv) 1.5, 3, 4.5

2. Observe each sequence and find the term asked for: (i) 50, 45, 40, 35, ... 12th term (ii) 2, 6, 18, 54, ... 6th (iii) 1, 4, 7, 10, 13, ... 15th (iv) 10, 5, 2.5, 1.25, ... 6th (v) 12, 18, 24, 30, 36, ... 8th (vi) 500, 450, 400, 350, ... 15th (vii) 20, 30, 40, 50, 60, ... 13th (viii) 1, 4, 9, 16, 25, ... 10th

First decide whether the sequence adds the same amount each time (arithmetic) or multiplies by the same amount (geometric). Subtract one term from the next to test.

For an arithmetic sequence the n th term is $\text{first} + \text{difference} \times (n - 1)$.

For a geometric one it is $\text{first} \times \text{ratio}^{(n-1)}$.

Counting the steps as $(n - 1)$ rather than n is what most answers turn on.

(i) 50, 45, 40, 35, ... — find the 12th term

$45 - 50 = -5$, so arithmetic with first = 50, difference = -5

12th = $\text{first} + \text{difference} \times (n - 1)$

$= 50 + (-5) \times (12 - 1)$

$= 50 + (-5) \times 11$

$= 50 - 55$

$= -5$

12th term = -5

(ii) 2, 6, 18, 54, ... — find the 6th term

$6 \div 2 = 3$, so geometric with first = 2, ratio = 3

6th = $\text{first} \times \text{ratio}^{(n-1)}$

$= 2 \times 3^{(6-1)}$

$= 2 \times 3^5$

$= 2 \times 243$

$= 486$

6th term = 486

(iii) 1, 4, 7, 10, 13, ... — find the 15th term

$4 - 1 = 3$, so arithmetic with first = 1, difference = 3

15th = $1 + 3 \times (15 - 1)$

$$\begin{aligned}
 &= 1 + 3 \times 14 \\
 &= 1 + 42 \\
 &= 43 \\
 &\text{15th term} = 43 \\
 \text{(iv) } &10, 5, 2.5, 1.25, \dots \text{ — find the 6th term} \\
 &5 \div 10 = \frac{1}{2}, \text{ so geometric — each term is half the one before} \\
 &6\text{th} = 10 \times \left(\frac{1}{2}\right)^{(6-1)} \\
 &= 10 \div 2^5 \\
 &= 10 \div 32 \\
 &= 0.3125 \\
 &6\text{th term} = 0.3125 \\
 \text{(v) } &12, 18, 24, 30, 36, \dots \text{ — find the 8th term} \\
 &18 - 12 = 6, \text{ so arithmetic with first} = 12, \text{ difference} = 6 \\
 &8\text{th} = 12 + 6 \times (8 - 1) \\
 &= 12 + 6 \times 7 \\
 &= 12 + 42 \\
 &= 54 \\
 &8\text{th term} = 54 \\
 \text{(vi) } &500, 450, 400, 350, \dots \text{ — find the 15th term} \\
 &450 - 500 = -50, \text{ so arithmetic with first} = 500, \text{ difference} = -50 \\
 &15\text{th} = 500 + (-50) \times (15 - 1) \\
 &= 500 + (-50) \times 14 \\
 &= 500 - 700 \\
 &= -200 \\
 &15\text{th term} = -200 \\
 \text{(vii) } &20, 30, 40, 50, 60, \dots \text{ — find the 13th term} \\
 &30 - 20 = 10, \text{ so arithmetic with first} = 20, \text{ difference} = 10 \\
 &13\text{th} = 20 + 10 \times (13 - 1) \\
 &= 20 + 10 \times 12 \\
 &= 20 + 120 \\
 &= 140 \\
 &13\text{th term} = 140 \\
 \text{(viii) } &1, 4, 9, 16, 25, \dots \text{ — find the 10th term} \\
 &1 = 1^2, 4 = 2^2, 9 = 3^2, 16 = 4^2 \text{ — these are the perfect squares, so the } n\text{th term is } n^2 \\
 &10\text{th} = 10^2 \\
 &= 10 \times 10 \\
 &= 100 \\
 &10\text{th term} = 100
 \end{aligned}$$

Answer (i) -5 (ii) 486 (iii) 43 (iv) 0.3125 (v) 54 (vi) -200 (vii) 140 (viii) 100

EXERCISE 6.3

1. Mark the following as true, false or open: (i) Five is greater than three (ii) Sunday is a working day (iii) Saudi Arabia is a Muslim country (iv) $8 \times 6 = 62$ (v) $9 \times () = 45$ (vi) $2 \times 3 = 6$ (vii) $() \times 4 = 28$ (viii) 18 is an even number (ix) Ten multiplied by five equals fifty (x) $7 + 4 = 10$

A statement is true or false if it can be judged as it stands.

It is open if it contains a blank or a variable, so that it cannot be judged until the blank is filled. An open sentence is neither true nor false by itself — that is the whole idea this exercise is introducing.

- (i) Five is greater than three

$5 > 3$, and this can be decided, so it is a statement with a definite truth value

True

- (ii) Sunday is a working day

- Sunday is a holiday
False
- (iii) Saudi Arabia is a Muslim country
a matter of fact
True
- (iv) $8 \times 6 = 62$
 $8 \times 6 = 48$, not 62
False
- (v) $9 \times () = 45$
it cannot be judged until the blank is filled. With 5 it is true, with anything else false — so it is an open sentence
Open
- (vi) $2 \times 3 = 6$
correct arithmetic
True
- (vii) $() \times 4 = 28$
again it depends on the blank; 7 makes it true
Open
- (viii) 18 is an even number
18 divides by 2 exactly
True
- (ix) Ten multiplied by five equals fifty
 $10 \times 5 = 50$
True
- (x) $7 + 4 = 10$
 $7 + 4 = 11$
False

Answer (i) True (ii) False (iii) True (iv) False (v) Open (vi) True (vii) Open (viii) True (ix) True (x) False

2. Find the number which satisfies each statement: (i) $() + 7 = 13$ (ii) $4 - () = 2$ (iii) $() - 8 = 18$ (iv) $95 - () = 70$ (v) $8 \times () = 48$ (vi) $() + \underline{\quad} = 10$ (vii) $4 \times () = 36$ (viii) $55 - () = 40$ (ix) $0.7 - () = 0.2$ (x) $60 \div () = 12$

Each is an open sentence; the task is to find the value that makes it true.

Undo whatever has been done to the blank — the opposite operation.

- (i) $() + 7 = 13$
subtract 7 from both sides
 $() = 13 - 7$
 $() = 6$
- (ii) $4 - () = 2$
the amount taken from 4 to leave 2
 $() = 4 - 2$
 $() = 2$
- (iii) $() - 8 = 18$
add 8 to both sides
 $() = 18 + 8$
 $() = 26$
- (iv) $95 - () = 70$
the amount removed from 95
 $() = 95 - 70$
 $() = 25$
- (v) $8 \times () = 48$
divide both sides by 8
 $() = 48 \div 8$
 $() = 6$

- (vi) $() + \underline{\hspace{1cm}} = 10$
 this part is incomplete as printed
 cannot be answered as printed
- (vii) $4 \times () = 36$
 divide both sides by 4
 $() = 36 \div 4$
 $() = 9$
- (viii) $55 - () = 40$
 the amount removed from 55
 $() = 55 - 40$
 $() = 15$
- (ix) $0.7 - () = 0.2$
 $0.7 - 0.5 = 0.2$
 $() = 0.7 - 0.2$
 $() = 0.5$
- (x) $60 \div () = 12$
 the number that divides 60 into 12
 $() = 60 \div 12$
 $() = 5$

Answer (i) 6 (ii) 2 (iii) 26 (iv) 25 (v) 6 (vi) — (vii) 9 (viii) 15 (ix) 0.5 (x) 5

Please check — part (vi) is printed as ' $() + = 10$ ' — a number is missing between the plus and the equals sign, so it cannot be answered.

3. Separate the variables and constants: (i) $3a + 2b + c$ (ii) $3x - y - 15$ (iii) $3x + 2y + 117$ (iv) $9/4$ (v) $x + 5$ (vi) $x + y + z$ (vii) $3l + 7m + 3$ (viii) $2p + 3q + r$

A variable is a letter — its value can change. A constant is a fixed number.

The number multiplying a variable is its coefficient; it is a constant, but it is attached to the variable rather than standing alone.

(iv) has no letter at all, so it is purely constant.

- (i) $3a + 2b + c$
 variables: a, b, c
 constants: 3, 2 (and the 1 understood before c)

- (ii) $3x - y - 15$
 variables: x, y
 constants: 3, 1 and 15

- (iii) $3x + 2y + 117$
 variables: x, y
 constants: 3, 2 and 117

- (iv) $9/4$
 variables: none
 constants: 9 and 4 — this is a constant, with no letter in it

- (v) $x + 5$
 variables: x
 constants: 1 and 5

- (vi) $x + y + z$
 variables: x, y, z
 constants: the understood 1s

- (vii) $3l + 7m + 3$
 variables: l, m
 constants: 3, 7 and 3

- (viii) $2p + 3q + r$
 variables: p, q, r
 constants: 2, 3 and the understood 1

Answer (i) variables a, b, c
 (ii) variables x, y
 (iii) variables x, y
 (iv) variables none
 (v) variables x
 (vi) variables x, y, z
 (vii) variables l, m
 (viii) variables p, q, r

■ EXERCISE 6.3 (SECOND)

1. Write the following as algebraic expressions: (i) the sum of $3a$ and 8 (ii) difference of $5p$ and 5 (iii) divide $5a$ by $8b$ (iv) product of a number n and four times a variable y (v) cost of n litres of cooking oil at Rs 370 per litre

Translate the words one phrase at a time. 'Sum' means add, 'difference' means subtract, 'product' means multiply, 'divide' means divide.

(iv) 'four times a variable y ' is $4y$, and the product of n with it is $n \times 4y$, written $4ny$ — number first, then letters in alphabetical order.

(v) Each litre costs Rs 370, so n litres cost $370 \times n$, written Rs $370n$.

(i) The sum of $3a$ and 8

$$3a + 8$$

(ii) Difference of $5p$ and 5

$$5p - 5$$

(iii) Divide $5a$ by $8b$

$$5a \div 8b, \text{ or } 5a/8b$$

(iv) Product of a number n and four times a variable y

$$n \times 4y = 4ny$$

(v) Cost of n litres of cooking oil at Rs 370 per litre

$$\text{Rs } 370n$$

Answer (i) $3a + 8$ (ii) $5p - 5$ (iii) $5a \div 8b$, or $5a/8b$ (iv) $n \times 4y = 4ny$ (v) Rs $370n$

2. Write the number of terms and separate them: (i) $5x + 3y + 2$ (ii) $7x + 37$ (iii) $7m^2 - 8n^2 - 2mn$ (iv) $y - 2x - 4y^2$ (v) $2x^2 + 3x^2 + 3y + 2$ (vi) $6p - 4q - 2$

Terms are the parts separated by $+$ and $-$ signs, and each term carries the sign in front of it.

A minus sign belongs to the term that follows it — in (iii) the terms are $7m^2$, $-8n^2$ and $-2mn$, not $8n^2$ and $2mn$.

(v) has four terms as written. $2x^2$ and $3x^2$ are like terms and could be collected into $5x^2$, but the question asks for the terms as they stand.

(i) $5x + 3y + 2$

term 1: $5x$

term 2: $3y$

term 3: 2

3 terms

(ii) $7x + 37$

term 1: $7x$

term 2: 37

2 terms

(iii) $7m^2 - 8n^2 - 2mn$

term 1: $7m^2$

term 2: $-8n^2$

term 3: $-2mn$

3 terms

(iv) $y - 2x - 4y^2$

term 1: y

- term 2: $-2x$
 term 3: $-4y^2$
 3 terms
 (v) $2x^2 + 3x^2 + 3y + 2$
 term 1: $2x^2$
 term 2: $3x^2$
 term 3: $3y$
 term 4: 2
 4 terms
 (vi) $6p - 4q - 2$
 term 1: $6p$
 term 2: $-4q$
 term 3: -2
 3 terms

Answer (i) 3 terms: $5x$, $3y$, 2
 (ii) 2 terms: $7x$, 37
 (iii) 3 terms: $7m^2$, $-8n^2$, $-2mn$
 (iv) 3 terms: y , $-2x$, $-4y^2$
 (v) 4 terms: $2x^2$, $3x^2$, $3y$, 2
 (vi) 3 terms: $6p$, $-4q$, -2

EXERCISE 6.4

1. Simplify: (i) $7y^2 + 4y^2$ (ii) $5x + 7y + 3x + 4y$

Only like terms can be collected — terms with exactly the same letters to exactly the same powers.

(i) $7y^2$ and $4y^2$ are alike, so add the numbers in front: $7 + 4 = 11$, giving $11y^2$. The y^2 does not change — it is not $11y^4$.

(ii) Collect the x terms and the y terms separately: $5x + 3x = 8x$, and $7y + 4y = 11y$.

So $8x + 11y$. These two cannot be combined any further, because x and y are different letters.

Answer (i) $11y^2$ (ii) $8x + 11y$

2. Add by the vertical method: (i) $3x + 2$, $3x$ and $4x + 1$ (ii) $6y + 3y$ and $7y + 4y$ (iii) $4x$ and $3x$ (iv) $3y^2 - 6x + 7$ and $2y^2 - 4x - 2$ (v) $4n + 7n + n$ (vi) $x + x + x + x$ (vii) $4m + 6m + 8m$ (viii) $7ab + 4ab$ (ix) $2a + 7a + 2b$ (x) $4y + 5y + 6y$ (xi) $m + 3n + 4n + 2n$

The vertical method means writing the expressions one under another with like terms in the same column, then adding each column on its own.

(i) $3x + 2 + 3x + 4x + 1$

x : $3x + 3x + 4x = 10x$

the constants: $2 + 1 = 3$

Answer: $10x + 3$

(ii) $6y + 3y + 7y + 4y$

y : $6y + 3y + 7y + 4y = 20y$

Answer: $20y$

(iii) $4x + 3x$

x : $4x + 3x = 7x$

Answer: $7x$

(iv) $3y^2 - 6x + 7 + 2y^2 - 4x - 2$

y^2 : $3y^2 + 2y^2 = 5y^2$

x : $-6x - 4x = -10x$

the constants: $7 - 2 = 5$

Answer: $5y^2 - 10x + 5$

(v) $4n + 7n + n$

- n: $4n + 7n + n = 12n$
 Answer: $12n$
- (vi) $x + x + x + x$
 x: $x + x + x + x = 4x$
 Answer: $4x$
- (vii) $4m + 6m + 8m$
 m: $4m + 6m + 8m = 18m$
 Answer: $18m$
- (viii) $7ab + 4ab$
 ab: $7ab + 4ab = 11ab$
 Answer: $11ab$
- (ix) $2a + 7a + 2b$
 a: $2a + 7a = 9a$
 b: $2b$ stands alone
 Answer: $9a + 2b$
- (x) $4y + 5y + 6y$
 y: $4y + 5y + 6y = 15y$
 Answer: $15y$
- (xi) $m + 3n + 4n + 2n$
 m: m stands alone
 n: $3n + 4n + 2n = 9n$
 Answer: $m + 9n$

Answer (i) $10x + 3$ · (ii) $20y$ · (iii) $7x$ · (iv) $5y^2 - 10x + 5$ · (v) $12n$ · (vi) $4x$ · (vii) $18m$ · (viii) $11ab$ · (ix) $9a + 2b$ · (x) $15y$ · (xi) $m + 9n$

3. Find the sum by the horizontal method: (i) $a^2 - 2ab + b^2$, $a^2 + 2ab + b^2$ and $a^2 - ab - b^2$ (ii) $4x + 6y$, $2x + 3y$ (iii) $3p + 2q$, $3q + 3r$ and $r + 4p$ (iv) $3x^3 + 3x^2 + 2x + 6$, $5x^3 + x^2 + 3x + 6$ and $5x^3 + 5x^2 + 3$ (v) $3ab + 4bc + 5ca$, $7ab - 2bc + 7ca$ and $5ab + 6ab + 3ca$

The horizontal method writes everything on one line and collects like terms as you go — one kind of term at a time.

- (i) $a^2 - 2ab + b^2 + a^2 + 2ab + b^2 + a^2 - ab - b^2$
 a^2 : $a^2 + a^2 + a^2 = 3a^2$
 ab : $-2ab + 2ab - ab = -ab$
 b^2 : $b^2 + b^2 - b^2 = b^2$
 Answer: $3a^2 - ab + b^2$
- (ii) $4x + 6y + 2x + 3y$
 x: $4x + 2x = 6x$
 y: $6y + 3y = 9y$
 Answer: $6x + 9y$
- (iii) $3p + 2q + 3q + 3r + r + 4p$
 p: $3p + 4p = 7p$
 q: $2q + 3q = 5q$
 r: $3r + r = 4r$
 Answer: $7p + 5q + 4r$
- (iv) $3x^3 + 3x^2 + 2x + 6 + 5x^3 + x^2 + 3x + 6 + 5x^3 + 5x^2 + 3$
 x^3 : $3x^3 + 5x^3 + 5x^3 = 13x^3$
 x^2 : $3x^2 + x^2 + 5x^2 = 9x^2$
 x: $2x + 3x = 5x$
 the constants: $6 + 6 + 3 = 15$
 Answer: $13x^3 + 9x^2 + 5x + 15$
- (v) $3ab + 4bc + 5ca + 7ab - 2bc + 7ca + 5ab + 6ab + 3ca$
 ab : $3ab + 7ab + 5ab + 6ab = 21ab$
 bc : $4bc - 2bc = 2bc$
 ac : $5ac + 7ac + 3ac = 15ac$

Answer: $21ab + 2bc + 15ac$

Answer (i) $3a^2 - ab + b^2$ · (ii) $6x + 9y$ · (iii) $7p + 5q + 4r$ · (iv) $13x^3 + 9x^2 + 5x + 15$ · (v) $21ab + 2bc + 15ac$

4. Simplify by the horizontal method: (i) $(4ab - 3bc) - (3ab + 2bc)$ (ii) $(3y^2 - 6x + 7) - (2y^2 - 4x - 2)$ (iii) $(x + 1) - (x - 1)$ (iv) $(m - n) - (m + n)$ (v) $3 - (x^2 + x^3 - 2x - 1)$ (vi) $(4x) - (3x)$ (vii) $(7x^2 + 5x + 3) - (5x^2 - 2x + 1)$ (viii) $7p - 3q - (11p + 2q)$

A minus sign in front of a bracket changes the sign of every term inside it. That one step accounts for almost every mark lost here.

(i) $(4ab - 3bc) - (3ab + 2bc)$

change every sign inside the bracket being taken away, then add:

$$4ab - 3bc - 3ab - 2bc$$

$$ab: 4ab - 3ab = ab$$

$$bc: -3bc - 2bc = -5bc$$

$$\text{Answer: } ab - 5bc$$

(ii) $(3y^2 - 6x + 7) - (2y^2 - 4x - 2)$

change every sign inside the bracket being taken away, then add:

$$3y^2 - 6x + 7 - 2y^2 + 4x + 2$$

$$y^2: 3y^2 - 2y^2 = y^2$$

$$x: -6x + 4x = -2x$$

$$\text{the constants: } 7 + 2 = 9$$

$$\text{Answer: } y^2 - 2x + 9$$

(iii) $(x + 1) - (x - 1)$

change every sign inside the bracket being taken away, then add:

$$x + 1 - x + 1$$

$$x: x - x = 0$$

$$\text{the constants: } 1 + 1 = 2$$

$$\text{Answer: } 2$$

(iv) $(m - n) - (m + n)$

change every sign inside the bracket being taken away, then add:

$$m - n - m - n$$

$$m: m - m = 0$$

$$n: -n - n = -2n$$

$$\text{Answer: } -2n$$

(v) $(3) - (x^2 + x^3 - 2x - 1)$

change every sign inside the bracket being taken away, then add:

$$3 - x^2 - x^3 + 2x + 1$$

$$x^3: -x^3 \text{ stands alone}$$

$$x^2: -x^2 \text{ stands alone}$$

$$x: 2x \text{ stands alone}$$

$$\text{the constants: } 3 + 1 = 4$$

$$\text{Answer: } -x^3 - x^2 + 2x + 4$$

(vi) $(4x) - (3x)$

change every sign inside the bracket being taken away, then add:

$$4x - 3x$$

$$x: 4x - 3x = x$$

$$\text{Answer: } x$$

(vii) $(7x^2 + 5x + 3) - (5x^2 - 2x + 1)$

change every sign inside the bracket being taken away, then add:

$$7x^2 + 5x + 3 - 5x^2 + 2x - 1$$

$$x^2: 7x^2 - 5x^2 = 2x^2$$

$$x: 5x + 2x = 7x$$

$$\text{the constants: } 3 - 1 = 2$$

$$\text{Answer: } 2x^2 + 7x + 2$$

$$(viii) (7p - 3q) - (11p + 2q)$$

change every sign inside the bracket being taken away, then add:

$$7p - 3q - 11p - 2q$$

$$p: 7p - 11p = -4p$$

$$q: -3q - 2q = -5q$$

$$\text{Answer: } -4p - 5q$$

Answer (i) $ab - 5bc$ · (ii) $y^2 - 2x + 9$ · (iii) 2 · (iv) $-2n$ · (v) $-x^3 - x^2 + 2x + 4$ · (vi) x · (vii) $2x^2 + 7x + 2$ · (viii) $-4p - 5q$

5. Subtract by the vertical method: (i) $2y^3 + x^2y^2 + x^2y$ from $2x^3 - 3x^2y^2 + x^2y$ (ii) $2x - 2y + 1$ from $11x + 5y + 7$ (iii) $a + 2b + c$ from $6a + 8b + 9c$ (iv) $3x + 4y$ from $6x + 8y$ (v) $5n^4 - 3n^3 + 9n^2$ from $6n^6 + 5n^2 - 8n + 7$ (vi) $8m^3 - 15m^2 + 2m + 20$ from $5m^3 + 12m^2 - 7m + 15$ (vii) $xy + yz + xz$ from $4xy + 5yz + 15xz$ (viii) $6n^4 + 2n^2 + 2$ from $6n^4 + 5n^3 + 10n^2 - 11n + 7$

'A from B' means $B - A$ — the second expression is the one being taken away from. Reversing them is the first thing to get right, so each part below is rewritten that way first.

$$(i) (2x^3 - 3x^2y^2 + x^2y) - (2y^3 + x^2y^2 + x^2y)$$

change every sign inside the bracket being taken away, then add:

$$2x^3 - 3x^2y^2 + x^2y - 2y^3 - x^2y^2 - x^2y$$

$$x^3: 2x^3 \text{ stands alone}$$

$$x^2y^2: -3x^2y^2 - x^2y^2 = -4x^2y^2$$

$$x^2y: x^2y - x^2y = 0$$

$$y^3: -2y^3 \text{ stands alone}$$

$$\text{Answer: } 2x^3 - 4x^2y^2 - 2y^3$$

$$(ii) (11x + 5y + 7) - (2x - 2y + 1)$$

change every sign inside the bracket being taken away, then add:

$$11x + 5y + 7 - 2x + 2y - 1$$

$$x: 11x - 2x = 9x$$

$$y: 5y + 2y = 7y$$

$$\text{the constants: } 7 - 1 = 6$$

$$\text{Answer: } 9x + 7y + 6$$

$$(iii) (6a + 8b + 9c) - (a + 2b + c)$$

change every sign inside the bracket being taken away, then add:

$$6a + 8b + 9c - a - 2b - c$$

$$a: 6a - a = 5a$$

$$b: 8b - 2b = 6b$$

$$c: 9c - c = 8c$$

$$\text{Answer: } 5a + 6b + 8c$$

$$(iv) (6x + 8y) - (3x + 4y)$$

change every sign inside the bracket being taken away, then add:

$$6x + 8y - 3x - 4y$$

$$x: 6x - 3x = 3x$$

$$y: 8y - 4y = 4y$$

$$\text{Answer: } 3x + 4y$$

$$(v) (6n^6 + 5n^2 - 8n + 7) - (5n^4 - 3n^3 + 9n^2)$$

change every sign inside the bracket being taken away, then add:

$$6n^6 + 5n^2 - 8n + 7 - 5n^4 + 3n^3 - 9n^2$$

$$n^6: 6n^6 \text{ stands alone}$$

$$n^4: -5n^4 \text{ stands alone}$$

$$n^3: 3n^3 \text{ stands alone}$$

$$n^2: 5n^2 - 9n^2 = -4n^2$$

$$n: -8n \text{ stands alone}$$

$$\text{the constants: } 7 \text{ stands alone}$$

$$\text{Answer: } 6n^6 - 5n^4 + 3n^3 - 4n^2 - 8n + 7$$

$$(vi) (5m^3 + 12m^2 - 7m + 15) - (8m^3 - 15m^2 + 2m + 20)$$

change every sign inside the bracket being taken away, then add:

$$5m^3 + 12m^2 - 7m + 15 - 8m^3 + 15m^2 - 2m - 20$$

$$m^3: 5m^3 - 8m^3 = -3m^3$$

$$m^2: 12m^2 + 15m^2 = 27m^2$$

$$m: -7m - 2m = -9m$$

$$\text{the constants: } 15 - 20 = -5$$

$$\text{Answer: } -3m^3 + 27m^2 - 9m - 5$$

$$(vii) (4xy + 5yz + 15xz) - (xy + yz + xz)$$

change every sign inside the bracket being taken away, then add:

$$4xy + 5yz + 15xz - xy - yz - xz$$

$$xy: 4xy - xy = 3xy$$

$$yz: 5yz - yz = 4yz$$

$$xz: 15xz - xz = 14xz$$

$$\text{Answer: } 3xy + 4yz + 14xz$$

$$(viii) (6n^4 + 5n^3 + 10n^2 - 11n + 7) - (6n^4 + 2n^2 + 2)$$

change every sign inside the bracket being taken away, then add:

$$6n^4 + 5n^3 + 10n^2 - 11n + 7 - 6n^4 - 2n^2 - 2$$

$$n^4: 6n^4 - 6n^4 = 0$$

$$n^3: 5n^3 \text{ stands alone}$$

$$n^2: 10n^2 - 2n^2 = 8n^2$$

$$n: -11n \text{ stands alone}$$

$$\text{the constants: } 7 - 2 = 5$$

$$\text{Answer: } 5n^3 + 8n^2 - 11n + 5$$

Answer (i) $2x^3 - 4x^2y - 2y^3$ · (ii) $9x + 7y + 6$ · (iii) $5a + 6b + 8c$ · (iv) $3x + 4y$ · (v) $6n^6 - 5n^4 + 3n^3 - 4n^2 - 8n + 7$ · (vi) $-3m^3 + 27m^2 - 9m - 5$ · (vii) $3xy + 4yz + 14xz$ · (viii) $5n^3 + 8n^2 - 11n + 5$

EXERCISE 6.5

1. Simplify: (i) $5x + 3(8x + 4y)$ (ii) $3a + [3b - \{5c + (2b + 4c)\}]$ (iii) $(l - 2m) - (2l - 3m)$ (iv) $[x^2 + \{2xy + (3y^2 - 2y^2)\}]$ (v) $(x^2 + y - 2xy) - (x^2 + y^2 + 2xy)$ (vi) $4a + \{2a - 3b - (4a - b)\}$ (vii) $[y - \{x - (y + x)\}]$ (viii) $[6x - \{4x + (3x - 2x)\}]$ (ix) $5x - (x + 3x + 4)$ (x) $2x - [4y - \{4x - 3y - 2x + 2y\}]$ (xi) $a + 1 - [a + 1 \{a + 1 - a + 1 - a + 1\}]$

Work from the innermost bracket outward — round brackets, then braces, then square brackets.

A minus sign before a bracket changes the sign of every term inside it.

(i)

$$\begin{aligned} &5x + 3(8x + 4y) \\ &= 5x + 24x + 12y \\ &= 29x + 12y \end{aligned}$$

(ii)

$$\begin{aligned} &3a + [3b - \{5c + (2b + 4c)\}] \\ &= 3a + [3b - \{5c + 2b + 4c\}] \\ &= 3a + [3b - \{2b + 9c\}] \\ &= 3a + [3b - 2b - 9c] \\ &= 3a + [b - 9c] \\ &= 3a + b - 9c \end{aligned}$$

(iii)

$$\begin{aligned} &(l - 2m) - (2l - 3m) \\ &= l - 2m - 2l + 3m \\ &= l - 2l - 2m + 3m \\ &= -l + m \end{aligned}$$

(iv)

$$\begin{aligned} &[x^2 + \{2xy + (3y^2 - 2y^2)\}] \\ &= [x^2 + \{2xy + y^2\}] \end{aligned}$$

$$\begin{aligned}
 &= x^2 + 2xy + y^2 \\
 &= (x + y)^2 \\
 \text{(v)} \quad &(x^2 + y - 2xy) - (x^2 + y^2 + 2xy) \\
 &= x^2 + y - 2xy - x^2 - y^2 - 2xy \\
 &= (x^2 - x^2) + y - y^2 + (-2xy - 2xy) \\
 &= y - 4xy - y^2 \\
 \text{(vi)} \quad &4a + \{2a - 3b - (4a - b)\} \\
 &= 4a + \{2a - 3b - 4a + b\} \\
 &= 4a + \{-2a - 2b\} \\
 &= 4a - 2a - 2b \\
 &= 2a - 2b \\
 \text{(vii)} \quad &[y - \{x - (y + x)\}] \\
 &= [y - \{x - y - x\}] \\
 &= [y - \{-y\}] \\
 &= y + y \\
 &= 2y \\
 \text{(viii)} \quad &[6x - \{4x + (3x - 2x)\}] \\
 &= [6x - \{4x + x\}] \\
 &= [6x - 5x] \\
 &= x \\
 \text{(ix)} \quad &5x - (x + 3x + 4) \\
 &= 5x - (4x + 4) \\
 &= 5x - 4x - 4 \\
 &= x - 4 \\
 \text{(x)} \quad &2x - [4y - \{4x - 3y - 2x + 2y\}] \\
 &= 2x - [4y - \{2x - y\}] \\
 &= 2x - [4y - 2x + y] \\
 &= 2x - [5y - 2x] \\
 &= 2x - 5y + 2x \\
 &= 4x - 5y \\
 \text{(xi)} \quad &\text{cannot be simplified as printed — see the note.}
 \end{aligned}$$

Answer (i) $29x + 12y$ (ii) $3a + b - 9c$ (iii) $-l + m$ (iv) $(x + y)^2$ (v) $y - 4xy - y^2$ (vi) $2a - 2b$ (vii) $2y$ (viii) x (ix) $x - 4$ (x) $4x - 5y$ (xi) —

Please check — part (xi) has no operator between 'a + 1' and the brace that follows it, so the expression is incomplete and cannot be simplified. The missing sign needs restoring.

EXERCISE 6.6

1. Simplify and evaluate: (i) $a + \{5b + (a - 2b - c)\}$ if $a = 1$, $b = 2$, $c = 3$ (ii) $(x - 4) - (y - z) - (2 - x)$ if $x = 2$, $y = -1$, $z = 0$ (iii) $(x - y) - (y - z) - (2x)$ if $x = 4$, $y = 2$, $z = 3$

Simplify first, then substitute — it is far less error-prone than putting the numbers in straight away.

(i) $a + 5b + a - 2b - c = 2a + 3b - c$. With $a = 1$, $b = 2$, $c = 3$: $2 + 6 - 3 = 5$.

(ii) $x - 4 - y + z - 2 + x = 2x - y + z - 6$. With $x = 2$, $y = -1$, $z = 0$: $4 + 1 + 0 - 6 = -1$.

(iii) $x - y - y + z - 2x = -x - 2y + z$. With $x = 4$, $y = 2$, $z = 3$: $-4 - 4 + 3 = -5$.

Watch the signs when substituting a negative: in (ii), $-y$ with $y = -1$ becomes $-(-1) = +1$.

Answer (i) $2a + 3b - c = 5$ (ii) $2x - y + z - 6 = -1$ (iii) $-x - 2y + z = -5$

2. Find the value of $(ac - b)/(b - a)$ if $a = 3$, $b = 2$, $c = 3$.

Substitute into the top and the bottom separately.

Top: $ac - b = 3 \times 3 - 2 = 9 - 2 = 7$.

Bottom: $b - a = 2 - 3 = -1$.

So the value is $7 \div (-1) = -7$.

A negative denominator makes the whole fraction negative — worth checking rather than dropping the sign.

Answer -7

3. Find the value of $(2a - 4de + 5b)/(cd)$ if $a = 3$, $b = -4$, $c = 2$, $d = -1$, $e = 0$.

Substitute carefully, taking the signs with the numbers.

Top: $2a = 6$; $4de = 4 \times (-1) \times 0 = 0$, so $-4de = 0$; $5b = 5 \times (-4) = -20$.

Top = $6 - 0 + (-20) = -14$.

Bottom: $cd = 2 \times (-1) = -2$.

Value = $-14 \div (-2) = 7$.

The $e = 0$ makes the whole middle term vanish, whatever d is — worth spotting before multiplying.

Answer 7



7

CHAPTER

Linear Expressions and Equations

EXERCISE 7.1

1. Separate the algebraic equations from the expressions: (i) $2x + 3 - 7$ (ii) $4l + m - n + 0$ (iii) $2x + 3x + 5 = 2x - 7$ (iv) $7x^3 - 3 - 7x + 2$ (v) $5x^2 - 7 + x^2 = 3x - 2$

The test is simple: an equation contains an equals sign; an expression does not.

An expression can be simplified but not solved — there is nothing to balance it against.

- (i) $2x + 3 - 7$ — expression, there is no equals sign
 (ii) $4l + m - n + 0$ — expression, again no equals sign
 (iii) $2x + 3x + 5 = 2x - 7$ — equation, it has an equals sign
 (iv) $7x^3 - 3 - 7x + 2$ — expression, no equals sign
 (v) $5x^2 - 7 + x^2 = 3x - 2$ — equation, it has an equals sign

Answer Equations: (iii) and (v) · Expressions: (i), (ii) and (iv)

2. Which of these are linear equations in one variable? (i) $7x - 8y = 0$ (ii) $5l - 5 + 3l = 0$ (iii) $7x^2 - 19z = 0$ (iv) $13t - 17 + 21t = 5t$ (v) $3a + 5b = 28$ (vi) $17p + 4 - 13p = 0$

Two conditions must both hold: exactly one variable, and that variable only to the first power.

A squared term makes it quadratic, not linear; a second letter makes it an equation in two variables.

- (i) $7x - 8y = 0$ — no, two variables, x and y
 (ii) $5l - 5 + 3l = 0$ — yes, one variable l , to the first power
 (iii) $7x^2 - 19z = 0$ — no, two variables, and x is squared
 (iv) $13t - 17 + 21t = 5t$ — yes, one variable t , first power
 (v) $3a + 5b = 28$ — no, two variables, a and b
 (vi) $17p + 4 - 13p = 0$ — yes, one variable p , first power

Answer (ii), (iv) and (vi) are linear in one variable

3. Write an equation for each statement: (i) subtracting 5 from a number gives 3 (ii) after two years Ali will be twice as old as he is now (iii) Ali and Mehmood's ages add to 30 (iv) the price of 10 books equals the price of 1 bag (v) a number increased by 4 is 6 (vi) the length of a rectangle is thrice its breadth

Choose a letter for the unknown and say what it stands for, then translate the sentence phrase by phrase.

(ii) is the one to think about: 'after two years' is $x + 2$, and 'twice as old as he is now' is $2x$ — the now matters. $x + 2 = 2x$ gives $x = 2$.

(iv) needs two letters because books and bags are different things.

- (i) Subtracting 5 from a number, the result is 3 → $x - 5 = 3$
 (ii) After two years Ali will be twice as old as he is now → $x + 2 = 2x$
 (iii) Ali and Mehmood's ages add to 30 years → $a + m = 30$
 (iv) The price of 10 books equals the price of 1 bag → $10b = g$
 (v) A number increased by 4 is 6 → $x + 4 = 6$
 (vi) The length of a rectangle is thrice its breadth → $l = 3b$

Answer (i) $x - 5 = 3$ (ii) $x + 2 = 2x$ (iii) $a + m = 30$ (iv) $10b = g$ (v) $x + 4 = 6$ (vi) $l = 3b$

4. Solve and verify: (i) $3x + 4 = 28$ (ii) $x + 2 = 2x - 1$ (iii) $2x + 12 = 32$ (iv) $1.5x + 3 = 7$ (v) $9b + 9 = 0$ (vi) $x + 5 = 39$ (vii) $n + 1 + 2n + 7 = 0$ (viii) $0.25x + 1.5 = 7.5$ (ix) $6x = 48$ (x) $2y/3 = -6$ (xi) $6x + 15 = 2x + 28$ (xii) $x/2 + 3x/2 + x/2 + 5x/2 = 25$ (xiii) $(x-7)/2 - (x+3)/3 = 2/3$ (xiv) $(7x-2)/3 - (2x+3)/5 - 5 = 0$ (xv) $(2x+15)/2 = 2(3+2x)/3$ (xvi) $(2x+5)/15 - (x+3)/3 = 2/3$

Three habits make all of these straightforward.

Collect the variable terms on one side and the numbers on the other, doing the same thing to both sides each time.

Clear fractions early by multiplying every term by the lowest common denominator — (xiii) to (xvi) become much easier that way.

Verify by putting the answer back into the original equation, which the question asks for explicitly.

(i) subtract 4, then divide by 3

$$3x + 4 = 28$$

$$3x = 28 - 4$$

$$3x = 24$$

$$x = 24 \div 3$$

$$x = 8$$

(ii) take x from both sides: $2 = x - 1$

$$x + 2 = 2x - 1$$

$$x - 2x = -1 - 2$$

$$-x = -3$$

$$x = 3$$

(iii) subtract 12, then halve

$$2x + 12 = 32$$

$$2x = 32 - 12$$

$$2x = 20$$

$$x = 20 \div 2$$

$$x = 10$$

(iv) subtract 3 to get $1.5x = 4$, then divide

$$1.5x + 3 = 7$$

$$1.5x = 7 - 3$$

$$1.5x = 4$$

$$x = 4 \div 1.5$$

$$x = 8/3$$

(v) subtract 9 to get $9b = -9$

$$9b + 9 = 0$$

$$9b = -9$$

$$b = -9 \div 9$$

$$b = -1$$

(vi) subtract 5

$$x + 5 = 39$$

$$x = 39 - 5$$

$$x = 34$$

(vii) collect first: $3n + 8 = 0$

$$n + 1 + 2n + 7 = 0$$

$$3n + 8 = 0$$

$$3n = -8$$

$$n = -8/3$$

(viii) subtract 1.5 to get $0.25x = 6$, then divide by 0.25 — or multiply through by 4 first

$$0.25x + 1.5 = 7.5$$

$$0.25x = 7.5 - 1.5$$

$$0.25x = 6$$

$$x = 6 \div 0.25$$

$$x = 24$$

(ix) divide by 6

$$6x = 48$$

$$x = 48 \div 6$$

$$x = 8$$

(x) multiply both sides by 3, then divide by 2

$$2y/3 = -6$$

$$2y = -6 \times 3$$

$$2y = -18$$

$$y = -18 \div 2$$

$$y = -9$$

(xi) gather the x terms: $4x = 13$

$$6x + 15 = 2x + 28$$

$$6x - 2x = 28 - 15$$

$$4x = 13$$

$$x = 13/4$$

(xii) the left side is $10x/2 = 5x$, so $5x = 25$

$$x/2 + 3x/2 + x/2 + 5x/2 = 25$$

$$(1 + 3 + 1 + 5)x/2 = 25$$

$$10x/2 = 25$$

$$5x = 25$$

$$x = 25 \div 5$$

$$x = 5$$

(xiii) multiply through by 6 to clear every fraction

$$(x - 7)/2 - (x + 3)/3 = 2/3$$

multiply every term by 6

$$3(x - 7) - 2(x + 3) = 4$$

$$3x - 21 - 2x - 6 = 4$$

$$x - 27 = 4$$

$$x = 4 + 27$$

$$x = 31$$

(xiv) multiply through by 15

$$(7x - 2)/3 - (2x + 3)/5 - 5 = 0$$

multiply every term by 15

$$5(7x - 2) - 3(2x + 3) - 75 = 0$$

$$35x - 10 - 6x - 9 - 75 = 0$$

$$29x - 94 = 0$$

$$29x = 94$$

$$x = 94/29$$

(xv) cross-multiply: $3(2x + 15) = 4(3 + 2x)$

$$(2x + 15)/2 = 2(3 + 2x)/3$$

cross-multiply

$$3(2x + 15) = 2 \times 2(3 + 2x)$$

$$6x + 45 = 4(3 + 2x)$$

$$6x + 45 = 12 + 8x$$

$$45 - 12 = 8x - 6x$$

$$33 = 2x$$

$$x = 33/2$$

(xvi) multiply through by 15

$$(2x + 5)/15 - (x + 3)/3 = 2/3$$

multiply every term by 15

$$(2x + 5) - 5(x + 3) = 10$$

$$2x + 5 - 5x - 15 = 10$$

$$-3x - 10 = 10$$

$$-3x = 20$$

$$x = -20/3$$

Four answers are fractions rather than whole numbers. That is allowed, but see the note.

Answer (i) 8 (ii) 3 (iii) 10 (iv) $8/3$ (v) -1 (vi) 34 (vii) $-8/3$ (viii) 24 (ix) 8 (x) -9 (xi) $13/4$
(xii) 5 (xiii) 31 (xiv) $94/29$ (xv) $33/2$ (xvi) $-20/3$

Please check — parts (xi), (xiv), (xv) and (xvi) give $13/4$, $94/29$, $33/2$ and $-20/3$. A solution of $94/29$ is very unusual at this level and suggests a mistyped coefficient. The working shown is correct for the equations as printed.

■ EXERCISE 7.2

1. Asghar's brother is 10 years less than twice Asghar's age. What will Asghar's age be when his brother is 20?

Let Asghar's age be x . His brother is $2x - 10$.

When the brother is 20: $2x - 10 = 20$.

Add 10: $2x = 30$.

Divide by 2: $x = 15$.

Check: twice 15 is 30, less 10 is 20 ✓

Answer Asghar will be 15

2. The sum of two numbers is 12. Find them, given one is twice the other.

Let the smaller be x ; then the other is $2x$.

$x + 2x = 12$, so $3x = 12$.

$x = 4$, and the other is $2 \times 4 = 8$.

Check: $4 + 8 = 12$ ✓ and 8 is twice 4 ✓

Answer 4 and 8

3. Nouman is 30 years older than his son. After 12 years he will be three times as old as his son. Find their present ages.

Let the son be x now; Nouman is $x + 30$.

In 12 years the son is $x + 12$ and Nouman is $x + 42$.

Then Nouman is three times the son: $x + 42 = 3(x + 12)$.

$x + 42 = 3x + 36$.

$42 - 36 = 3x - x$, so $6 = 2x$ and $x = 3$.

So the son is 3 and Nouman is 33.

Check: in 12 years they will be 15 and 45, and $45 = 3 \times 15$ ✓

Answer The son is 3 and Nouman is 33

4. After 20 years Alia will be 5 times as old as she is now. What is her present age?

Let her present age be x .

In 20 years she is $x + 20$, and that equals 5 times her age now: $x + 20 = 5x$.

Take x from both sides: $20 = 4x$.

$x = 5$.

Check: in 20 years she will be 25, and $25 = 5 \times 5$ ✓

Answer She is 5 now

5. The sum of two consecutive numbers is 37. Find them.

Consecutive numbers differ by 1, so let them be x and $x + 1$.

$x + (x + 1) = 37$, so $2x + 1 = 37$.

$2x = 36$, giving $x = 18$.

The numbers are 18 and 19.

Check: $18 + 19 = 37$ ✓

Answer 18 and 19

6. A man is 40 years older than his son. After 12 years he will be three times as old as his son. Find their present ages.

Let the son be x ; the man is $x + 40$.

In 12 years: son $x + 12$, man $x + 52$.

$$x + 52 = 3(x + 12) = 3x + 36.$$

$$52 - 36 = 2x, \text{ so } 16 = 2x \text{ and } x = 8.$$

The son is 8 and the man is 48.

Check: in 12 years they will be 20 and 60, and $60 = 3 \times 20$ ✓

This is the same question as number 3 with 40 in place of 30 — worth noticing how the answer shifts.

Answer The son is 8 and the man is 48



8

CHAPTER

Surface Area and Volume

EXERCISE 8.1

- Find the area and perimeter of the given squares and rectangles. (six figures are printed, with the first worked as an example: 3 ft by 2 ft, area $3 \times 2 = 6 \text{ ft}^2$)

For every rectangle, two formulas are needed and no others:

$$\text{Area} = \text{length} \times \text{breadth}$$

$$\text{Perimeter} = 2 \times (\text{length} + \text{breadth})$$

For a square both sides are the same, so these become Area = side $\times$ side and Perimeter = $4 \times$ side.

(i) is worked in the book: 3 ft by 2 ft gives Area = $3 \times 2 = 6 \text{ ft}^2$, and Perimeter = $2 \times (3 + 2) = 2 \times 5 = 10 \text{ ft}$.

The remaining five carry their measurements on the printed figures — read the two numbers off each one and apply the same two formulas.

Always write the unit: a length is in ft, an area in ft^2 . Marks are lost for the missing square.

Answer (i) Area 6 ft^2 , Perimeter 10 ft. The rest follow the same two formulas from the measurements on each figure.

Please check — the measurements for (ii) to (vi) are printed on the figures, so those need reading from the page.

- Find the perimeter and area of squares whose dimensions are: (i) 16 cm (ii) 5.3 m (iii) 2 m 37 cm (iv) 86 dm

A square has four equal sides, so:

$$\text{Perimeter} = 4 \times \text{side}$$

$$\text{Area} = \text{side} \times \text{side}$$

Where the measurement is given in two units, convert to one unit first — that is part (iii).

(i) Side = 16 cm.

$$\text{Perimeter} = 4 \times \text{side} = 4 \times 16 = 64 \text{ cm}$$

$$\text{Area} = \text{side} \times \text{side} = 16 \times 16 = 256 \text{ cm}^2$$

(ii) Side = 5.3 m.

$$\text{Perimeter} = 4 \times \text{side} = 4 \times 5.3 = 21.2 \text{ m}$$

$$\text{Area} = \text{side} \times \text{side} = 5.3 \times 5.3 = 28.09 \text{ m}^2$$

(iii) Side = 2 m 37 cm. (first convert: $2 \text{ m } 37 \text{ cm} = 200 \text{ cm} + 37 \text{ cm} = 237 \text{ cm}$)

$$\text{Perimeter} = 4 \times \text{side} = 4 \times 237 = 948 \text{ cm}$$

$$\text{Area} = \text{side} \times \text{side} = 237 \times 237 = 56,169 \text{ cm}^2$$

(iv) Side = 86 dm.

$$\text{Perimeter} = 4 \times \text{side} = 4 \times 86 = 344 \text{ dm}$$

$$\text{Area} = \text{side} \times \text{side} = 86 \times 86 = 7,396 \text{ dm}^2$$

Notice that (ii) gives an area with two decimal places: $5.3 \times 5.3 = 28.09$. Multiplying two numbers each with one decimal place always gives two.

Answer (i) P = 64 cm, A = 256 cm^2

(ii) P = 21.2 m, A = 28.09 m^2

(iii) P = 948 cm, A = $56,169 \text{ cm}^2$

(iv) P = 344 dm, A = $7,396 \text{ dm}^2$

- Find the perimeter and area of these rectangles: (i) length 35 cm, breadth 12 cm (ii) length 12 m, breadth 5 m (iii) length 24 m, breadth 7 m (iv) length 289 cm, breadth 111 cm

Perimeter = $2 \times (\text{length} + \text{breadth})$; Area = length $\times$ breadth.

Add inside the bracket first, then double — that is what the bracket is telling you to do.

(i) Length = 35 cm, breadth = 12 cm.

$$\text{Perimeter} = 2 \times (\text{length} + \text{breadth}) = 2 \times (35 + 12) = 2 \times 47 = 94 \text{ cm}$$

$$\text{Area} = \text{length} \times \text{breadth} = 35 \times 12 = 420 \text{ cm}^2$$

(ii) Length = 12 m, breadth = 5 m.

$$\text{Perimeter} = 2 \times (\text{length} + \text{breadth}) = 2 \times (12 + 5) = 2 \times 17 = 34 \text{ m}$$

$$\text{Area} = \text{length} \times \text{breadth} = 12 \times 5 = 60 \text{ m}^2$$

(iii) Length = 24 m, breadth = 7 m.

$$\text{Perimeter} = 2 \times (\text{length} + \text{breadth}) = 2 \times (24 + 7) = 2 \times 31 = 62 \text{ m}$$

$$\text{Area} = \text{length} \times \text{breadth} = 24 \times 7 = 168 \text{ m}^2$$

(iv) Length = 289 cm, breadth = 111 cm.

$$\text{Perimeter} = 2 \times (\text{length} + \text{breadth}) = 2 \times (289 + 111) = 2 \times 400 = 800 \text{ cm}$$

$$\text{Area} = \text{length} \times \text{breadth} = 289 \times 111 = 32,079 \text{ cm}^2$$

Answer (i) $P = 94 \text{ cm}$, $A = 420 \text{ cm}^2$

(ii) $P = 34 \text{ m}$, $A = 60 \text{ m}^2$

(iii) $P = 62 \text{ m}$, $A = 168 \text{ m}^2$

(iv) $P = 800 \text{ cm}$, $A = 32,079 \text{ cm}^2$

4. A rectangular lawn 50 m long and 35 m wide is surrounded externally by a path 2 m wide. Find the cost of turfing the path at \$3 per m^2 .

The path goes around the outside, so picture two rectangles: the lawn, and the bigger rectangle made by lawn plus path. The path is the difference between them.

Step 1 — the lawn's area.

$$\text{Area} = 50 \times 35 = 1,750 \text{ m}^2$$

Step 2 — the outer rectangle's dimensions. The path adds 2 m at each end, so each dimension grows by $2 + 2 = 4 \text{ m}$. That doubling is the step most often missed.

$$\text{Outer length} = 50 + 4 = 54 \text{ m}$$

$$\text{Outer breadth} = 35 + 4 = 39 \text{ m}$$

Step 3 — the outer area.

$$\text{Area} = 54 \times 39 = 2,106 \text{ m}^2$$

Step 4 — the path's area is the difference.

$$2,106 - 1,750 = 356 \text{ m}^2$$

Step 5 — the cost, at \$3 for every square metre.

$$356 \times 3 = \$1,068$$

Answer The path is 356 m^2 and costs \$1,068

5. The area of a square field is 100 acres. Find its perimeter.

To get a perimeter we need a side, and to get the side from the area we take the square root.

But the area is in acres, and an acre is not a square of any simple length — so $\sqrt{100}$ acres is not 10 of anything useful.

The area must first be put into square metres.

$$1 \text{ acre} = 4,046.86 \text{ m}^2$$

$$100 \text{ acres} = 100 \times 4,046.86 = 404,686 \text{ m}^2$$

$$\text{Side} = \sqrt{404,686} \approx 636.15 \text{ m}$$

$$\text{Perimeter} = 4 \times 636.15 \approx 2,544.6 \text{ m}$$

If instead the question meant an area of 100 m^2 , the side would be $\sqrt{100} = 10 \text{ m}$ and the perimeter $4 \times 10 = 40 \text{ m}$ — which is almost certainly what was intended.

Answer $\approx 2,544.6 \text{ m}$ if the area really is 100 acres; 40 m if the area is 100 m^2

Please check — a perimeter cannot be found from acres without converting to square metres, which is beyond this grade. The question probably means 100 m^2 , giving a side of 10 m and a perimeter of 40 m .

6. Find the perimeter of a square whose area is 625 cm^2 .

Area of a square = side $\times$ side, so the side is the square root of the area.

Step 1 — find the side.

$$\text{side} \times \text{side} = 625$$

$$\text{side} = \sqrt{625} = 25 \text{ cm (because } 25 \times 25 = 625)$$

Step 2 — find the perimeter.

$$\text{Perimeter} = 4 \times \text{side} = 4 \times 25 = 100 \text{ cm}$$

Check: a 25 cm square has area $25 \times 25 = 625 \text{ cm}^2$ ✓

Answer 100 cm

7. The perimeter of a rectangle is 326 cm and its length is 98 cm. Find its breadth and area.

Start from Perimeter = $2 \times (\text{length} + \text{breadth})$ and work backwards.

Step 1 — halve the perimeter to get length + breadth.

$$326 \div 2 = 163 \text{ cm}$$

Step 2 — subtract the length to get the breadth.

$$163 - 98 = 65 \text{ cm}$$

Step 3 — find the area.

$$\text{Area} = 98 \times 65$$

$$= 98 \times 60 + 98 \times 5$$

$$= 5,880 + 490 = 6,370 \text{ cm}^2$$

Check the perimeter: $2 \times (98 + 65) = 2 \times 163 = 326 \text{ cm}$ ✓

Answer Breadth 65 cm, area 6,370 cm^2

8. A rectangular park is 615 m long and 550 m wide. Find the cost of fencing it at Rs 9.25 per metre.

Fencing runs round the edge, so it is the perimeter that is needed — not the area.

Step 1 — find the perimeter.

$$\text{Perimeter} = 2 \times (615 + 550)$$

$$= 2 \times 1,165 = 2,330 \text{ m}$$

Step 2 — multiply by the rate.

$$\text{Cost} = 2,330 \times 9.25$$

$$= 2,330 \times 9 + 2,330 \times 0.25$$

$$= 20,970 + 582.50$$

$$= \text{Rs } 21,552.50$$

Answer Rs 21,552.50

9. Find the area of a square whose perimeter is 440 cm.

Perimeter = $4 \times \text{side}$, so the side is a quarter of the perimeter.

Step 1 — find the side.

$$440 \div 4 = 110 \text{ cm}$$

Step 2 — find the area.

$$\text{Area} = 110 \times 110 = 12,100 \text{ cm}^2$$

Check: a 110 cm square has perimeter $4 \times 110 = 440 \text{ cm}$ ✓

Answer 12,100 cm^2

10. The perimeter of a rectangle is 50 m and its breadth is 12 m. Find its length and area.

Step 1 — halve the perimeter.

$$50 \div 2 = 25 \text{ m} = \text{length} + \text{breadth}$$

Step 2 — subtract the breadth.

$$25 - 12 = 13 \text{ m} = \text{length}$$

Step 3 — find the area.

$$\text{Area} = 13 \times 12 = 156 \text{ m}^2$$

Check: $2 \times (13 + 12) = 2 \times 25 = 50 \text{ m}$ ✓

Answer Length 13 m, area 156 m^2

11. Find the length and perimeter of a rectangle whose breadth is 19 m and area is 475 m².

This time the area is given rather than the perimeter, so start from Area = length × breadth.

Step 1 — find the length by dividing.

$$\text{length} = 475 \div 19 = 25 \text{ m}$$

$$(19 \times 25 = 475, \text{ so this is exact})$$

Step 2 — find the perimeter.

$$\text{Perimeter} = 2 \times (25 + 19)$$

$$= 2 \times 44 = 88 \text{ m}$$

Check the area: $25 \times 19 = 475 \text{ m}^2$ ✓

Answer Length 25 m, perimeter 88 m

EXERCISE 8.2

1. Find the missing value for each parallelogram: (i) base 18 cm, height 5 cm, area ? (ii) base 8 m, height ?, area 56 sq m (iii) base ?, height 17 mm, area 221 sq m

For a parallelogram the formula is Area = base × height. Note there is no $\frac{1}{2}$ — that belongs to the triangle, and confusing the two is the commonest error in this exercise.

The height is the perpendicular height, measured at a right angle to the base, not the slanting side.

(i) Both base and height are given, so multiply.

$$\text{Area} = 18 \times 5 = 90 \text{ cm}^2$$

(ii) The area and base are given, so divide.

$$\text{height} = \text{Area} \div \text{base} = 56 \div 8 = 7 \text{ m}$$

$$\text{Check: } 8 \times 7 = 56 \text{ sq m } \checkmark$$

(iii) The area and height are given, so divide again.

$$\text{base} = \text{Area} \div \text{height} = 221 \div 17 = 13$$

But the units clash — see the note.

Answer (i) 90 cm² (ii) 7 m (iii) 13 (units need correcting)

Please check — in (iii) the height is in millimetres but the area in square metres. A length in mm multiplied by a length in mm gives mm², not m², so the two figures cannot belong to the same shape as printed. The arithmetic $221 \div 17 = 13$ is right whatever unit is intended.

2. Find the area and perimeter of each parallelogram. (three figures are printed, with their measurements)

Two formulas are needed, and they use different measurements:

$$\text{Area} = \text{base} \times \text{perpendicular height}$$

$$\text{Perimeter} = 2 \times (\text{base} + \text{slanting side})$$

The perimeter uses the slanting side because that is the actual edge you walk along. The area uses the perpendicular height because that is the true distance between the two parallel sides. So a parallelogram drawing usually carries three numbers, and you must pick the right two for each formula.

$$(a) \text{ base } 7 \text{ cm, height } 4 \text{ cm, slant } 6 \text{ cm} \rightarrow \text{Area} = 7 \times 4 = 28 \text{ cm}^2; \text{ Perimeter} = 2 \times (7 + 6) = 26 \text{ cm.}$$

$$(b) \text{ base } 5 \text{ cm, height } 4 \text{ cm, slant } 3 \text{ cm} \rightarrow \text{Area} = 5 \times 4 = 20 \text{ cm}^2; \text{ Perimeter} = 2 \times (5 + 3) = 16 \text{ cm.}$$

$$(c) \text{ base } 3.5 \text{ cm, height } 2.5 \text{ cm, slant } 3 \text{ cm} \rightarrow \text{Area} = 3.5 \times 2.5 = 8.75 \text{ cm}^2; \text{ Perimeter} = 2 \times (3.5 + 3) = 13 \text{ cm.}$$

Answer Area = base × height; Perimeter = 2 × (base + slant side). Read the three measurements off each figure and use the right two for each formula.

Please check — the measurements are printed on the figures, so the readings above follow the labels as the scan shows them.

■ EXERCISE 8.3

1. Find the perimeter of the triangle whose sides are: (i) 3 m, 4 m, 5 m (ii) 11 m, 17 m, 19 m (iii) 20 cm, 20 cm, 15 cm (iv) 6 cm, 8 cm, 10 cm

The perimeter of any triangle is simply the sum of its three sides — there is no formula to remember beyond adding.

$$\text{Perimeter} = a + b + c$$

$$\text{(i) Perimeter} = 3 + 4 + 5$$

$$= 7 + 5 = 12 \text{ m}$$

$$\text{(ii) Perimeter} = 11 + 17 + 19$$

$$= 28 + 19 = 47 \text{ m}$$

$$\text{(iii) Perimeter} = 20 + 20 + 15$$

$$= 40 + 15 = 55 \text{ cm}$$

$$\text{(iv) Perimeter} = 6 + 8 + 10$$

$$= 14 + 10 = 24 \text{ cm}$$

Worth noticing: (i) 3, 4, 5 and (iv) 6, 8, 10 are both right-angled triangles, since $3^2 + 4^2 = 5^2$ and $6^2 + 8^2 = 10^2$. And (iii) has two equal sides, so it is isosceles.

Answer (i) 12 m (ii) 47 m (iii) 55 cm (iv) 24 cm

2. Find the area of a triangle having: (a) base 18 cm, height 12 cm (b) base 6.5 m, height 5 cm (c) base 7.2 m, height 7 dm (d) base 10.5 m, height 8 mm

Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height}$. The half is there because a triangle is exactly half of the rectangle that encloses it.

Before multiplying, the base and height must be in the same unit. Three of these four are given in mixed units, so each needs converting first.

(a) Both in cm already.

$$\text{Area} = \frac{1}{2} \times 18 \times 12$$

$$= \frac{1}{2} \times 216 = 108 \text{ cm}^2$$

(b) Convert 6.5 m to cm: $6.5 \times 100 = 650 \text{ cm}$.

$$\text{Area} = \frac{1}{2} \times 650 \times 5$$

$$= \frac{1}{2} \times 3,250 = 1,625 \text{ cm}^2$$

(c) Convert 7 dm to m: $1 \text{ dm} = 0.1 \text{ m}$, so $7 \text{ dm} = 0.7 \text{ m}$.

$$\text{Area} = \frac{1}{2} \times 7.2 \times 0.7$$

$$= \frac{1}{2} \times 5.04 = 2.52 \text{ m}^2$$

(d) Convert 8 mm to m: $1 \text{ mm} = 0.001 \text{ m}$, so $8 \text{ mm} = 0.008 \text{ m}$.

$$\text{Area} = \frac{1}{2} \times 10.5 \times 0.008$$

$$= \frac{1}{2} \times 0.084 = 0.042 \text{ m}^2$$

(d) describes a triangle 10.5 m along the base and 8 mm tall — a sliver, not a shape anyone would draw.

Answer (a) 108 cm^2 · (b) $1,625 \text{ cm}^2$ · (c) 2.52 m^2 · (d) 0.042 m^2

Please check — parts (b), (c) and (d) give the base and height in different units, which must be made to match before multiplying. In (d) the resulting triangle is 10.5 m wide and 8 mm high, which suggests a mistyped unit.

3. Find the height of a triangle whose: (a) area 420 cm^2 , base 60 cm (b) area 1500 mm^2 , base 7.5 cm (c) area 64 dm^2 , base 1.6 m

Start from $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$ and rearrange for the height.

Multiply both sides by 2: $2 \times \text{Area} = \text{base} \times \text{height}$.

Then divide by the base: $\text{height} = (2 \times \text{Area}) \div \text{base}$.

The step people forget is the $\times 2$ — dividing the area by the base alone gives half the right answer.

(a) Both already in cm.

$$\text{height} = (2 \times 420) \div 60$$

$$= 840 \div 60 = 14 \text{ cm}$$

(b) Convert the base: $7.5 \text{ cm} = 75 \text{ mm}$, to match the mm^2 area.

$$\begin{aligned}\text{height} &= (2 \times 1,500) \div 75 \\ &= 3,000 \div 75 = 40 \text{ mm}\end{aligned}$$

(c) Convert the base: $1.6 \text{ m} = 16 \text{ dm}$, to match the dm^2 area.

$$\begin{aligned}\text{height} &= (2 \times 64) \div 16 \\ &= 128 \div 16 = 8 \text{ dm}\end{aligned}$$

$$\text{Check (a): } \frac{1}{2} \times 60 \times 14 = \frac{1}{2} \times 840 = 420 \text{ cm}^2 \checkmark$$

Answer (a) 14 cm (b) 40 mm (c) 8 dm

4. Find the base of a triangle whose: (a) area 300 cm^2 , height 7.5 cm (b) area 3.6 dm^2 , height 90 cm (c) area 3.6 m^2 , height 1.8 m

Same rearrangement, the other way round: $\text{base} = (2 \times \text{Area}) \div \text{height}$.

(a) Both in cm.

$$\begin{aligned}\text{base} &= (2 \times 300) \div 7.5 \\ &= 600 \div 7.5 = 80 \text{ cm}\end{aligned}$$

(b) Convert the height: $90 \text{ cm} = 9 \text{ dm}$, to match the dm^2 area.

$$\begin{aligned}\text{base} &= (2 \times 3.6) \div 9 \\ &= 7.2 \div 9 = 0.8 \text{ dm}\end{aligned}$$

(c) Both in m.

$$\begin{aligned}\text{base} &= (2 \times 3.6) \div 1.8 \\ &= 7.2 \div 1.8 = 4 \text{ m}\end{aligned}$$

$$\text{Check (c): } \frac{1}{2} \times 4 \times 1.8 = \frac{1}{2} \times 7.2 = 3.6 \text{ m}^2 \checkmark$$

Answer (a) 80 cm (b) 0.8 dm (c) 4 m

5. Find the area of a triangle whose sides are 24 cm , 32 cm and 40 cm .

Only the three sides are given — no height. So first test whether it is right-angled, because that gives the height free.

Check with Pythagoras on the two shorter sides against the longest:

$$\begin{aligned}24^2 &= 576 \\ 32^2 &= 1,024 \\ 576 + 1,024 &= 1,600 \\ 40^2 &= 1,600 \checkmark\end{aligned}$$

They are equal, so the triangle is right-angled, with the right angle between the 24 cm and 32 cm sides.

In a right-angled triangle those two sides are the base and the height.

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 24 \times 32 \\ &= \frac{1}{2} \times 768 = 384 \text{ cm}^2\end{aligned}$$

Answer 384 cm^2

6. Find the area of a triangle whose sides are 10 cm and 9 cm and whose perimeter is 36 cm .

Step 1 — find the third side from the perimeter.

$$\text{third} = 36 - 10 - 9 = 17 \text{ cm}$$

Step 2 — check it is a possible triangle: the two shorter sides must beat the longest.

$$9 + 10 = 19, \text{ which is more than } 17 \checkmark$$

Step 3 — it is not right-angled ($9^2 + 10^2 = 81 + 100 = 181$, but $17^2 = 289$), so use Heron's formula.

$$s = \text{half the perimeter} = 36 \div 2 = 18$$

$$\begin{aligned}\text{Area} &= \sqrt{[s(s-a)(s-b)(s-c)]} \\ &= \sqrt{[18 \times (18-10) \times (18-9) \times (18-17)]} \\ &= \sqrt{[18 \times 8 \times 9 \times 1]} \\ &= \sqrt{1,296} \\ &= 36 \text{ cm}^2 \text{ (since } 36 \times 36 = 1,296)\end{aligned}$$

Answer 36 cm^2

7. Find the height of a triangle whose base is 50 cm and whose area is 500 cm².

$$\begin{aligned}\text{height} &= (2 \times \text{Area}) \div \text{base} \\ &= (2 \times 500) \div 50 \\ &= 1,000 \div 50 = 20 \text{ cm} \\ \text{Check: } \frac{1}{2} \times 50 \times 20 &= \frac{1}{2} \times 1,000 = 500 \text{ cm}^2 \checkmark\end{aligned}$$

Answer 20 cm

8. A triangle has a perimeter of 50 cm. If two sides are 15 cm and 19 cm, what is the third side?

The perimeter is the sum of all three sides, so subtract the two known ones.
 $15 + 19 = 34 \text{ cm}$
 $\text{third} = 50 - 34 = 16 \text{ cm}$
 Check it can exist: the two shortest are 15 and 16, and $15 + 16 = 31$, which is more than 19 ✓

Answer 16 cm

9. A field is a right triangle with sides in the ratio 5 : 12 : 13 and a perimeter of 900 units. Find its area.

Step 1 — find what one part of the ratio is worth.
 $\text{Total parts} = 5 + 12 + 13 = 30$
 $\text{One part} = 900 \div 30 = 30$
 Step 2 — find the three sides.
 $5 \text{ parts} = 5 \times 30 = 150$
 $12 \text{ parts} = 12 \times 30 = 360$
 $13 \text{ parts} = 13 \times 30 = 390$
 $\text{Check: } 150 + 360 + 390 = 900 \checkmark$
 Step 3 — confirm the right angle.
 $150^2 = 22,500$ and $360^2 = 129,600$
 $22,500 + 129,600 = 152,100$
 $390^2 = 152,100 \checkmark$
 Step 4 — the two shorter sides are the base and height.
 $\text{Area} = \frac{1}{2} \times 150 \times 360$
 $= \frac{1}{2} \times 54,000 = 27,000 \text{ square units}$

Answer 27,000 square units

EXERCISE 8.4

1. Find the surface area of each cube: (a) side 2 cm (b) side 5 cm (c) side 7 mm

A cube has six identical square faces, so:
 $\text{Surface area} = 6 \times \text{side} \times \text{side}$
 Work it in two steps — one face first, then six of them. Doing $6 \times \text{side}$ and then squaring gives a quite different (and wrong) answer.
 (a) Side = 2 cm.
 $\text{Area of one face} = 2 \times 2 = 4 \text{ cm}^2$
 $\text{Surface area} = 6 \times 4 = 24 \text{ cm}^2$
 (b) Side = 5 cm.
 $\text{Area of one face} = 5 \times 5 = 25 \text{ cm}^2$
 $\text{Surface area} = 6 \times 25 = 150 \text{ cm}^2$
 (c) Side = 7 mm.
 $\text{Area of one face} = 7 \times 7 = 49 \text{ mm}^2$
 $\text{Surface area} = 6 \times 49 = 294 \text{ mm}^2$
 Notice how fast this grows: the side from (a) to (b) goes up $2\frac{1}{2}$ times, but the surface area goes up more than six times, because the side is squared.

Answer (a) 24 cm² (b) 150 cm² (c) 294 mm²

2. Find the surface area of each cuboid. (six cuboids are printed with their measurements)

A cuboid has six rectangular faces in three matching pairs — top and bottom, front and back, the two ends.

$$\text{Surface area} = 2 \times (lw + lh + wh)$$

where l is the length, w the width and h the height.

Take it in four steps every time:

Step 1 — work out $l \times w$ (the top face)

Step 2 — work out $l \times h$ (the front face)

Step 3 — work out $w \times h$ (the end face)

Step 4 — add the three and double

Worked for (a), 5 cm by 4 cm by 2 cm:

$$lw = 5 \times 4 = 20$$

$$lh = 5 \times 2 = 10$$

$$wh = 4 \times 2 = 8$$

$$\text{Sum} = 20 + 10 + 8 = 38$$

$$\text{Surface area} = 2 \times 38 = 76 \text{ cm}^2$$

The remaining five carry their three measurements on the printed figures; apply the same four steps to each.

Answer Surface area = $2 \times (lw + lh + wh)$. For (a), $5 \times 4 \times 2$ cm: $2 \times (20 + 10 + 8) = 76 \text{ cm}^2$.

Please check — the measurements for the other cuboids are printed on the figures, so those need reading from the page. The method is identical for every one.





CHAPTER

3D Shapes and Symmetry

EXERCISE 9.1

1. Write the number of vertices, faces and edges of the given shapes. (four solids are printed in the book)

Three words to be clear about first:

A face is a flat or curved surface — a side of the solid.

An edge is the line where two faces meet.

A vertex is a corner — a point where edges meet. (The plural is vertices.)

A reliable way to count faces without missing any: name them as you go — top, bottom, front, back, left, right.

For edges, count the edges of one face, multiply by the number of faces, then halve — because every edge is shared by exactly two faces. For a cube: $4 \times 6 = 24$, and $24 \div 2 = 12$.

Cube — 6 faces, 12 edges, 8 vertices: 6 square faces — top, bottom and four sides. Each face has 4 edges, giving 24, but every edge is shared by exactly 2 faces, so $24 \div 2 = 12$. The corners are 4 on top and 4 beneath: 8.

Cuboid — 6 faces, 12 edges, 8 vertices: the same counts as a cube — only the faces are rectangles instead of squares. Shape of face does not change how many there are.

Cylinder — 3 faces, 2 edges, 0 vertices: 2 flat circular faces plus 1 curved surface = 3. The 2 edges are the circles where the curved surface meets each flat end. There are no corners at all.

Cone — 2 faces, 1 edges, 1 vertices: 1 flat circular base plus 1 curved surface = 2. The single edge is the circle round the base. The single vertex is the point at the top.

Sphere — 1 faces, 0 edges, 0 vertices: one continuous curved surface, with nothing to form an edge and no corner anywhere.

Square pyramid — 5 faces, 8 edges, 5 vertices: 1 square base plus 4 triangular faces = 5. The base has 4 edges and 4 more run up to the apex: 8. The 4 base corners plus the apex: 5.

A check worth knowing: for any solid with flat faces, Faces + Vertices – Edges = 2. For a cube: $6 + 8 - 12 = 2$ ✓ For a square pyramid: $5 + 5 - 8 = 2$ ✓ If your three counts do not satisfy it, one of them is wrong.

The rule does not apply to the cylinder, cone or sphere, because their surfaces are curved.

Answer Cube and cuboid: 6 faces, 12 edges, 8 vertices · Cylinder: 3 faces, 2 edges, 0 vertices · Cone: 2 faces, 1 edge, 1 vertex · Sphere: 1 face, 0 edges, 0 vertices · Square pyramid: 5 faces, 8 edges, 5 vertices

Please check — the four solids are printed as pictures, so the counts above cover the shapes this exercise normally uses. Match each picture to its name and read off the row.

2. Define these terms of 3-D shapes: (a) Vertex (b) Edge (c) Face

(a) A vertex is a point where two or more edges meet — a corner of the solid. A cube has 8; a sphere has none.

(b) An edge is the straight or curved line where two faces meet. A cube has 12; a sphere has none.

(c) A face is one of the surfaces that bounds the solid. It may be flat, as on a cube, or curved, as on a cylinder.

They build on one another: faces meet at edges, and edges meet at vertices.

Answer Vertex — a corner, where edges meet. Edge — the line where two faces meet. Face — a flat or curved surface of the solid.

3. Draw these 3-D shapes: (a) Sphere (b) Cone (c) Cylinder (d) Cube (e) Cuboid

Each is drawn the same way: show the front faces solid and the hidden edges as dashed lines, so the solid reads as three-dimensional on flat paper.

- (a) Sphere — a circle, with an ellipse drawn across the middle to suggest roundness.
- (b) Cone — an ellipse for the base, then two straight lines from its ends meeting at a point above.
- (c) Cylinder — two ellipses, one above the other, joined by two vertical lines. The far half of each ellipse is dashed.
- (d) Cube — draw one square, then a second square offset diagonally behind it, and join the matching corners. The three hidden edges are dashed.
- (e) Cuboid — exactly the same construction, but start from a rectangle instead of a square.

Answer Five drawings, each with the hidden edges shown dashed.

EXERCISE 9.2

1. Complete the following shapes according to the given mirror line or line of symmetry. (two part-drawn figures are printed)

A reflection flips the shape across the mirror line. Two things stay the same and one changes:

The size stays the same.

The shape stays the same.

The orientation reverses — left becomes right, or top becomes bottom.

Work one point at a time rather than trying to draw the whole image at once:

Step 1 — take a corner of the given half.

Step 2 — count the squares from that corner straight across to the mirror line, at a right angle to it.

Step 3 — count the same number of squares again on the far side and mark the point there.

Step 4 — repeat for every corner, then join the new points in the same order as the original.

The key is that each point and its image are the same distance from the mirror line, measured perpendicular to it. A point sitting on the line does not move at all.

Answer Each corner is reflected to the same perpendicular distance on the other side of the mirror line, then the points are joined in order.

Please check — the part-drawn figures are printed on grid paper in the book.

2. Draw the line of symmetry for each of the following shapes on grid paper. (three shapes are printed)

A line of symmetry is a line that cuts the shape so that one half is the mirror image of the other.

Fold along it and the two halves land exactly on top of one another.

To find one, look for a fold that would make the two sides match corner for corner.

How many there are depends on the shape:

A square has 4 — two through the middles of opposite sides, two along the diagonals.

A rectangle has only 2 — through the middles of opposite sides. Its diagonals are not lines of symmetry, which surprises people.

An equilateral triangle has 3, an isosceles triangle has 1, and a scalene triangle has none.

A circle has infinitely many — every line through the centre.

Check any line you draw by testing a few corners: each should have a matching corner the same distance away on the other side.

Answer A square has 4 lines of symmetry, a rectangle 2, an equilateral triangle 3, an isosceles triangle 1, and a circle infinitely many.

Please check — the three shapes are printed on grid paper in the book.



10

CHAPTER

Practical Geometry

EXERCISE 10.1

1. Identify lines, line segments and rays from the figures and write them in notation form. (eight figures are printed)

The three are told apart by their ends:

A line has no ends — it goes on for ever both ways. Arrowheads at both ends. Written AB with a double-headed arrow above.

A line segment has two ends — a definite length you could measure. Written AB with a plain bar above.

A ray has one end and goes on for ever the other way. One arrowhead. Written AB with a single arrow above, and the order matters: the fixed end is named first.

So to identify each figure, count its arrowheads:

two arrowheads → a line

one arrowhead → a ray

no arrowheads → a line segment

A note on naming: for a line or a segment, AB and BA mean the same thing. For a ray they do not — ray AB starts at A, ray BA starts at B, and they point opposite ways.

Answer Two arrowheads = line, one arrowhead = ray, none = line segment. For rays the order of the letters matters; for lines and segments it does not.

Please check — the eight figures are printed in the book, so each must be matched by counting its arrowheads.

2. Identify the parallel, perpendicular, transversal and intersecting lines. (six figures are printed)

Parallel lines run in the same direction and never meet, however far they are extended. They stay the same distance apart. Marked with matching arrowheads on each line; written $AB \parallel CD$.

Intersecting lines cross at a point. Any two lines that are not parallel must intersect somewhere.

Perpendicular lines are a special case of intersecting: they cross at exactly 90° . Marked with a small square at the corner; written $AB \perp CD$.

Transversal — a line that cuts across two or more other lines. It is usually drawn crossing a pair of parallel lines, and the angles it makes are what the next topic is about.

So the order to check each figure:

Do the lines meet? If never → parallel.

If they meet at a right angle → perpendicular.

If they meet at any other angle → simply intersecting.

If one line crosses two others → that one is a transversal.

Note that perpendicular lines are also intersecting lines — the categories overlap.

Answer Parallel: never meet. Intersecting: cross at a point. Perpendicular: cross at 90° . Transversal: crosses two or more other lines.

Please check — the six figures are printed in the book.

EXERCISE 10.2

1. Draw a perpendicular bisector on a line segment 4.8 cm long.

'Perpendicular bisector' does two things at once: it cuts the segment exactly in half, and it crosses it at 90° .

Step 1 — draw $AB = 4.8$ cm with a ruler.

Step 2 — open the compass to more than half of AB . More than 2.4 cm will do; about 3 cm is comfortable. This matters: if the radius were less than half, the two arcs would never meet.

Step 3 — put the compass point on A and draw an arc above the line and another below it.

Step 4 — without changing the compass opening, put the point on B and draw two more arcs, crossing the first two.

Step 5 — join the two crossing points with a ruler. That line is the perpendicular bisector.

Why it works: both crossing points are the same distance from A as from B . Every point equally distant from A and B lies on the perpendicular bisector, so two such points fix it.

Check: the bisector should cut AB at 2.4 cm from each end, and a protractor should read 90° .

Answer A line through the two arc crossings, cutting AB at 90° and 2.4 cm from each end.

2. Draw a perpendicular bisector on a line segment 5 cm long.

Exactly the same construction, with different numbers.

Step 1 — draw the segment 5 cm long.

Step 2 — open the compass to more than half of 5 cm, that is more than 2.5 cm. About 3.5 cm is convenient.

Step 3 — arcs above and below from one end.

Step 4 — the same radius, arcs from the other end.

Step 5 — join the crossings.

Check: it should meet the segment at 2.5 cm from each end, at a right angle.

Answer A line cutting the segment at 90° , 2.5 cm from each end.

3. Draw a perpendicular LM on AB at any point on AB .

Here the perpendicular must pass through a given point on the line, so it is not a bisector — the point need not be the middle.

Step 1 — draw AB and mark the point, call it L .

Step 2 — with the compass point on L and any convenient radius, draw arcs cutting AB on both sides of L . Call these P and Q . Now L is the midpoint of PQ .

Step 3 — open the compass wider, put the point on P and draw an arc above the line.

Step 4 — same radius, point on Q , draw an arc crossing it. Call the crossing M .

Step 5 — join LM . That is the perpendicular.

Why it works: step 2 turns the problem into bisecting PQ , and the perpendicular bisector of PQ passes through its midpoint — which is L .

Answer $LM \perp AB$, passing through the chosen point L .

4. Draw a perpendicular from a point on each given line. (four figures are printed, each with a marked point)

Each is the same construction as question 3, with the point already marked on the line.

Step 1 — compass on the marked point, arcs cutting the line on both sides.

Step 2 — wider compass, arcs from each of those two crossings, meeting above the line.

Step 3 — join the marked point to that meeting point.

The position of the point along the line makes no difference to the method — only to where the perpendicular ends up.

Answer A perpendicular through each marked point, built by the same three steps.

Please check — the four figures with their marked points are printed in the book.

5. Draw a line MN. Take a point O outside it and draw a perpendicular to MN from O.

This time the point is off the line, which changes the first step.

Step 1 — draw MN, and mark O somewhere above or below it.

Step 2 — put the compass point on O and open it wide enough to reach past the line. Draw an arc cutting MN at two points; call them P and Q.

Step 3 — put the compass on P, open it to more than half of PQ, and draw an arc on the far side of MN from O.

Step 4 — same radius, compass on Q, draw an arc crossing it. Call the crossing R.

Step 5 — join O to R with a ruler. The line OR cuts MN at a right angle.

Why it works: O is the same distance from P as from Q (step 2 made it so), and R is too (steps 3 and 4). Two points equally distant from P and Q determine the perpendicular bisector of PQ — and a perpendicular bisector meets the line at 90° .

Answer $OR \perp MN$, where R is the crossing of the two far-side arcs.

EXERCISE 10.3

1. Construct the following angles: (i) 90° (ii) 60° (iii) 105° (iv) 75° (v) 120°

Two angles come straight from the compass, and the rest are built from those two.

(ii) 60° — the foundation.

Draw a ray OA. With O as centre and any radius, draw an arc cutting OA at P.

With the same radius and P as centre, draw an arc cutting the first at Q.

Join OQ. Angle AOQ = 60° .

Why: OP, PQ and OQ are all the same radius, so triangle OPQ is equilateral — and every angle of an equilateral triangle is 60° .

(v) 120° — two steps of the same radius.

Carry on from the 60° construction: step the same radius once more along the arc, from Q to R.

Angle AOR = $60^\circ + 60^\circ = 120^\circ$.

(i) 90° — bisect the 60° to 120° gap.

Mark both Q (60°) and R (120°) as above.

From Q and R in turn, with equal radii, draw arcs meeting at S.

Join OS. Angle AOS = $60^\circ + (120^\circ - 60^\circ) \div 2 = 60^\circ + 30^\circ = 90^\circ$.

(iv) 75° — halfway between 60° and 90° .

Construct 60° and 90° from the same ray, then bisect the 30° between them.

$60^\circ + 15^\circ = 75^\circ$.

(iii) 105° — halfway between 90° and 120° .

Construct 90° and 120° , then bisect the 30° gap.

$90^\circ + 15^\circ = 105^\circ$.

Every angle in this list is 60° or 90° with halvings. That is the whole toolkit — which is also why 20° cannot be constructed this way at all.

Answer (i) 90° by bisecting 60° – 120° · (ii) 60° from one step of the radius · (iii) $105^\circ = 90^\circ + 15^\circ$ · (iv) $75^\circ = 60^\circ + 15^\circ$ · (v) 120° from two steps of the radius

2. Measure the following angles with a protractor. (four angles are printed)

Step 1 — place the protractor's centre mark exactly on the vertex of the angle.

Step 2 — line the protractor's base line up along one arm of the angle.

Step 3 — read where the other arm crosses the scale.

Step 4 — choose the right scale. A protractor carries two sets of numbers running opposite ways. Use the one that starts at 0 on the arm you lined up.

A sanity check before writing the answer: if the angle looks sharper than a square corner it must be less than 90° , and if it looks wider it must be more. Reading the wrong scale turns 40° into 140° , and that check catches it every time.

If an arm is too short to reach the scale, extend it with a ruler first — lengthening an arm does not change the angle.

Answer Each angle read from the scale that starts at 0° on the arm laid along the base line.

Please check — the four angles are printed in the book, so the readings must be taken from the page.

3. Construct an angle of 45° by bisecting a 90° angle.

Step 1 — construct a right angle, as in question 1(i): 60° , then 120° , then bisect between them to reach 90° . Call it angle AOB.

Step 2 — with O as centre and any radius, draw an arc cutting both arms of the right angle. Call the crossings X and Y.

Step 3 — with X as centre and a radius more than half XY, draw an arc inside the angle.

Step 4 — with the same radius and Y as centre, draw an arc crossing it. Call the crossing Z.

Step 5 — join OZ. It bisects angle AOB.

Each half = $90^\circ \div 2 = 45^\circ$.

Check with a protractor: both halves should read 45° , and together they must come back to 90° .

Bisecting again would give $22\frac{1}{2}^\circ$, and once more $11\frac{1}{4}^\circ$ — each bisection halves what came before.

Answer 45° — half of the constructed right angle



11

CHAPTER

Data Management

EXERCISE 11.1

1. Which of the following collections shows grouped data and which ungrouped? (i) 25, 15, 11, 11, 15, 18, 20, 22, 11, 9, 8, 7, 18 (ii) a table of groups 1–4, 5–9, 10–14, ... with numbers of students (iii) a table of groups 10–12, 13–15, 16–18 with numbers of chairs (iv) a table of groups 5–6, 7–8, 9–10, 11–12 with numbers of books

Ungrouped data is the raw list, exactly as it was collected — every single reading written out, in no particular order.

Grouped data has been sorted into bands, with a count against each band. The individual readings can no longer be seen.

(i) is a bare list of thirteen numbers with no bands, so it is ungrouped.

(ii), (iii) and (iv) each show bands with a frequency against each, so all three are grouped.

Note the book letters its four parts i, ii, ii, iii, iv — 'ii' appears twice. See the note.

Answer (i) ungrouped · (ii), (iii) and (iv) grouped

Please check — the book labels two different tables as 'ii'. They are lettered i to iv above.

2. The weights (in kg) of 25 students are given below. Convert this ungrouped data into grouped data.
15, 39, 40, 30, 32, 10, 16, 25, 26, 33, 35, 46, 48, 49, 22, 23, 18, 15, 14, 11, 18, 14, 23, 29, 30.

Step 1 — find the smallest and largest readings, so the bands can be made to cover everything.

Smallest = 10 kg, largest = 49 kg.

Range = $49 - 10 = 39$ kg.

Step 2 — choose bands of equal width that cover that range. Bands of 10 kg give four of them:
10–19, 20–29, 30–39, 40–49.

Step 3 — go through the list once, putting a mark against the band each reading falls in. Working through the list once is what makes a tally reliable; searching the list again for each band is where miscounts come from.

10–19: 9 students

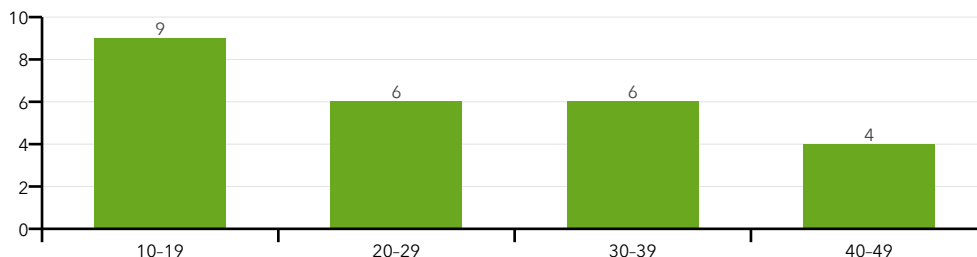
20–29: 6 students

30–39: 6 students

40–49: 4 students

Step 4 — check. The frequencies add to 25, which matches the 25 students given. Nothing has been missed or counted twice.

No. of students



Weight (kg)

Weight (kg)	Frequency
10 – 19	9
20 – 29	6
30 – 39	6
40 – 49	4
Total	25

3. The heights (in cm) of 18 students are given below. Convert this ungrouped data into grouped data.
109, 110, 115, 123, 102, 127, 119, 118, 113, 107, 103, 120, 121, 124, 129, 113, 121, 109.

Step 1 — find the extremes.

Smallest = 102 cm, largest = 129 cm.

Range = $129 - 102 = 27$ cm.

Step 2 — bands of 10 cm cover that range in three: 100–109, 110–119, 120–129.

Step 3 — tally through the list once.

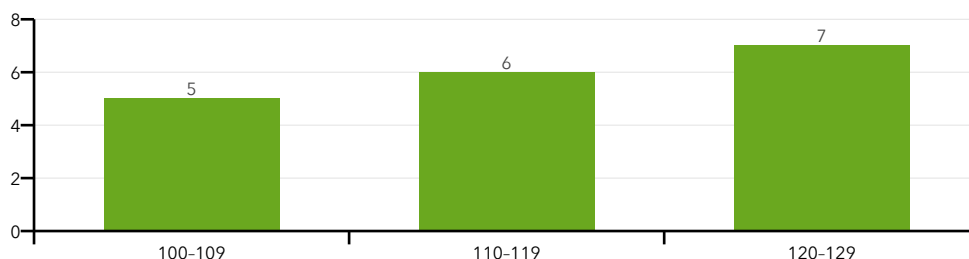
100–109: 5 students

110–119: 6 students

120–129: 7 students

Step 4 — check: $5 + 6 + 7 = 18$, matching the 18 students ✓

No. of students



Height (cm)

Answer	Height (cm)	Frequency
	100 – 109	5
	110 – 119	6
	120 – 129	7
	Total	18

4. Ali recorded the shirt sizes of students in his school. Convert his findings into grouped form and answer: (i) how many have sizes 4 and 5? (ii) how many have size 8 or more? (iii) how many have size less than 6? (iv) how many students did Ali talk to?

4, 5, 7, 8, 5, 6, 7, 8, 9, 7, 9, 10, 7, 8, 6, 5, 4, 7, 8, 9, 6, 7, 8, 9, 5, 4, 7, 6, 8, 9, 10, 9, 8, 7, 5, 6, 4, 7, 5, 4.

Here the readings only take seven different values, so each size can be its own group — no banding is needed.

Step 1 — tally each size through the list once.

Size 4: 5 students

Size 5: 6 students

Size 6: 5 students

Size 7: 9 students

Size 8: 7 students

Size 9: 6 students

Size 10: 2 students

Step 2 — check the total: $5 + 6 + 5 + 9 + 7 + 6 + 2 = 40$ ✓

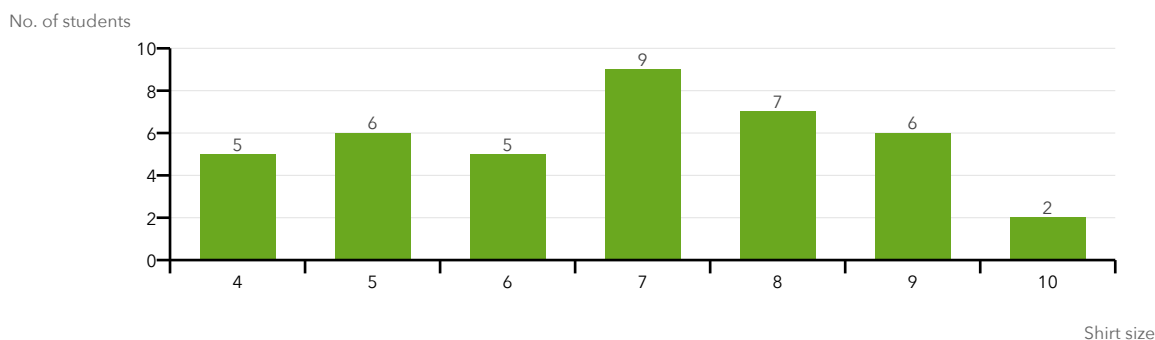
(i) Sizes 4 and 5 together: $5 + 6 = 11$ students.

(ii) 'Size 8 or more' means 8, 9 and 10 — the word equal or more includes 8 itself: $7 + 6 + 2 = 15$.

(iii) 'Less than 6' means 4 and 5 only, not 6: $5 + 6 = 11$.

(iv) The total number of students is the sum of every frequency: 40.

Notice (i) and (iii) give the same answer here, because 'sizes 4 and 5' and 'less than 6' describe the same group.



Answer (i) 11 · (ii) 15 · (iii) 11 · (iv) 40

EXERCISE 11.2

1. Fill in the blanks. (i) Heights (cm) of 6 plants are 100, 115, 120, 135, 145, 111. The mean height is ____ (ii) Ages of 5 students are 9, 12, 10, 9, 11. The mode is ____ (iii) Mean of 3, 5, 7, 9 is ____ (iv) Median of 13, 15, 17, 14, 10 is ____ (v) Mode of 1, 2, 2, 6, 8, 2, 9 is ____

Three different averages are being asked for, and each is found a different way:

Mean = add everything, divide by how many there are.

Median = put them in order, take the middle one.

Mode = the value that appears most often.

- (i) Add the six heights:

$$100 + 115 + 120 + 135 + 145 + 111 = 726$$

$$\text{Mean} = 726 \div 6 = 121 \text{ cm}$$

- (ii) Count how often each age appears: 9 twice, 10 once, 11 once, 12 once.

The most frequent is 9.

- (iii) Mean = $(3 + 5 + 7 + 9) \div 4 = 24 \div 4 = 6$

- (iv) Put them in order first — this is the step people skip:

10, 13, 14, 15, 17

There are 5 values, so the middle one is the 3rd: 14

- (v) Count the appearances: 1 once, 2 three times, 6 once, 8 once, 9 once.

Mode = 2

Answer (i) 121 cm · (ii) 9 · (iii) 6 · (iv) 14 · (v) 2

2. State whether each is true or false: (i) Mode is the sum of all the values in a set of data (ii) Median is the sum of the greatest and the least value (iii) If the number of values is even, its median can be easily found (iv) Mode cannot be more than 1 (v) Median of 13, 10, 16, 25, 30 is 10

(i) False. The mode is the most frequent value, not a sum of anything. Adding everything up is the first step towards the mean, not the mode.

(ii) False. The median is the middle value once the data is in order. Adding the greatest and least would give something quite different.

(iii) True. With an even number of values there is no single middle one, so you take the two middle values and average them — still straightforward. For 4, 6, 8, 10 the median is $(6 + 8) \div 2 = 7$.

(iv) False. A set can have two or more modes if several values tie for most frequent. In 2, 2, 5, 5, 7 both 2 and 5 are modes.

(v) False. Sort them first: 10, 13, 16, 25, 30. There are 5 values, so the median is the 3rd: 16, not 10. 10 is the smallest value.

Answer (i) False · (ii) False · (iii) True · (iv) False · (v) False





CHAPTER

Probability

EXERCISE 12.1

1. A vessel contains 4 blue balls, 5 red balls and 11 white balls. Three balls are drawn at random. What is the probability that the first is red, the second blue and the third white?

Step 1 — count the balls altogether.

$$4 \text{ blue} + 5 \text{ red} + 11 \text{ white} = 20 \text{ balls}$$

Step 2 — the first draw. There are 5 red balls out of 20.

$$P(\text{first is red}) = 5/20 = 1/4$$

Step 3 — the second draw. One ball has already gone and has not been put back, so only 19 remain. The 4 blue balls are all still there, because the one removed was red.

$$P(\text{second is blue}) = 4/19$$

Step 4 — the third draw. Two balls have gone, leaving 18. All 11 white balls remain.

$$P(\text{third is white}) = 11/18$$

Step 5 — for all three to happen in that order, multiply the three probabilities.

$$\begin{aligned} P &= 5/20 \times 4/19 \times 11/18 \\ &= (5 \times 4 \times 11) \div (20 \times 19 \times 18) \\ &= 220 \div 6,840 \\ &= 11/342 \end{aligned}$$

The shrinking totals — 20, then 19, then 18 — are the whole point. If the balls were replaced each time the totals would stay at 20.

Answer 11/342

2. Two dice are rolled. Find the probability that the sum is (i) equal to 1 (ii) equal to 4 (iii) less than 13

Step 1 — find the sample space. Each die can show 6 faces, and the two are independent.

$$\text{Total outcomes} = 6 \times 6 = 36$$

- (i) The smallest possible sum is $1 + 1 = 2$, so a sum of 1 can never happen.

$$\text{Favourable outcomes} = 0$$

$$P = 0 \div 36 = 0$$

A probability of 0 means an impossible event.

- (ii) List the ways to make 4: (1,3), (2,2), (3,1).

That is 3 outcomes.

$$P = 3 \div 36 = 1/12$$

- (iii) The largest possible sum is $6 + 6 = 12$, and 12 is less than 13 — so every outcome qualifies.

$$\text{Favourable outcomes} = 36$$

$$P = 36 \div 36 = 1$$

A probability of 1 means a certain event.

These three together show the whole range: 0 for impossible, something in between, and 1 for certain. No probability can ever lie outside 0 to 1.

Answer (i) 0 (ii) 1/12 (iii) 1

3. Find the probability of getting a number less than 5 when a dice is rolled.

Step 1 — the sample space. A dice has 6 faces: {1, 2, 3, 4, 5, 6}, so there are 6 outcomes.

Step 2 — the favourable outcomes. 'Less than 5' means 1, 2, 3 and 4 — it does not include 5.

That is 4 outcomes.

Step 3 — apply the formula.

$$P = \text{favourable} \div \text{total} = 4 \div 6$$

$$= 2/3 \text{ (dividing both by 2)}$$

Including the 5 by mistake would give 5/6 — the commonest slip, and why 'less than' is worth reading twice.

Answer 2/3

4. A bag contains 50 counters numbered 1 to 50. One is drawn at random. What is the probability that it has (i) an even number (ii) a 2-digit number (iii) a multiple of 5 (iv) a perfect square?

The total number of outcomes is 50 every time — only the favourable count changes.

- (i) The even numbers from 1 to 50 are 2, 4, 6, ... 50.

That is $50 \div 2 = 25$ numbers.

$$P = 25/50 = 1/2$$

- (ii) The 2-digit numbers run from 10 to 50.

Count them: $50 - 10 + 1 = 41$ numbers. (The '+ 1' is needed because both ends count.)

$$P = 41/50$$

The other 9 — the numbers 1 to 9 — are single-digit.

- (iii) The multiples of 5 are 5, 10, 15, ... 50.

That is $50 \div 5 = 10$ numbers.

$$P = 10/50 = 1/5$$

- (iv) A perfect square is a whole number times itself. List them up to 50:

$$1 \times 1 = 1, 2 \times 2 = 4, 3 \times 3 = 9, 4 \times 4 = 16, 5 \times 5 = 25, 6 \times 6 = 36, 7 \times 7 = 49.$$

$8 \times 8 = 64$, which is past 50, so stop.

That is 7 numbers: 1, 4, 9, 16, 25, 36, 49.

$$P = 7/50$$

Answer (i) 1/2 · (ii) 41/50 · (iii) 1/5 · (iv) 7/50

REVIEW EXERCISE

- i. What is the probability of an event that is certain to happen?

- a) 0 b) 0.5 c) 1 d) 0.75

Probability runs from 0 to 1 and never outside that range.

0 means the event cannot happen at all.

1 means it is bound to happen — every outcome in the sample space is a favourable one, so the fraction is 36/36, or 6/6, or whatever it may be — always equal to 1.

Example: rolling a number less than 7 on an ordinary dice.

Answer c) 1

- ii. If a coin is tossed, how many outcomes are in the sample space?

- a) 1 b) 2 c) 3 d) 4

The sample space is the list of everything that could happen.

A coin can land heads or tails: {H, T}.

That is 2 outcomes.

(Landing on its edge is not counted in school probability.)

Answer b) 2

iii. An event that cannot happen has a probability of:

- a) 1 b) 0 c) 0.5 d) 0.75

If no outcome is favourable, the favourable count is 0.

$$P = 0 \div \text{total} = 0.$$

Question 2(i) above is exactly this case: a sum of 1 on two dice is impossible, so its probability is 0.

Answer b) 0

iv. Which of the following is an example of a single event?

- a) Tossing two coins b) Rolling a die once c) Drawing two cards from a deck d) Selecting two marbles from a bag

A single event involves one action with one outcome.

Tossing two coins, drawing two cards and taking two marbles each involve two things happening, so they are compound events.

Rolling a die once is the only one with a single action.

Answer b) Rolling a die once

v. What is the sample space when rolling a six-sided die?

- a) {1, 2, 3} b) {1, 2, 3, 4, 5, 6} c) {Even, Odd} d) {1, 3, 5, 7, 9}

The sample space lists every possible outcome, not a selection of them.

A six-sided die can show any of 1, 2, 3, 4, 5 or 6.

{1, 2, 3, 4, 5, 6} — six outcomes.

{Even, Odd} describes two events, not the outcomes themselves, and {1, 3, 5, 7, 9} contains 7 and 9, which are not on a die at all.

Answer b) {1, 2, 3, 4, 5, 6}